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MECHANICS OF MACHINERY

MECHANISM

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MECHANICS OF MACHINERY

MECHANISM

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PREFACE

The purpose of this book is to cover the whole field of motions and forces in machines, as described in Sect. 1 (*g*), page 6. It aims to put this division of engineering knowledge into more effective shape for study and teaching. Combination into a related and consistent whole is offered in place of scattered and isolated branches.

The subject is too big to be confined within a single volume, therefore Part I, predominantly on Mechanism, will be followed by a second volume dealing with Kinematics and Dynamics of Machines. To get a quick glance at what Part I covers read the chapter introductions on pages 60, 97, 248, 348 and 448, also the table of contents for Chap. IV and V.

A first controlling idea has been to arrange and divide material according to the kind of problems presented in mechanism or mechanics, rather than to relate it to different kinds of machines. Using the term broadly, kinematics is the first and larger portion of the subject and underlies the whole of it. Based upon kinematical considerations, the main division is into machinery for uniform motion, for intermittent motion and for cyclical or reciprocating motion.

The first, introductory chapter contains the initial elements of nearly every branch of the subject. Uniform motion is presented in a general way by Chap. II, carried through specialized lines of gear trains in Chap. IV and V, and nearly completed by the study of gear teeth in Chap. VII and VIII.

Mechanisms for intermittent motion are sufficiently represented by the examples in Chap. III. Simple linkages and their movements are given in Chap. VI, with a mere beginning of the subject of velocity.

The kinematics of variable motion go forward into Part II, including all determination of velocities and accelerations, also the study of force actions both transmissive and kinetic.

The textbook viewpoint predominates, although the ground is covered with a degree of completeness that makes the book a fairly complete treatise. The experienced reader will find several

matters that have not had previous adequate presentation in elementary and systematic form, notably the geometric-series gear train, the quick-change screw-cutting train with its binary-series element, the flexible involute system and the hobbing train for helical teeth.

The embodiment of two educational ideas, at first sight conflicting but really complementary, may make the selection of material seem erratic in amount. One idea is to use a series of different typical examples; generally prevalent, this scheme is particularly exemplified by the series of gear trains in Sect. 10 and of stop and reverse mechanisms in Sect. 14. The other idea is to take some simple principle or purpose and show a large number of ways in which it is applied or realized. Notable examples of this much fuller presentation are, the clutches in Sect. 13 and the speed-changing gear trains in Chap. IV.

At the beginning of each division of the subject, where the student is encountering new ideas and concepts, the treatment is full and detailed; then as the discussion advances it becomes more concise, with less of verbal statement and explanation to supplement drawings, diagrams and sample computations.

The arrangement is such as to facilitate selection for courses of less or greater length; and judicious choice as to what shall be closely studied and what hastily run over for breadth of view will be an important function of the teacher.

A very useful educational feature is the large provision of examples and problems, both computational and graphical. The latter type will more largely prevail in the kinematics and graphics of Part II.

Robert C. H. Heck.

New Brunswick, N. J.
July, 1923.

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MECHANICS OF MACHINERY
MECHANISM

CHAPTER I

ELEMENTS AND PRINCIPLES OF MACHINES

SECTION 1. GENERAL IDEAS AS TO MACHINES

A novice in the subject may find that some parts of this introductory section are difficult reading, because so many new ideas are casually introduced. By no means should he be asked to recite upon it as a first lesson; rather, the teacher can well use it for an opening talk, giving a general view of the field which may be inspiring even though many of the details are hazy. Close reading will come better in review, whether of the book or of the reader's experience.

(*a*) **A Machine** is a combination of rigid or resistant bodies, so formed and connected that they can move upon each other but only in a definite way.

If the final clause of this definition were changed to, "*so formed and connected that they will not move upon each other,*" it would describe a **structure.** Definite **relative motion** of its members, **constrained** by form and contact, is an essential characteristic of the **machine.**

The word *rigid* implies fixity of form under the impression of forces. In adding *resistant* as an alternative term we recognize the fact that rigidity is never perfect or absolute; and also cover machine members (such as belts and chains) that are of fixed form in but one direction.

(*b*) **Machine Motion** is commonly accompanied by force actions greater than would be needed simply to keep the parts moving upon each other. The general scheme of operation of machines is as follows:

1. Upon a primary moving member is received a **driving force,** the combination of force and motion constituting an **input** of work or power.

2. This work or power is **transmitted** through the machine, with concordant changes in its force and motion factors, some work being lost in the overcoming of internal (frictional) resistances.

3. As **output** the final member exerts a **working force** in definite motion, against an external resistance and to the accomplishment of a useful effect.

If the force action is no greater than is needed to produce relative movement—in other words, if change of position of members is the useful effect and there is no performance of external work—then the machine is likely to be called an **instrument.**

Under the last statement (but perhaps after reading several of the articles that follow) consider and discuss the self-aligning telescope, the clock, a system of interlocking railroad switches

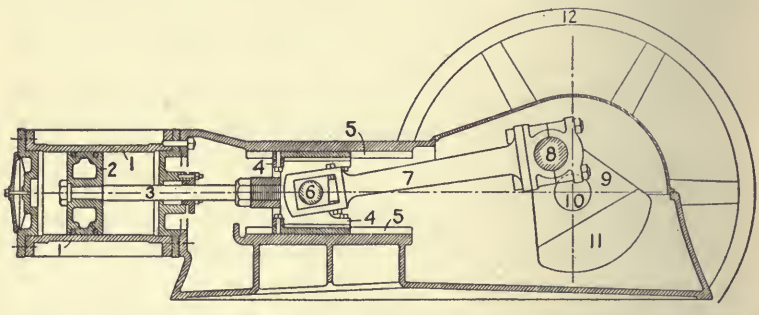

FIG. 1.—Sectional drawing of a steam engine. (1) Cylinder. (2) Piston. (3) Piston rod. (4) Crosshead. (5) Guides for crosshead. (6) Wrist pin. (7) Connecting rod. (8) Crank pin. (9) Crank. (10) Shaft. (11) Counterweight. (12) Flywheel.

and signals, the adding machine, the typewriter, the printing press and the weaving loom. How far is the distinction just suggested practical and satisfactory? How would you classify each example?

(*c*) **Machine Examples.**—Consider the steam engine, shown in section by Fig. 1. The driving pressure of steam acts upon piston 2 and is concentrated into a strong force along rod 3 to crosshead 4. By cylinder walls 1, 1 and guide surfaces 5, 5 piston-slide member 2-3-4 is constrained to straight-line movement; through the action of connecting rod 7 and crank 9 that movement is positively limited in range; and by the proper admission and discharge of steam these three members are given a continuous motion which is controlled in marked degree by the inertia-

force action of flywheel 12. In this machine back-and-forth or reciprocating motion that is convenient for receiving steam pressure is converted to rotary motion that is useful for driving machinery. The connecting rod, with the center of wrist pin 6 constrained to a rectilinear path and the center of crank pin 8 to a circular path, has motion of a somewhat complex type.

In Fig. 2 is outlined the hoist gearing of a traveling crane, by top view (*a*) and side view (*b*); it begins at electric motor M and ends with rope drum H. The driving effort is distributed around the armature of the motor, producing the pure turning moment to which the name **torque** is given. Through the **gear train** composed of three pairs A-B, C-D, E-F of toothed

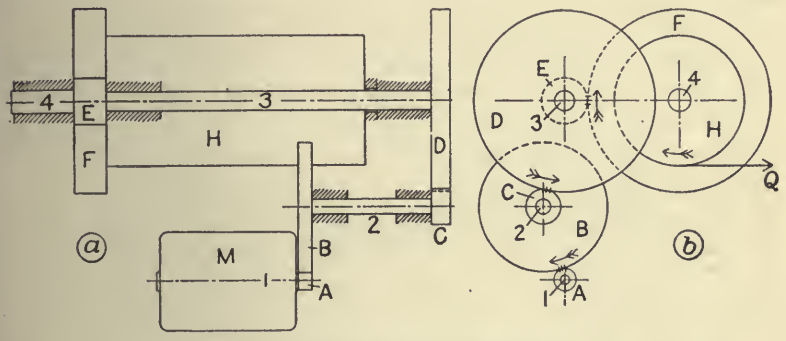

FIG. 2.—Outline of crane hoist gearing.

wheels, with intermediate shafts 2 and 3, there is a progressive change from small force at high speed to large force at low speed. As regards kind of motion conditions are much simpler than in the engine: when fully established it is uniform rotation for all members concerned; and in starting and stopping all receive angular acceleration or retardation of the same kind.

(*d*) **Types of Machinery.**—The first of these examples represents the class of **prime movers** or power developers, whose function is to receive a natural driving force and transmit or convert it without completing the useful application. At the input end of such machines the types of action are few and simple: beyond the limited muscular effort of men or animals, the driving force is either the static pressure of a confined fluid, the kinetic pressure of a stream of fluid, or electromagnetic attraction and repulsion; and the machine motions associated with the whole force action are simple. The electric motor in the second exam-

ple is what may well be called a secondary prime mover, since its driving force results from a preliminary machine action—except in the minor case of battery current.

Sometimes the whole operation of receiving and applying work is performed within one compact machine: the locomotive, with input at the engine pistons and output at the drawbar, is an excellent example. Again, the prime mover and working machine, while distinct or distinguishable, are closely combined or coupled together: besides Fig. 2 various examples are, the steam-driven pump or air compressor, the engine or turbine directly connected to an electric generator, the engine combined with a mine hoist or coupled to a rolling mill, and the electric motor applied to a single machine tool.

Very often the **working machine** is not only distinct but distant from the prime mover, being driven (commonly in multiple) through a system of **transmission machinery**: typically this is true of a factory plant where power from the engine is transmitted by belts, pulleys and shafting to the working units. The use of electrical transmission breaks such a system into separate parts, with conversion of power to and from a nonmechanical form.

The word *work* is here used in two senses: the first is the strictly physical or mechanical action of overcoming resistance through distance, the second the accomplishing of a useful effect in moving, shaping or fabricating materials. At the place of output in such operations conditions vary widely, ranging from big resistance and simple motion to highly complex motion generally against light forces.

(*e*) **Machine Representation.**—So far as may be done in a small drawing, Fig. 1 shows the exact form of the machine and of its parts with a fair amount of detail. Back of such an "assembly" lies a set of detailed shop drawings, which show the full design of the machine and guide and control its construction. Into the design, besides the general matters of strength and facility of construction, have entered considerations of adaptation to steam action (including the prevention of leakage), accessibility and ease of adjustment, lubrication, cleanliness and safety in operation.

In Fig. 2 we pass from this fuller representation to a mere outline that can show the form and carry the dimensions needed for a study of motions and force actions. Bearings are suggested in view (*a*), but in (*b*) the fixity of a shaft axis is taken to be

sufficiently indicated by crossed center lines. Toothed wheels or gears are represented by their pitch cylinders—see Sect. 4 (*e*). Where these appear as circles, view (*b*), the convention is introduced of indicating teeth "in mesh" by three little cross lines at the place of tangency. For some purposes a bare skeleton is sufficient, as seen in Figs. 92–94, 101, etc., showing merely the essential geometrical form of the machine members.

(*f*) **Machine Action.**—To gather up and carry forward the ideas suggested in the last two articles, consider Figs. 3 to 5, which form a sequence with Fig. 1. As to manner of representation Fig. 3 parallels Fig. 2, in that it shows no more than the form outline of the engine. Upon this outline is now diagrammed

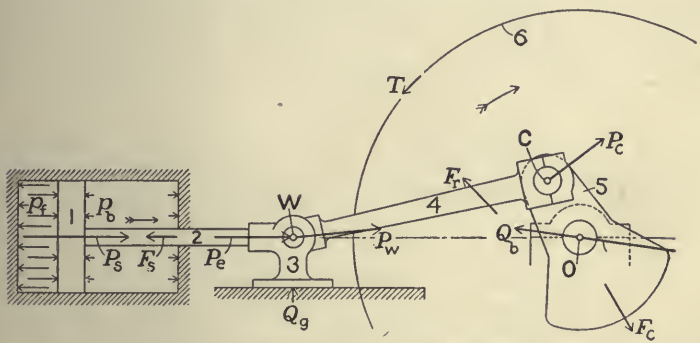

FIG. 3.—Outline of the engine, with representation of force action.

the force action described in Art. (*c*), with elaboration. Understanding that this is wholly an illustrative example, showing results far in anticipation of the methods of getting them, it suggests the kind of problems that will arise when the subject of force action is developed in Part II.

The driving steam force P_s is seen to be the concentrated resultant of forward pressure p_f and back pressure p_b, both of which are distributed over the piston faces and react upon the cylinder heads. Because the piston-slide mass has a continually varying motion and in the position shown is being accelerated toward the right, it has an inertia force F_s. The resultant of P_s and F_s is carried over to wrist pin W as an effective driving force P_e. A guide-bar pressure Q_g constrains the crosshead to its path, and the resultant of P_e and Q_g is the pressure P_w which the wrist pin exerts upon the connecting rod. Transmitted to the crank, with modification by the rod inertia force F_r, this becomes the

pin pressure P_c which acts to turn the crankshaft. The output of delivered work consists in the overcoming of a load resistance here represented as a force T tangential to the rim of belt wheel 6. The centrifugal force F_c of the counterweight in large degree balances recoils F_s and F_r, neutralizing their tendency to vibrate or shake the whole engine. A heavy bearing pressure Q_b opposes the tendency of all the other forces on the shaft to move its axis from the fixed position in the body of the engine.

In this machine both motions and forces run through a cycle of change, the period of which is the double stroke of the piston or the complete turn of the crank, called the "revolution" of the engine. The necessity of following each quantity through its whole range of variation makes this a much more complex problem than one with uniform motion: the simpler type of condition will, of course, be studied first.

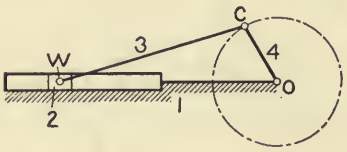

FIG. 4.

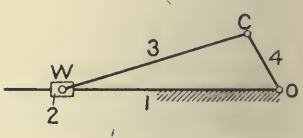

FIG. 5.

Kinematic skeletons of the engine mechanism.

While dropping structural detail, Fig. 3 continues to show a fairly full outline of the machine and its members, especially as to the contact surfaces. For purposes of motion study or kinematics a further condensation is convenient and desirable, to the mere geometric outline or skeleton in Figs. 4 and 5. Particularly, the piston-crosshead member is contracted to a simple slide block 2. Figure 4 simulates actual form of block and guides but Fig. 5 is the more convenient drawing, with little block 2 conceived as pierced for and sliding on line or rod 1. A standard notation for such kinematic skeletons consists in numbering the members in serial order, putting 1 on the fixed member or frame and following (so far as possible) the order of driving effect.

(*g*) **Place and Scope of Subject.**—We are now ready to take a general view of the place which our subject occupies in the broad field of Mechanics as a whole and to see what are the boundaries of its portion of that field. The matter can be covered by a series of concise statements, as follows:

1. **Mechanics** is the general science of force, motion, work and energy. As types of condition to which its laws and principles apply we may distinguish,

Bodies moving freely in space, under forces not impressed by contact, along paths determined wholly by these forces—astronomical or celestial mechanics;

The motions of free nature, as in winds, waves, streams, land slides, wholly or in large degree uncontrolled and indefinite;

Conditions and problems in which definiteness and constraint of motion are aimed at and more or less completely attained—in the field of engineering.

2. **Mechanics of engineering** may conveniently be divided into three main sections, characterized by no motion, by constrained motion and by controlled motion.

A principal function of static structures, of their component units and of the definitely formed bodies that serve as machine members, is to resist change of form or the relative motion of combined parts. **Mechanics of materials** relates internal stress or resistance to external loads which tends to change form; and in structural design and machine design this branch of mechanics is applied to the proportioning of parts for needed strength and stiffness.

Constraint of motion as an essential of the machine has been fully set forth. Its several parts or members are compelled, by form and contact, to move in a certain determinate way relative to each other. Within the limits of strength and rigidity, change in applied force will not cause a change in relative motion. Under this constraint of motion, the problems in **mechanics of machinery** are far easier of solution than would be questions involving equally complex systems of bodies in free or force-controlled motion.

The third division is more varied in its content than the other two. Mechanics of Machinery extends into it when taking up such problems as the control of motion by flywheels and the working of speed governors: here a system with constrained relative motion has its total or combined motion determined by free or live forces. Another example is the constraining of fluids to flow in prescribed channels: the simple motion of the current as a whole may cover complex internal motions. Sometimes it is a question of determining and allowing for the influence of a natural force, as of air resistance upon the trajectory of a projec-

tile, in ballistics. Again, the production of a required motion may be a matter of skilful balancing of natural forces, as in the navigation of a ship and, far more, in the management of an airplane. It is evident that **hydrodynamics** and **aerodynamics** constitute a major portion of this section, while **hydrostatics** would fall into the first.

Of course, the division of applied mechanics into these three departments is not complete and absolute, nor is it the only or even perhaps the best division that might be made. But it is convenient for defining the scope of this book, which covers the middle section and laps over a little way into the other two.

(*h*) **Machine Study and Design.**—The first step in the study of an existing machine or the design of a new one is to consider it from without and get a clear idea of its external relations. On the one hand is the question of manner of drive, on the other that of output action. In the case of prime movers a full study of input force extends into fields of thermodynamics, fluid-dynamics and electrodynamics. On the output side of work machines there is a great variety of function, motion and force action, conditions being most complex in automatic manufacturing machines and in those which imitate and supersede intricate manual operations. Between these two departments, knowledge of the development and control of natural driving forces and knowledge of the art or craft of shaping and fabricating materials, lies the field of study of the principles, working and design of the machine itself.

Knowing what motions the machine is to have with reference to driving force and to working action, the designer must first lay out the form and motions of its members. Having this geometric outline or kinematic skeleton and data as to force actions or reactions from without, the forces impressed upon the members can be determined. Then follows the proportioning of those members for resisting capacity and of the contact surfaces for proper intensity of pressure. If there are heavy members in fluctuating motion their kinetic forces must be determined, since these forces may modify design for strength or have to be compensated if they tend to shake the machine, causing vibration: even the plain centrifugal force of a uniformly rotating body can have a big effect in both the directions named.

Now the intelligent and effective performance of the operation just outlined is possible only as a sequel to training and experi-

ence. The program is synthetical, but calls for the application of knowledge and capacity gained by methods predominantly analytical. Into it enter, first, acquaintance with the form and action of the various machine members and simple combinations in use for transmitting and modifying force and motion; second, the principles and methods of mechanics of machinery, to determine definite motion and force action; third, knowledge not only of the properties and strength of materials but also of the constructive processes involved, to the end of facility and economy in manufacture; fourth, but by no means last in importance, ingenuity, inventiveness and sense of proportion.

From the preceding program it might be inferred that our whole subject is one for the comparatively small group of designing engineers. That would be an erroneous view, because the purpose is *understanding* of machines and their action, gained by analysis. From this general knowledge, which at least every mechanical engineer ought to have, the designer goes forward into his special field of quantitative synthesis.

(*i*) **Classification by Service.**—The following schedule arranges machines in appropriate classes, and may serve to systematize the ideas of the reader in regard to typical uses of machinery.

1. Prime movers: piston and rotary engines, steam and water turbines, water and wind wheels, electric motors. In these the principal motions are simple, but with a fluid as driving agent the control of driving force involves an auxiliary mechanism (valve gear and governing device) more complex than the main part of the machine. Except in direct-acting pumps and compressors, the output of work is delivered by means of a uniform rotary motion.

2. Prime movers reversed: this title covers pumps, compressors, blowers and electric generators, in all of which the load force is of the same kind as the driving force in the corresponding prime movers.

3. Transmission machinery: shafting and bearings, pulleys and wheels, belts, ropes and chains, toothed wheels, etc.

4. Transportation machinery: steam and electric locomotives and cars, self-propelled road vehicles, ship propelling machinery, etc. Conditions range from the very direct connection of prime mover and propelling element in locomotives and steamships to the interposition of a fairly extensive transmission gearing in the automobile.

5. Machines for moving materials: hoists, cranes, conveyors, etc. With these may be associated excavators, dredges and structure-building machines such as pile drivers and concrete mixers.

6. Agricultural machinery forms an extensive class, with functions so varied as to borrow types of motion and of working action from several of the other classes here named.

7. Machines for crushing, grinding and screening materials, used in the preparation of foodstuffs and in many lines of mineral industry.

8. Machines for shaping wood, metals and other resistant materials: these cover a great variety of cutting and pressing operations. The metal-cutting machines are especially characterized by the output requirement of exact motion accompanied by large work performance.

9. Textile machinery, for preparing fibers, spinning, thread and rope making, weaving, knitting, sewing, etc.

10. Machines for making paper and other sheet fabrics and for printing, embossing, etc.

11. Miscellaneous manufacturing machines, used in a wide variety of industries: many of these are to a high degree automatic in their operation.

12. Manual machines and instruments, such as typewriters and typesetters, counting and adding machines, railway signal systems, and the like. In many of these the work-performance function nearly or quite disappears, since the output is in the form of certain motions or combinations of position, with very small force factors.

The examples of machine members, mechanisms, etc., used in this book to illustrate the discussion of mechanical principles will be taken mostly from the more general classes of machinery, Nos. 1 to 5 and No. 8 in the schedule just completed. The controlling idea is not to consider machines in classes based on service, but to take up in order the various types of problems in the transmission and modification of motion and of force.

(*j*) **Divisions of Machine Study.**—Mechanics of Machinery has two main branches, **kinematics** and **dynamics,** concerned with motions and with forces respectively. Because machine motion is constrained by form of rigid bodies rather than left to be determined by resultant force effect, kinematics is largely independent of dynamics: that is, motion relations can be determined

by purely geometric methods from the form of the machine, without taking forces into account. This statement involves no subversion of the general principles of mechanics, but the practical difference is that with freely moving bodies motion is wholly dependent upon impressed forces, while in machines the forces necessary to a particular motion are automatically developed in or supplied by the machine members.

The study of kinetic forces such as F_s, F_r and F_c in Fig. 3, is a part of dynamics, but depends upon kinematics also for determination of the accelerations which those forces must produce or against which they react. In view of this double relationship, **kinetics of machinery** may be considered either a third division of the general subject or a combined extension of Kinematics and Dynamics, as happens to be convenient.

The general scheme of procedure in this book is, kinematics first, then dynamics; but the treatment of the separate divisions is not exclusive, cross references and general statements being introduced as occasion arises.

SECTION 2. KINEMATIC IDEAS AND SIMPLE FORMS

(*a*) **Machine and Mechanism.**—Between these terms there is a distinction that must now be made clear. Within itself the machine has the two functions of producing definite relative motion and of transmitting forces. When attention is concentrated upon the first of these—needed strength and rigidity for maintenance of form being assumed and then given no further thought—the name **mechanism** is applied to the combination of geometrical bodies constituting the machine. These may be shown in actual outline (Fig. 3) or reduced to simplest essential form (Figs. 4 and 5); but if we look at Fig. 3 it must be understood that the *mechanism of the steam engine* begins with the crosshead (not including the piston for steam action) and ends with the crank arm (not going on to the delivery of power by shaft or wheel). This contraction to no more than is really necessary for motion study is a part of the scheme of the kinematic skeleton as in Figs. 4 and 5.

Emphasizing and expressing the idea of transmission and conversion from primary to final motion, the name "train of mechanism" is sometimes given to the series of moving parts of a machine: and it is convenient to distinguish the **main train,**

as of the engine in Figs. 3 to 5, from a **secondary train** such as the valve gear. **Gear train** or **train of gearing** is the standard name for a wheel mechanism of the type represented by Fig. 2.

In brief summary, the mechanism is the geometric skeleton of the machine, or its **form for motion.** When embodied (for strength and rigidity) and put to work the mechanism becomes a machine. The final step in generalizing the representation of mechanisms—the kinematic-chain diagram—is described in Art. (*g*).

(*b*) **Kinematic Elements.**—The smallest combination within which definite relative motion can occur is a pair of solid bodies. Such motion is constrained or made definite by the contact of surfaces formed upon the two bodies. These contact surfaces, with so much of the bodies as is needed to support them, constitute a pair of elements or a **kinematic pair,** which is the ultimate unit in the analysis of mechanisms. The machine member carries and connects halves of two or more pairs, and by the combination of such members the machine is built up. Thus the engine mechanism, Fig. 3, has four pairs and four members; there are three turning pairs and one sliding pair; two of the members enter each into two turning pairs, the other two each into a turning and a sliding pair.

There are two possible types of moving contact for machine members, one distributed over complementary surfaces which have equal and uniform curvature, the other concentrated at the line or point in which dissimilar surfaces touch each other. Kinematic pairs having contact of the first type are called **lower pairs,** those with contact of the second type are **higher pairs.**

Four kinds of surface meet the requirement for lower-pair contact, namely, (*a*) The cylinder and other surfaces of revolution, permitting relative rotation about a common axis; (*b*) The prism (curvature zero), of whatever cross section, permitting straight-line motion along an axis; (*c*) The screw and nut, with simultaneous rotation about and shifting along an axis; (*d*) The sphere, allowing rotation in any direction about a point or center. In any case, one of the two like surfaces forms the outer boundary of a contained solid, the other the inner boundary of a surrounding solid. A complete lower pair fully constrains its members to the motion named, excluding any other; but the higher pairs, which we shall take up presently, do not have this full effect.

(*c*) **Turning Pairs.**—The examples in Figs. 6 to 13 are shown with much slurring of constructional detail, but with variations determined largely by force action. Considering turning pairs first, we have the plain journal and bearing of Fig. 6 to cover the case of fixed axis, the pin-and-eye joint of Fig. 7 for that of

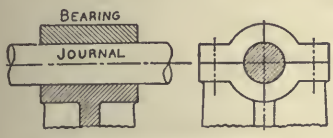

FIG. 6.—Plain bearing.

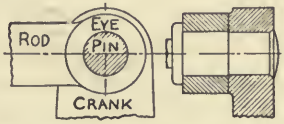

FIG. 7.—Pin joint.

moving axis. In Fig. 6 there is no constraint against endwise movement, along the axis. When forces in that direction are incidental and secondary, light collars as in Fig. 8, are enough. When forces along the axis are of major magnitude we have the pivot bearing (Fig. 9) and the collar bearing (Fig. 10): the latter

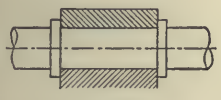

FIG. 8.—Journal bearing with collars.

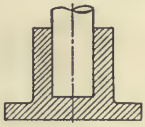

FIG. 9.—Pivot bearing.

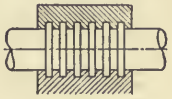

FIG. 10.—Collar bearing.

has a larger and more flexible load capacity and does not bring the shaft to a dead end.

Besides the cylinder and the plane (on the side of collars), the only other surface of revolution at all common in journal-bearing pairs is the cone, of which an example is given in Fig. 11. The

FIG. 11.—Machine-tool spindle with cone bearing.

purpose there is to maintain a close running fit, any looseness due to wear being taken up by a slight endwise shifting, effected by changing or adjusting the loose collars (shaded black) which sustain endwise thrust.

For complete constraint, without restriction as to where force shall be applied, a turning pair must have all the elements that

appear in Fig. 11, as also in Fig. 2. In terms of solid geometry, a straight line is held fixed in space if two points of the line are fixed: this requirement is met by two bearings at a considerable distance apart, to hold the axis to a certain fixed line, and two collars to keep it from moving endwise. With a definitely located plane of force action and fixed shaft axis, as in an engine, the single short pin bearing of Fig. 7 is sufficient.

(*d*) **Sliding Pairs.**—These play a prominent part in some kinds of machines, notably in piston engines and pumps and in machine tools. Two groups of typical cross-section profiles are given in Figs. 12 and 13, from the two classes of machines. Figure 12 (*a*) shows the cylindrical fit of a piston in its cylinder, with no provision for the prevention of incidental turning: and where, as in some close-coupled small pumps, there is no connection to

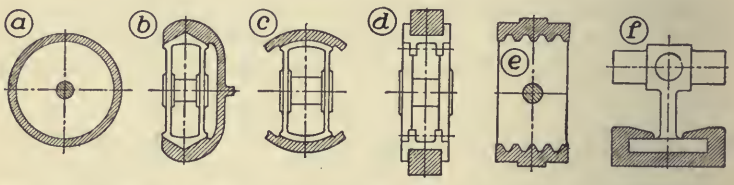

FIG. 12.—Cross sections of sliding pairs for the engine. (*a*) Cylinder and piston. (*b*) Crosshead and V guides. (*c*) Crosshead in bored guides. (*d*) Flat guide bars. (*e*) Steam-hammer guides. (*f*) Slipper guide.

external mechanism, no need exists for constraint against such turning. At (*b*) is given the profile of an older type of engine crosshead, with V-shaped guides. This form of guide, necessarily finished by planing, gives full constraint against turning; but it has been superseded by the bored-out cylindrical guides at (*c*), which can more easily be machined in true alignment. It is found that incipient turning is quite sufficiently prevented by the reaction of the connecting rod on the wrist pin.

Figure 12 (*d*) shows a very common form of locomotive crosshead, with separate shoes bolted fast to the body of the piece. The multiple V guide at (*e*) is from a large steam hammer, where strong constraint against the twisting tendency of a glancing blow is needed. Finally, the occasionally used marine-engine crosshead at (*f*) illustrates a guide which embraces and nearly surrounds the flat sole-plate of the moving piece.

All the slide blocks in Fig. 12, except the last, are symmetrical about a main axis of force transmission; and even the last is associated with a piston which is thus symmetrical. But in

machine tools the principal forces are frequently so far out of line with the guiding surfaces that a decided tendency to tip or overturn the sliding body is set up. This added element in the problem of constraint of motion is considered, as to detail of force action, in Part II.

In planing machines, represented by the first two examples in Fig. 13, sliding is the main working motion; more generally the

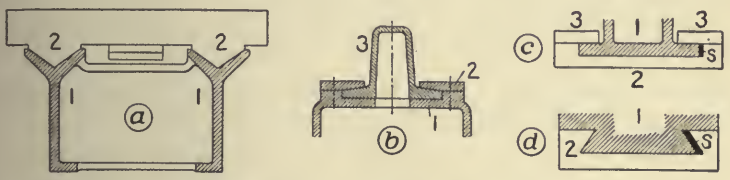

FIG. 13.—Machine-tool slides. (*a*) Planer table. (*b*) Ram of shaper. (*c*) Types of guiding.

sliding movements are those of feeding and adjustment. The planer "table" or "platen," piece 2 at (*a*), lacks complete mechanical constraint, dependence resting upon the always sufficient weight (gravity force) of the table and the piece of "work" upon it; but the relatively light ram or tool bar 3 of the shaper is fully constrained by its guides. Through the use of very long guides in these machines the tipping tendency of one-sided force action is balanced without excessive variation of pressure along the sliding surfaces.

FIG. 14.—Sliding on an arc.

A pair such as Fig. 13 (*a*), structurally incomplete or "open" but held together by an assumed and always predominant force, is said to be **force-closed.**

The last two cross sections in Fig. 13 show typical shapes of guide, with the necessary adjustments for close fit and taking up wear. The thin wedge strips shaded black, called "gibs," are held fast to either slide or guide by adjusting screws. With the dovetail form at (*d*) movement of the wedge gives full adjustment. With the stronger rectangular section at (*c*) one direction of

adjustment would require that bars 3, 3, be taken off and their seating on piece 2 be filed down a little.

A block sliding in a curved slot or on a curved guide bar, as in Fig. 14, is an element of certain mechanisms, mostly valve gears for engines. For true contact the curvature must be uniform or the guide profiles parts of circles. Structurally this is a sliding pair, but for kinematic purposes it is to be considered a turning pair, with axis at the center of curvature C.

(*e*) **Coplanar Motion.**—With but a few exceptions machine motions are planar or coplanar. This means that the machine members move in a plane or in parallel planes. Thus the engine has its stroke line (the axis on which the piston-slide reciprocates) in the plane of connecting rod motion, while the crank arm turns in another plane and the wheel in a third—all these being perpendicular to the shaft axis. Again, the hoist gearing of Fig. 2 has a set of parallel planes for the successive wheel pairs. Even with direction-changing devices such as bevel gears and twisted or guided belts—see Figs. 64–67 and 111—there is no more than a change from one plane of motion to another at an angle with it.

Back of the prevalence of coplanar motion lies the fundamental fact of the general use of turning pairs, which constrain to pure rotation without any motion along the axis. The angular transmission devices just cited have their place in wheel mechanisms. In linkage mechanisms—see Art. (*g*)—not only are the pairs coplanar but the axes are nearly always parallel: only under a few special conditions can they be made nonparallel without locking the linkage or restraining it from motion.

Having coplanar motion or parallel axes, it is a simple matter to project the whole mechanism upon a single plane and treat its problems of motion by methods of plane geometry. That is, while machine members are necessarily three-dimensional bodies their form for motion (and also to a considerable degree their form for force action) can be fully represented by plane outlines. In the projection upon a single plane the axes of pairs or joints are reduced to mere points or centers—see Fig. 2 (*b*).

(*f*) **Working Pair Units.**—At this point it is appropriate to call attention to the development of the commonest kinematic pairs into the working units most used in prime movers and allied machines. The sliding pair elaborates itself into the cylinder-and-piston combination, which is shown in two typical forms by Figs. 15 and 16. The disc piston, fast to a rod of rela-

tively small diameter, is generally double-acting: that is, it can receive or perform work on both sides, alternately. The single-faced trunk piston usual in small and medium sized gas engines is like the pump plunger of Fig. 16 in being single-acting.

The turning pair underlies the rotor-and-casing combination which constitutes the working unit in electric motors and generators, in water and steam turbines, and in centrifugal pumps and blowers: always a force action, driving or resisting, is exerted between the two members of the pair. No example of this type

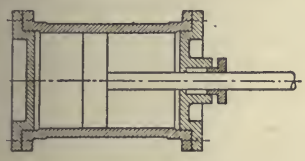

FIG. 15.—Cylinder and piston.

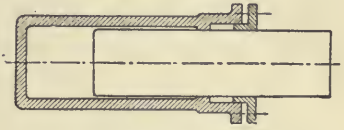

FIG. 16.—Cylinder and plunger.

of unit is here sketched, because upon its essential kinematic simplicity there is built a complexity of form or structure, having to do with the natural forces involved, which lies outside the field of present purposes.

(*g*) **Evolution of Mechanisms.**—Begin with a single pair, taking the sliding pair of Figs. 15 and 16 as represented in Fig. 17 (*a*). Extend each of the members so that it carries the half of another pair, Fig. 17 (*b*). Complete each new pair by adding two new members with a joint or pair between them, getting the

FIG. 17.—Evolution of the quadric chain.

full engine mechanism as in Fig. 17 (*c*). Then from one pair we have derived a four-member mechanism by adding two more members and three pairs.

In order to show the makeup of the mechanism in simplified and general terms, sketch it as a mere combination of elements in Fig. 17 (*d*). This is not strictly a diagram, for it need not show true proportions, but is rather a graphical description, paralleling the verbal description in the first paragraph of Art. (*b*). Cutting loose thus from definite proportions and for clearness separating pairs that overlap, as on piece 2, we pass from the mechanism to

the **kinematic chain.** This represents the mechanism as a series of **links** connected by pairs or **joints,** with the notation of designating each joint by the combined numbers of its links. Read these pair numbers "one-two," "two-three," etc., not "twelve" or "twenty-three."

If we take the final step of allowing free choice as to which link of the chain shall be selected for fixed link or frame member, the conclusion is reached that the same chain may represent a number of different mechanisms or machines. This phase of the matter will be fully dealt with later, and for the present it will be enough to complete the scheme of representation and analysis begun with Figs. 3 to 5 and carried forward in Art. (*a*), stating it in synthetical order as follows:

A kinematic chain is a combination of links or members, arranged and connected for definite relative motion.

A mechanism is got by selecting one link of the chain for frame and giving definite proportions and arrangement to the whole combination.

The **machine** is obtained by giving to each geometric link the substance and strength of a machine member.

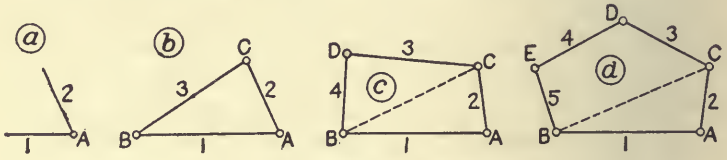

FIG. 18.—Locked and loose chains.

To mechanisms made up of levers, cranks, rods, etc., with turning and sliding joints, the name **linkage** or linkage mechanism is given, in contrast with trains of gearing, cam gearing, etc.

(*h*) **Constraint and Extension of Linkages.**—Starting with pair 12 in Fig. 18 (*a*) and extending it to B and C at (*b*), suppose that we connect these pairs by a single link 3: then the resulting jointed triangle is rigid or locked and is, in fact, the unit from which jointed structures (trusses) are built up.

The four-link chain, Fig. 18 (*c*), is the fundamental form of constrained linkage. Constraint of motion means that if two of the members are held in a certain definite relative position the other members cannot move. Here if we hold 2 on 1 so that line BC cannot change, it is evident that the remaining triangle BCD is fixed.

At (*d*) pairs B and C are connected by three links, and now the whole linkage is loose or unconstrained: with length BC established there remains a four-link chain BCDE free to move. Such a loose or free combination is not suitable for use in mechanism.

To see how more extensive mechanisms are built up, consider Figs. 19 and 20, both of which start with the engine mechanism of four links: a new item of notation is the representation of a sliding pair by a blacked circle in chain diagram (*b*). A joint is added to a moving member in each case—to crosshead and to connecting rod respectively—and from this two links are run to a new bearing on the frame. These particular additions are both

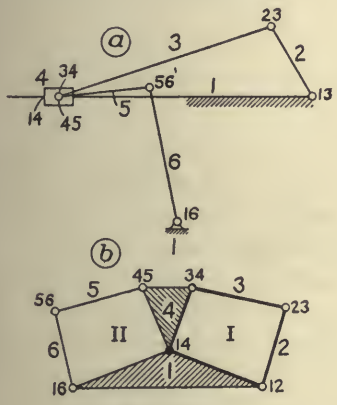

FIG. 19.

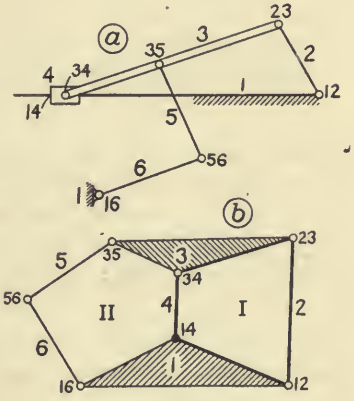

FIG. 20.

Extension to six-link chains.

practical, lever 6 of Fig. 19 being used to drive the condenser pump on many older steam engines, while 5 and 6 in Fig. 20 are the beginning of the Joy valve gear, described in Part II. The chain diagrams (*b*), (*b*) show the same initial four-link or *quadric* unit 1234 or I, but differ in the added unit chain. When adjacent or paired links are extended, as 1 and 4 in Fig. 19, new unit II or 1456 is quadric. With extension of nonadjacent links 1 and 3, Fig. 20, the addition 14356 contains five links or is a *quintic* unit. It appears that while an isolated quintic chain is loose, such a unit may borrow needed constraint from an associated quadric unit.

(*i*) **Three-link Chains.**—An exception to the rigidity of the three-member lower-pair linkage, Fig. 18 (*b*), is found in the case

of three sliding pairs, or in the wedge mechanism of Fig. 21. This is a unique form or a special single combination; and it must contain three distinct pairs, no two being parallel.

The screw pair finds its application, for motion as against fastening, in a similar three-link combination, Fig. 22. If we think of the screw as an inclined plane wrapped about a cylinder, the screw mechanism is seen to be a direct derivative from the wedge mechanism. There is good reason for showing chain

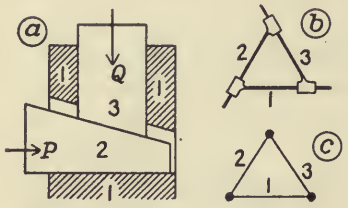

FIG. 21.—Wedge mechanism.

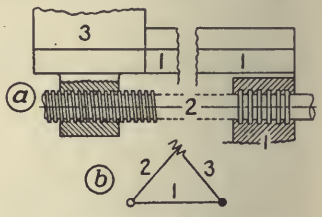

FIG. 22.—Screw mechanism.

sketches with Fig. 21, because that unit can enter into linkages; but the suggested screw-chain outline is of no practical value for two reasons: it gives a false impression of coplanarity, and this unit of mechanism is mostly employed with wheel trains (not in linkages) which cannot well be simplified beyond the full layout as in Fig. 2 (*b*).

(*j*) **Spheric Motion.**—Two instances of the ball-and-socket joint are shown in Figs. 23 and 24. The first is the "universal"

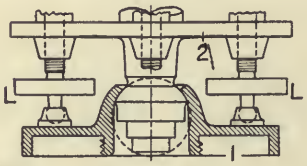

FIG. 23.—Universal joint, base of surveyor's instrument.

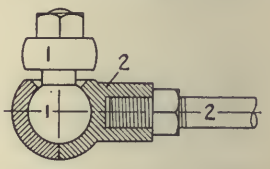

FIG. 24.—Ball and socket joint.

joint in the base of a surveyor's instrument (level or transit): the spheric pair is here an isolated unit, with its movement controlled by four separately manipulated leveling screws. Note the ball-and-socket heads on these screws. As a linkage element this pair has almost its sole application in the rod joint of Fig. 24, used on rods that connect cranks or arms with different planes of motion.

The spheric pair finds little use in mechanism, partly because it does not give sufficient constraint, partly because envelopment of the solid sphere interferes with movement of the part to which it is attached. Both defects are illustrated by the arrangement sketched in Fig. 25. Bar 2 can not only be swung into any angular position but can also rotate on its own axis; and this is a greater degree of freedom than is likely to be desirable. Again, even with the contraction of bearing 1 to a narrow zone the movement range of 2 is a good deal restricted; and with the bearing thus narrowed there is poor support, the ball having a tendency to wedge fast under load.

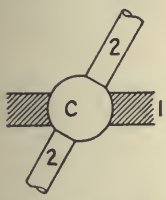

FIG. 25.—Universal spheric joint.

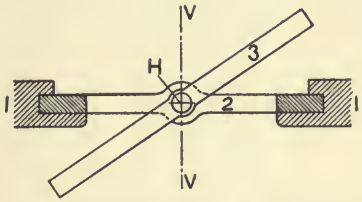

FIG. 26.—Rotation on crossed axes.

In practical mechanism the kind of axis movement that a spheric pair would give—as for a telescope, for a fully mounted gun or in a machine for cutting bevel gears—is secured by the combination of two coplanar rotations, generally about axes perpendicular to each other, as outlined in Fig. 26. Ring 2 turns in base 1 about vertical axis VV and carries bearings for trunnions on bar 3, which has its rotation about horizontal axis H. The two motions are independently controlled, whether by hand or by mechanism.

(*k*) **Kinds of Machine Motion.**—Having coplanarity of motion imposed by controlling requirements which are embodied in construction—see Sec. 29 (*a*) for a collected statement of conditions—body motion is brought to the state of two-point determination. That is, the position of a body is fixed by locating two of its points, its movement is made definite by fixing the paths of two points. We are not now concerned with problems requiring full two-point determination, of which the engine connecting rod presents a typical example: but there is a yet simpler state that is of immediate interest.

In many cases the motion of a machine member can be either completely or sufficiently defined by the motion of some one

point upon it. If the body has straight-line sliding one point makes complete showing because all points move alike. If the body rotates on a fixed axis one point, on its circle path, is sufficient: either by linear movement of the point or by angular movement of a radius to the point the rotation is fully measured.

The motion of point along path can be effectively represented by a diagram in which time t is base and distance s is ordinate. Then with uniform rotation any chosen point on the rim of a wheel or the obviously indicated center of a crank pin will have, along its circular path, the uniform progression shown by inclined straight line AB in Fig. 27.

The reciprocating movement of an engine piston is similarly shown in Fig. 28, points A and C being at the same distance

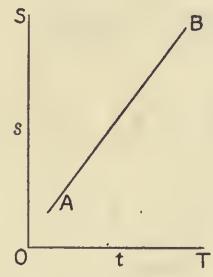

FIG. 27.—Continuous.

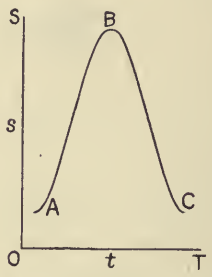

FIG. 28.—Reciprocating.

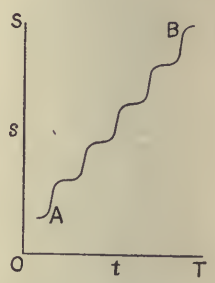

FIG. 29.—Intermittent.

Types of machine motion.

from base OT. The production of such cyclical motion is the usual function of linkages and of the cam drives to be given in Sect. 5. In general the motion curve will be less simply symmetrical than here and the cycle may even be a set or series of different reciprocations. Commonly the primary motion is rotation; and cams have the advantage of much greater flexibility for accommodation to a desired motion of the driven member.

The third type of motion, intermittent progression as indicated by Fig. 29, is given by ratchet devices, escapements and some other mechanisms of similar effect. A group of examples under this classification is gathered together in Sect. 12. The motion line may show sharp corners or abrupt starts and stops, but the easier action represented by rounded corners is much preferable, especially with a heavy mass or for high speed.

The curves in Figs. 27 and 29 are given in illustration, not as models for practical use; but the one in Fig. 28 is a regular method

of showing cyclical reciprocation. With the driver in uniform rotation a scale of its angular position may be substituted for the time scale; then we may even drop the idea of uniform drive and consider that the curve shows simply relative movement.

SECTION 3. LINKAGE MEMBERS

(*a*) **Cranks.**—The subject of linkages and their motions is carried forward in Chap. VI, after the uniform motion of gear trains has been dealt with. Before going on to higher pairs we shall here give, however, a few examples of the constructive form of linkage members, with some allusions to force action. These working parts may be classified under the titles cranks, levers, rods or struts and slides—the last group having already been sufficiently covered in Sect. 2 (*d*).

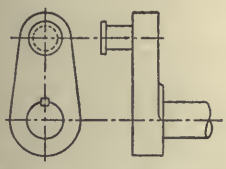

FIG. 30.—Outside crank.

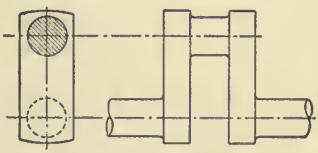

FIG. 31.—Inside crank.

The two typical forms of engine crank are shown in Figs. 30 and 31, and their distinctive features are apparent. The outside or end crank belongs especially to the single engine, although regularly doubled in locomotives, hoisting engines and duplex power engines. The inside or center crank is often used in single engines for the sake of its symmetrical bearing arrangement, but has its special field in multiple-unit engines and pumps.

Kinematically the simple crank arm, reducible to a mere radius as when we skeletonize the engine mechanism, may sufficiently represent the whole machine member. But in structure and in function with regard to force action the crank proper is only a part of a larger moving piece, which includes also the shaft and generally some form of wheel or rotor. Various conditions as to force action are shown in Figs. 32 to 34. The first and simplest case has a uniform tangential resistance distributed around a circle which may stand for the rotor of an electric generator or of a centrifugal pump or for a screw propeller. The direct push of crank-pin pressure P is balanced by an equal, opposite and parallel holding pressure B of bearing on journal; and the driving

moment of force P about axis O (or the moment of couple P-B) equals the resisting moment of distributed force.

In Fig. 33 circle AE is a belt wheel and the frictional pull of the belt is distributed over arc ADE. As before, the driving moment of P about O equals the resisting moment of belt friction or of net tension $(T_1 - T_2)$; but bearing pressure B must now balance both those tensions as well as pin pressure P. In Fig.

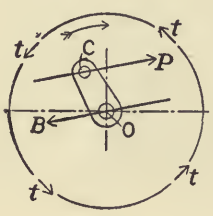

FIG. 32.

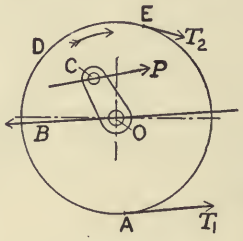

FIG. 33.

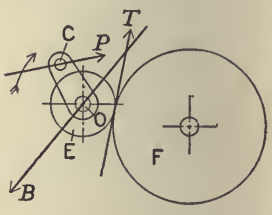

FIG. 34.

Typical crankshaft conditions.

34, with circles E and F representing a pair of gears, resistance T is a single concentrated force. The moments of P and T about O are equal and force B balances the resultant of P and T.

The principal force action upon the crank arm alone is shown in Fig. 35, which comes directly from Fig. 32 and just as truly, if not quite so evidently, from Figs. 33 and 34. A similar layout of the lever, for the special case of parallel forces, is drawn in

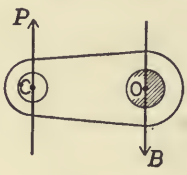

FIG. 35.

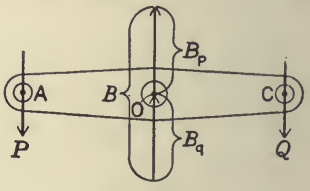

FIG. 36.

Force action on crank and on lever.

Fig. 36. To make clear the resemblance of either arm of the lever to the single arm of the crank, the whole bearing pressure is shown in two portions, B_p equal to P and B_q equal to Q. The distinction is that the force couple P-B on the crank is balanced by torsional stress of the shaft, while the balanced couples on the lever get at each other entirely through the medium of bending stress in the bar itself.

(*b*) **The Eccentric.**—Before going on with the lever a kinematically identical but structurally different form of the crank must be gotten out of the way. The problem is to combine a short length or radius of inside crank with a large diameter of crank shaft. In cross section (*a*), Fig. 37, O is axis of shaft and E axis of crank pin. To cut down the shaft to the little pin sectioned at (*b*) would take away all its strength; while a crank like Fig. 31, as suggested by the dotted outline, would be expensive to make

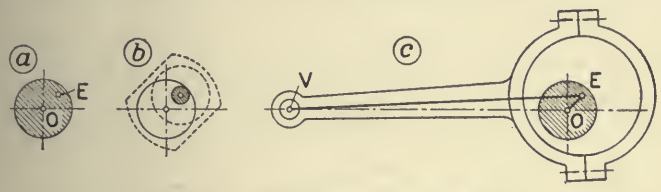

FIG. 37.—The eccentric and its rod.

and would take up a great deal of room in the direction of shaft axis. The practical solution is the eccentric, shown at (*c*) as used in the common form of steam-engine valve gear. The crank pin on axis E is simply enlarged into a disc big enough to

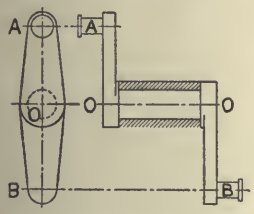

FIG. 38.—Reversing rock shaft.

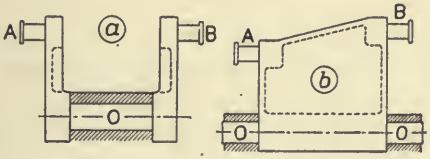

FIG. 39.—Direct rocker arm.

go clear around the shaft, with sufficient strength at its thinnest place; and simultaneously the rod eye is enlarged into the eccentric "strap," here shown as an integral part of rod EV.

(*c*) **Various Levers.**—Returning to the showing of Figs. 35 and 36 that there is no real difference between a crank and one arm of a lever, consider the cranks combined into levers in Figs. 38 and 39. Both examples are from valve gears, the first belonging to what was formerly the universal arrangement on American locomotives, the second type coming from stationary engines. With one pin driven by an eccentric the lever has an oscillating or rocking motion on axis O, hence the titles as given. Note in Fig. 39 the transition from a pure crank construction to a box

casting which will resist its twisting load through a complicated distribution of internal stress.

Levers such as Figs. 38 and 39, having widely separated planes of action for the forces on the two pins, are much less simple in construction and in design for strength than are those with arms, motions and force actions all in one plane. In defining the shape of such simple levers it is well to consider the two straight forms, (*a*) and (*d*) in Fig. 40, as particular and limiting cases in the possible fixing of any angle between arms OA and OB. For equilibrium of any lever the moments on the two arms must oppose each other. With parallel forces on the straight levers in Fig. 40 it is made apparent that with arms in opposite directions the forces point the same way, while with like arms the forces must be opposite. Note that cases (*b*) and (*c*) are similar to (*a*), and that (*c*) will be transformed toward (*d*) by swinging the force line

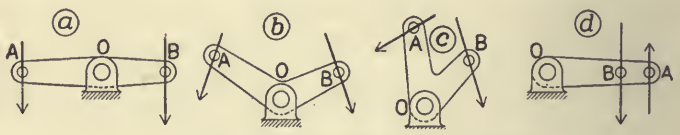

FIG. 40.—Various angles between arms.

through B so that it will cross arm OA—this throwing the intersection of force lines outside of acute angle AOB.

As a member of the main mechanism of heavy machines the plain lever is comparatively rare. It had prominent place in the obsolete beam engine, of which an outline will be found in Fig. 481. Its occasional use in heavy shearing machines is represented by Fig. 73, and it has one regular machine-tool use in the "shaper." Mostly, however, it has been relegated to the secondary mechanisms of the heavier machines, such as the valve gears of engines. But in conjunction with the cam it is a principal member in many of the lighter automatic machines designed for manufacturing operations.

(*d*) **Virtual Levers.**—The true lever, a beam or bar subjected to a bending load, is restricted to oscillating motion. The function of carrying such a load with continuous motion is taken by the compound wheel or by the wheel and crank. A combination of two wheels or cylinders, fastened together or keyed to a shaft so that they must turn together, is called a compound wheel —the name being less appropriate as the wheels are farther apart

axially. In Fig. 41 tangential forces P and Q are constant in direction and intensity, and their application points A and B maintain unchanged positions by travelling along the circumferences of the wheels as these turn. The portion of the two wheels instantaneously within the bounds of the shaded strip having radii OA and OB for center lines may be conceived as an equivalent or virtual lever. This active portion of the wheels is continually changing, or the virtual lever is turning on the wheel piece just as fast as the latter turns with reference to frame of machine.

With crank and wheel as in Fig. 42, which represents a locomotive driving wheel and so much of the force action upon it as is concerned with propulsion, this concept of the virtual lever

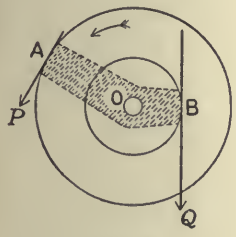

FIG. 41.

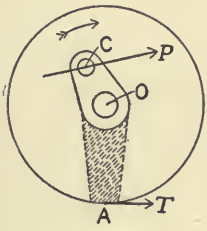

FIG. 42.

Virtual levers.

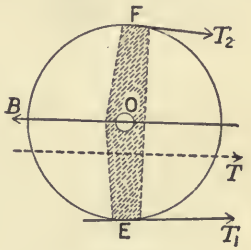

FIG. 43.

leads to the peculiar condition of a continual change of angle between arms. Real arm OC turns with the wheel while imagined arm OA runs always to the place of application of force T. This T is the holding friction which prevents slip of wheel on rail.

In Fig. 43 the idea of the virtual lever is applied to the belt pulley. The unequal belt tensions T_1 and T_2 have a resultant which does not pass through center O but lies nearer to its larger component T_1; and with bearing pressure B this T forms a couple that must be overcome by the driving moment or torque of the shaft.

Relative to the locomotive, the rail in Fig. 42 is continually running backward beneath the wheel, and in Fig. 43 the belt is winding on and off the pulley at E and F. Paralleling these physical actions by one purely geometrical, we may think of the force lines in Fig. 41 as similarly rolling on or off the wheel rims.

(*e*) **Rods and Struts.**—These names cover machine members in which the principal force (or stress) is a simple tension or com-

pression transmitted along a center line. Almost necessarily they are two-pair kinematic links, because the addition of a third joint is pretty sure to introduce a bending force action. The representative example of a rod to take either tensile or compressive load is the engine connecting rod. A sample is detailed in Fig. 44, with the especial purpose of showing the construction of eyes or bearings and how they are adjusted to a good fit on

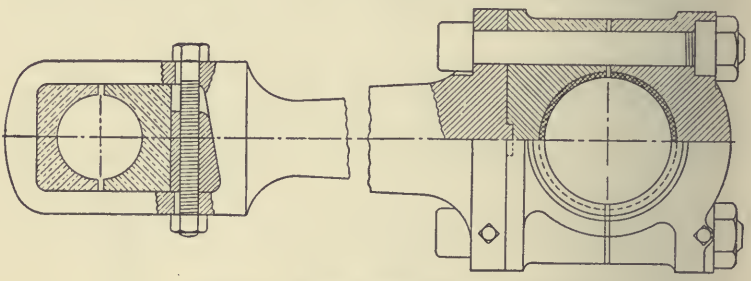

FIG. 44.—A steam-engine connecting rod.

the pins. Such a rod is divisible into three portions, the body or shank and the two heads.

In the less general case of subjection to compressive load only the special name strut is given to a rod-like piece. Not infrequently the connecting rod of an engine is mainly a compression member, as in the single-acting gas engine and yet more when the same mechanism is used in a power press; but so long as there is

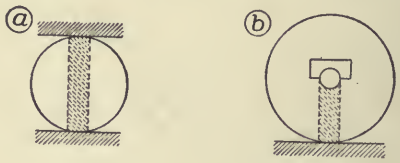

FIG. 45.—Virtual struts.

a small alternating tension and full kinematic provision is made for pulling action on the return stroke, the name strut hardly applies.

Following the analogy of the virtual lever, Fig. 45 shows how the roller or ball and the carrying wheel, from Figs. 46 and 47, may be considered as virtual struts. The supporting rod is maintained, of course, by having a continually changing part of the whole piece come into action, just as in Figs. 41 to 43.

SECTION 4. HIGHER PAIRS FOR UNIFORM MOTION

(*a*) **Higher Pairs.**—Generally the lower pair completely constrains its members to a certain relative motion, one upon the other. The higher pair is less complete in its effect, requiring either that it be used in multiple or that its members be lower-paired with a third piece. For this reason the examples that follow will show or imply more than the simple pair, commonly representing an elementary mechanism: this may be used either by itself or as a unit in more extended mechanisms.

Beside combinations which meet the definition of line or point contact of surfaces, the name higher pair is taken to cover also arrangements with wrapping contact, as in cord-and-pulley and belt-and-pulley devices. These differ from turning pairs in that there is no slipping of one member upon the other, except incidentally and as a secondary action.

(*b*) **Rolling Pairs.**—The simple function of carrying a load, without motion in the direction of the load force, is performed by the arrangements outlined in Figs. 46 and 47. The first repre-

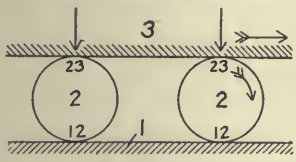

FIG. 46.—Roller or ball support.

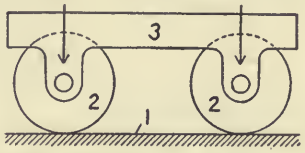

FIG. 47.—Carrying wheels.

sents either the roller bearing (line contact) or the ball bearing (point contact). Each unit contains two higher pairs 12 and 23, and at least two rollers or three balls are required for complete support of a load: the minimum of three balls is found under typewriter carriages, but most bearings have a far higher degree of multiplicity. There is always additional constraint of a secondary type, for balls by the form of tracks or "races" and by spacing cages, for rollers by aligning cages or equivalent devices to hold them in correct relation.

The carrying wheel or vehicle wheel, Fig. 47, rolls on its path or track, but for its complete function requires a turning pair at the axle, where the load body is supported. At least two broad rollers or three disc wheels not in line are necessary for complete support. The added function of driving or propelling the vehicle, by means of friction between wheel and path, is considered in Art. (*d*).

(*c*) **Wrapping Pairs.**—The simple loaded pulley, of the kind used in hoisting rigs, is sketched in Fig. 48, while Fig. 50 outlines a pair of transmission pulleys with their belt. Between hoisting arrangements and those for the transmission of power by means of ropes, chains or belts two distinctions may be drawn. In Fig. 48 there is substantial equality of the two tensions P and Q, with only the small difference due to journal friction and stiffness of rope or chain; while in Fig. 50, when the belt is transmitting a

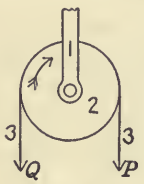

FIG. 48.

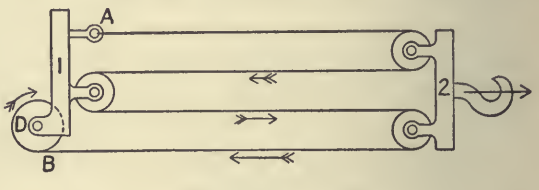

FIG. 49.

Cord and pulley: Element and application.

driving force, tension T_1 is decidedly larger than T_2. Again as shown by the diagram of a block-and-tackle hoist in Fig. 49 hoisting effect is secured by shortening a rope or chain which may be taken to have definite ends at anchorage A and at drum contact B, this shortening being accompanied by small and slow rotations of the pulleys. The belt, on the other hand, is endless and has a continuous rapid motion about its pulleys.

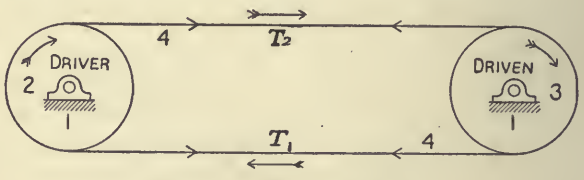

FIG. 50.—Pulleys and belt.

It is hard to say just how pairs are to be counted in combinations of wheels and tension members; but there is no real need of a definite formulation of method or convention, since occasion is not likely to arise for applying to such devices an analysis of the kind developed in Sect. 2 for linkages.

(*d*) **Friction Wheels.**—This might be taken as a general title for all wheels that work with a tangential resistance or driving force due to friction. So understood it would cover belt pulleys and the driving wheels of locomotives and other self-propelled

vehicles. In practice, however, the name is used only for wheels rolling together as in Figs. 51 and 52. As suggested by the first of these examples, the "frame" must include some device for maintaining (in spite of wear) the pressure contact which produces the friction of transmission.

In order to increase the resistance to slip, one wheel face is usually made of a material that has a high coefficient of friction, as wood, compressed paper or leather. Logically this goes on the driver, which is commonly the smaller wheel, in order to obviate local wear of the softer material in case of bad slippage. A different scheme for securing greater friction is shown by Fig. 53 (*a*), the wheel rims having V-shaped grooves and ridges which mate with a wedging action. Wheels for textile or fiber (not wire) rope are regularly made so as to get this wedging effect, with the form of groove in Fig. 53 (*b*).

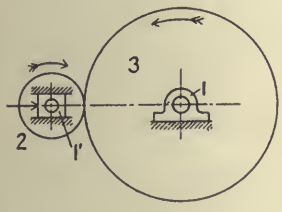

FIG. 51.—Friction wheels.

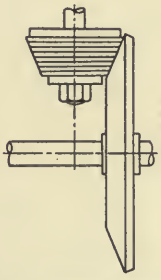

FIG. 52.—Bevel wheels.

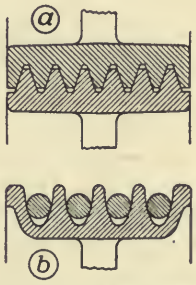

FIG. 53.—V grooves and rope grooves.

Friction wheels are not effective for the transmission of large working forces, because the useful friction is only a fraction (say 0.1 to 0.2) of the pressure which must be exerted at the bearings in order to produce it. Also the motion is not positive and definite, since there is possibility and high probability of slip.

Conditions as to friction are much more favorable with belt or rope contact than with the mere tangency of solid cylinders or cones, so that belt gearing occupies by far the more important place in machinery of transmission. There is, however, one class of service in which frictional resistance to slip of wheels is regularly employed for heavy work, and that is in the tractive action of driving wheels. Aside from the very obvious considerations of practical simplicity, convenience and sufficiency, conditions are there more favorable than in power transmission, because the

carrying of the heavy load which produces the needed friction or "adhesion" is itself a useful function. But even in that service there is a limit, passed in steep mountain railways, beyond which it is necessary to resort to the gear-on-rack drive.

(*e*) **Toothed Wheels.**—Suppose that the cylinders or discs 2 and 3 in Fig. 54 turn together on their axes A and B, rolling on each other at contact line C without slip. In order to make this relative motion positive and to give capacity for transmission of force and work, form teeth by cutting in and building out upon the cylinders and thus get the pair of toothed wheels or **spur gears** in Fig. 55. Physically the original cylinders or circles have disappeared; geometrically they remain as the **pitch sur-**

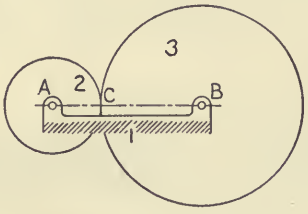

FIG. 54.—Friction-wheel pair.

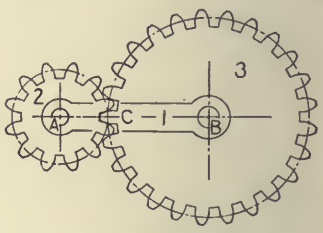

FIG. 55.—Toothed pair.

faces or **circles** of the gears. The teeth must be so formed that they will produce and insure perfect rolling of these circles. If they are inaccurately spaced or of incorrect profile, so that the driven wheel or follower receives an irregular motion, the gears will run noisily, especially at high speed. The tooth surfaces both roll and slip upon each other, so that relative motion in the tooth pair is more complex than in the simple rolling pairs thus far considered.

The two wheels with the frame bar connecting their axes constitute a very important three-member unit mechanism. Not only must there be two lower pairs to hold the wheels together—compare Art. (*a*)—but two higher pairs are necessary for complete constraint of motion. That is, there must be two contacts on opposite sides of one tooth in order to insure perfectly definite relative positioning of the gears. If the tooth is narrower than the groove or space into which it enters there will be some lost motion or **backlash** in the pair.

(*f*) **Parts and Dimensions of Teeth.**—To the process of building out and cutting in limits are set by the **addendum** and **root**

circles, Fig. 56, at respective distances h and d from the pitch circle. **Dedendum** or root depth d is made greater than **addendum** h so as to leave a certain bottom or radial **clearance,** which appears in Fig. 57. As parts of the tooth we distinguish **tip** (height h) outside the pitch circle and **root** (depth d) inside the pitch circle. Corresponding parts of the working surface and of its profile are named **face** and **flank,** as marked on Fig. 56, divided by the **pitch line** or **pitch point.** For the pair of wheels the total **working depth** or the depth of **mesh** is the sum of the two addenda.

The primary tooth dimension is **circular pitch** p, the length of pitch-circle arc belonging to one complete tooth-and-space and divided into parts t and s. With accurately formed teeth

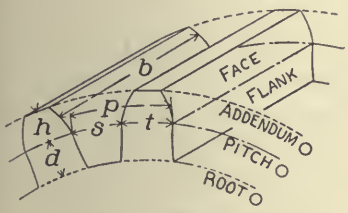

FIG. 56.—Parts of teeth.

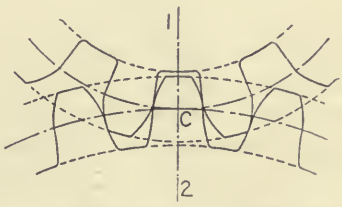

FIG. 57.—Tooth profiles.

thickness t and **space** s are made equal, so that tooth is neat fit in groove. With rougher teeth or after wear there will be some side clearance or **backlash,** so called because it allows the driven gear or train to swing back a little when the driver is held. Having diameter D and circumference πD of the pitch circle, together with the number z of teeth on a wheel, the length of arc p is

$$p = \frac{\pi D}{z}. \tag{1}$$

In early practice, when all gear teeth were cast in sand molds from wooden patterns, it was usual to take for linear pitch p a value in common (binary) fractions of the inch, leaving D expressible only by a decimal. From the desire for more simply exact diameters and axis distances, and with the adoption of more accurate methods of cutting teeth, has arisen the practice (now standard and almost universal) of rating tooth size by a ratio called the **diametral pitch.** This is defined by the equation

$$p_d = \frac{z}{D}, \tag{2}$$

or as *the number of teeth per inch of diameter.* Between the two pitches exists the simple relation

$$p \times p_d = \pi, \tag{3}$$

got by multiplying together Eqs. (1) and (2).

Under the metric system this ratio designation is inverted to what is called the **module,**

$$m = \frac{D}{z}, \tag{4}$$

which is *the number of millimeters of diameter per tooth.* Distinguishing D'' (inches) and D_{mm}, we have $D_{mm} = 25.4D''$, whence

$$m = \frac{D_{mm}}{z} = \frac{25.4D''}{z} = \frac{25.4}{p_d} \text{ and } m \times p_d = 25.4. \tag{5}$$

Note that a large numerical value of p_d means a small tooth and vice versa, while m varies directly with the size of tooth. It is usual to take only integral values of p_d down to 3, then introduce simple fractional numbers, as $2\frac{1}{2}$, $1\frac{3}{4}$, $1\frac{1}{2}$, etc.; and after a similar fashion the standard metric sizes are fixed by integral or simple-fractional values of m.

There is a very generally prevalent "standard" system of tooth proportions, used especially for sets of wheels that must interchange, in which the addendum is taken as

$$h = 1/p_d \text{ in inches} = m \text{ in millimeters;} \tag{6}$$

then the overall diameter of a gear, or the diameter to which a blank must be turned before the teeth are cut, is

$$D' = \frac{z + 2}{p_d} \text{ in.} = m(z + 2) \text{ mm.} \tag{7}$$

For some purposes shorter teeth called **stub teeth** are used; these will be considered in connection with the shape and action of tooth profiles, which forms a separate subject.

(*g*) **Velocity Relations.**—The commonest kind of uniform linear motion is that of the rim or periphery of a wheel or pulley or of the pitch circle of a gear, all turning at a constant rate. If R be radius in feet, D diameter in inches and N speed of rotation in revolutions per minute (r.p.m.), the linear velocity of circumference is,

$$V = 2\pi RN = \pi DN/12 \text{ ft. per min.} \tag{8}$$

For quantitative work with wheel pairs (and later with wheel trains) we change to the notation on Figs. 58 and 59, different from that used on mere mechanism outlines. In this system numbers are applied to rotating members or to axes of turning, capital letters A, B, etc. are placed on wheels and the same letters (small) stand for either radii, diameters or numbers of teeth; the reason why the same symbols may be used for these different dimensions will appear at once.

With positive drive by teeth and with frictional drive under the assumption of no slip, the two wheels or pulleys of a pair must have the same velocity of circumference. Then if a and N_1, b and N_2 are the respective radii and r.p.m's,

$$V = 2\pi a N_1 = 2\pi b N_2; \text{ whence } \frac{N_1}{N_2} = \frac{b}{a}, \tag{9}$$

or **speeds are inversely as radii.** It is because the wheel dimensions thus enter into a ratio that they can be taken in any convenient (and consistent) terms, or that any consistent dimensions may be used.

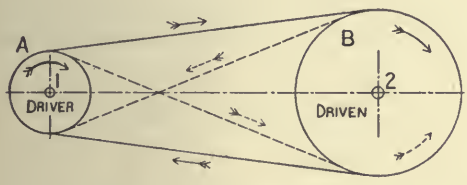

FIG. 58.—Belt-pulley pairs.

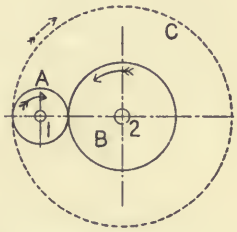

FIG. 59.—Spur-gear pairs.

Figures 58 and 59 show also relative directions of turning. With the usual open belt (full lines in Fig. 58) the two shafts turn alike; crossing the belt (dotted lines) gives reversal. Gears with outside contact have opposite rotation, while the pinion and internal gear turn in the same direction.

Prob. 1. Given pulley diameters D_1 and D_2 with speed N_1, find N_2 and velocity V of belt in feet per minute:

	a	b	c	d	e	f	g	h
D_1	60	48	24	144	6	10	15	24 in.
D_2	12	9	3	27	25	54	80	56 in.
N_1	240	220	450	120	1,320	1,000	600	420 r.p.m.

Prob. 2. For one of the following double groups of data find speed N_2 of second wheel and linear velocity V of pitch-circle circumferences:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	
z_1	24	27	56	23	24	37	40	25	teeth
z_2	54	56	84	67	45	90	64	96	teeth
N_1	300	400	175	210	240	330	160	260	r.p.m.
p_d	3	5	2	6	3	8	4	2	d.p.

Prob. 3. Given teeth z and module m, find diameter and circular pitch in both millimeters and inches:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
z	80	25	72	37	96	220	156	65	48	56
m	4	8	8	6	4	2	3	5	16	12

Prob. 4. Look up in several mechanical engineers' hand books the subject of gear tooth dimensions, and write a brief report as to what quantities are given by tables.

Prob. 5. Look up in the Brown & Sharpe catalog the list of diametral pitches for which standard tooth-milling cutters are supplied.

Prob. 6. For a speed of M miles per hour, of K kilometers per hour and of N knots, find factors by which to get velocity V' in feet per minute and V'' in feet per second. (One knot equals 6,080 ft. per hour.)

$$V'' = 1.46\dot{6}\,M \text{ or } 1.4\tfrac{2}{3}\,M.$$

Prob. 7. If t is the time between mile posts (in seconds) as observed from a railway train, prove that the speed in miles per hour is given by the formula $M = 3{,}600 \div t$; what is the corresponding constant for K with posts one kilometer apart?

Prob. 8. If M is the speed of a vehicle in miles per hour and D the diameter of its wheels in inches, prove that their turning speed is

$$N = 336.13\frac{M}{D} \text{ r.p.m.} \qquad (10)$$

Then find the corresponding constant with speed K in kilometers per hour, for D first in millimeters, then in inches.

(*h*) **Various Gear Pairs.**—The name spur gear implies, first, that axes are parallel or pitch surfaces cylindrical; second, that teeth are solid with rim, as against the old-fashioned cog wheel having inserted wooden teeth; third, that the teeth are axial or stand squarely across the rim. If gears have inclined or helical teeth, as in Fig. 61, but are to be run as spur gears (in contrast with Fig. 67), they may be called spur-helical gears. The purpose in the use of such teeth is to secure smooth and quiet running under severe conditions of pressure and speed.

In a pair like 2 and 3 of Fig. 55 or Fig. 60 the small wheel is called a **pinion.** The straight toothed bar 4 in Fig. 60 is a **rack.** A large toothed ring, made in segments to be bolted to a turn-

table base or to a warship turret, is often called (properly enough) a circular rack. One concept of the straight rack is to consider it a segment of a gear rim of infinite radius. Figure 62 shows the internal gear and pinion, the larger wheel having its teeth formed on the inside of an annular rim with side support.

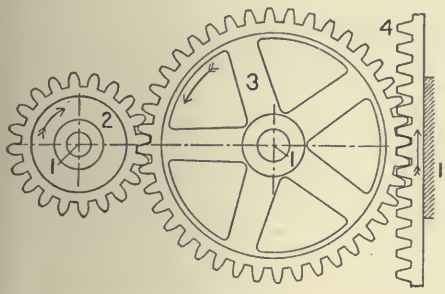

FIG. 60.—Spur gears and rack.

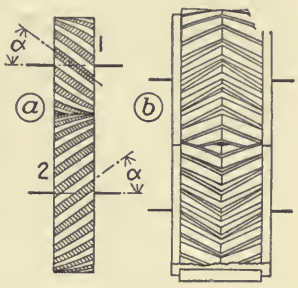

FIG. 61.—Oblique or helical teeth.

The scheme of representation for spur-helical gears in Fig. 61 (*a*) is to use sections made by the pitch cylinders, showing the teeth cross-hatched, the spaces white. Evidently the inclination of teeth tends to make the gears move sidewise, requiring considerable side pressures at the bearings to hold them in plane. For heavy service this difficulty is overcome and the pair

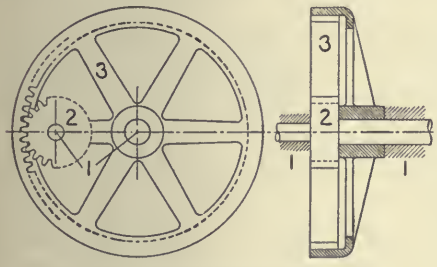

FIG. 62.—Pinion and annular gear.

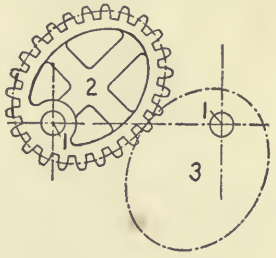

FIG. 63.—Elliptical gears.

made self-contained as to force action by using double-inclined teeth and getting the "herringbone" gears in Fig. 61 (*b*). This example is a pair of heavy rolling-mill pinions; and an added detail is the "shrouding" of the teeth, by casting with them a side flange that comes halfway up their ends for greater strength.

The commonest example of non-circular gears is the pair of equal elliptical wheels shown in Fig. 63. Geometrically, the

class which they represent is the general one, with circular pitch curves as a particular case. From the viewpoint of construction and use, however, the terms should be reversed and noncircular gears counted the special class. They can be made in a great variety of forms and are a good deal used in some kinds of machinery.

(*i*) **Gears with Axes Not Parallel.**—Bevel gears, shown in Figs. 64 and 65, are used for transmission between shafts that

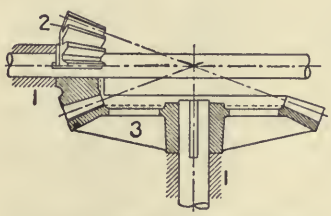

FIG. 64.—Bevel gear and pinion.

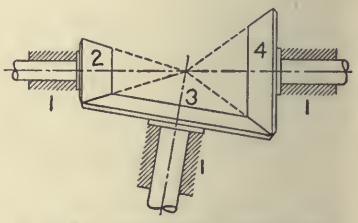

FIG. 65.—Gears with oblique axes.

lie in the same plane and have intersecting axes. The pitch surfaces of these gears are cones and the teeth taper across the rim, all lines converging to the intersection point. Generally the shafts are at right angles, and the speeds of rotation are either unequal with a pair like Fig. 64 or equal with the "miter" gears as shown in Fig. 68. The less frequent condition of oblique axes is represented by Fig. 65. In skeleton outlines of gear

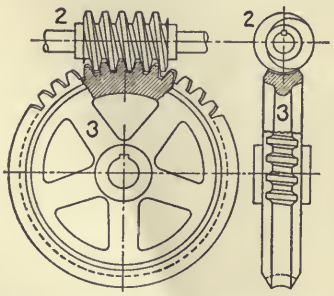

FIG. 66.—Worm and worm wheel.

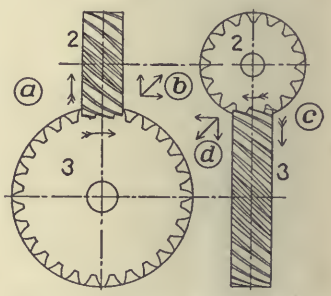

FIG. 67.—Helical or spiral gears.

trains the wheels are drawn without teeth, or are shown in their pitch-surface form, either as in Fig. 65 or after the more elementary fashion of Fig. 68.

Between shafts that cross without intersecting motion may be transmitted by means of the gear pairs in Figs. 66 and 67. The old-style worm and worm-wheel pair generally gives a big

reduction of speed; here a turn of worm 2 advances wheel 3 by only one tooth. With helical gears (often inaccurately called "spiral") the speed ratio is not so great, but has the same range of magnitude as with spur and bevel gears. The real distinction is not, however, in speed ratio—for the newer type of worm pair used in automobile transmissions may have as low a ratio as 4 or 5 to 1—but rather in the way that the rim of the worm wheel is curved or hollowed out so as to conform to the worm: as a result properly formed worm-wheel teeth have line contact, while the helical teeth in Fig. 67 have only spot contact.

With neither of these forms is it necessary for the axes to be at right angles, although that is the usual condition. There is yet another type of gears for axes which cross without intersecting, namely the hyperboloidal "skew bevel" pair. These are geometrically correct but are so difficult to form accurately

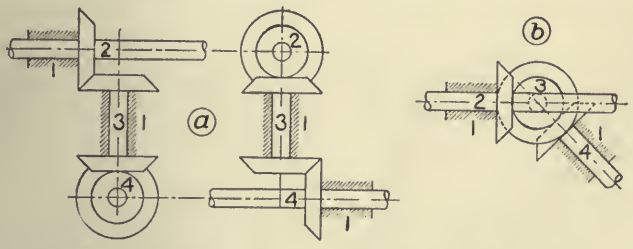

FIG. 68.—Bevel-gear drive for shafts that do not intersect.

that they rank rather as mechanical curiosities than as a practical form. They are discussed in Chap. VIII as the most general case of gear teeth in space.

The relation of inclination of teeth to directions of turning in a pair of helical gears is likely to be a puzzling matter at first sight, but the scheme used at (*b*) and (*d*) of Fig. 67 clears it up. In view (*a*) the feathered arrows show the direction of motion of the front of 2 and top of 3; and when these arrows are drawn from a common point, at (*b*), the diagonal of their rectangle points across the teeth of the wheel (No. 2) that appears "edge-on" in the drawing.

A much used and very flexible way of connecting shafts that do not intersect, shown in Fig. 68, is appropriately given here even though it is really a short gear train, involving two pairs of bevels and a short intermediate shaft. Sketch (*a*) shows two views of the arrangement, for the particular case of shafts 2 and 4 at

right angles. Top view (*b*) shows how the angle between 2 and 4 can be changed, provided that the common perpendicular to their axes remains in the axis of intermediate shaft 3.

(*j*) **Chain Gearing** bears the same relation to rope and belt gearing that toothed wheels do to friction wheels, substituting positive for uncertain motion. This is even done with common oval-link chain, in hand-power hoists and in hand drives for the horizontal movements of small cranes: the wheel rim is formed with pockets into which the links fit, so that wheel and chain have a definite hold upon each other.

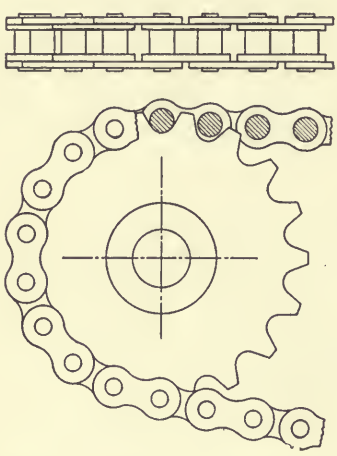

FIG. 69.—Pin chain and sprocket.

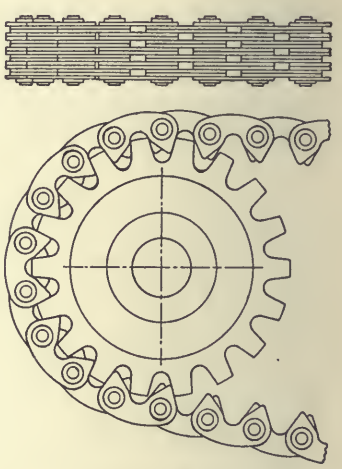

FIG. 70.—Tooth chain and wheel.

For transmission of power specially adapted chains are used, of the two types shown by Figs. 69 and 70: note the difference in chain construction, best seen in the top views. The teeth of chain wheels or **sprocket wheels** have a mere holding function, for there is no such action of rolling and slipping under pressure throughout a range of contact as with gear teeth. Pins or teeth of chain settle into place as chain wraps on wheel, swing out as it runs off. The problem of design and construction is to have this action take place quietly, at high speed and after long service: it has been successfully solved and chain drives are very effective. The figures show only general form, omitting much important detail, especially of chain joints.

With chain gearing shafts may be much closer together than with belts: in fact this close connection is strongly indicated,

for moving weight is a technical and cost of chain an economic argument against long drives.

SECTION 5. CAMS AND CAM GEARING

As stated in Sect. 2 (*k*), the function of a cam is to produce a reciprocating motion, with great flexibility of adaption to requirement. We shall first show how this is done, by a number of examples, then formulate a descriptive definition.

(*a*) **Rotating Cams.**—In Figs. 71 and 72 are shown typical arrangements of cams for actuating gas-engine valves; disc 2,

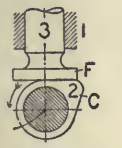

FIG. 71.

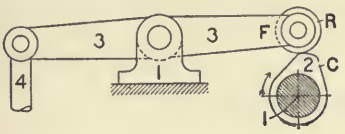

FIG. 72.

Cams used with gas-engine valves.

of irregular (noncircular) profile, is the cam and piece 3 to which it gives motion is the follower. The latter has a flat foot and slipping contact in Fig. 71, but by the use of a roller the friction and wear due to slipping can be eliminated, as in Fig. 72. These two cams are formed by building out a symmetrical extension on one side of a circular disc: they give the follower a quick up-and-down movement, then a longer period of rest or "dwell." Note the two types of follower, sliding and oscillating. Cam, follower and frame make up a three-member unit, roller R in

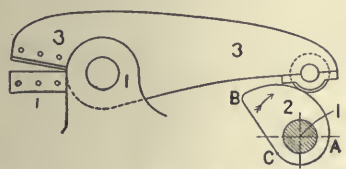

FIG. 73.—Cam-driven alligator shear.

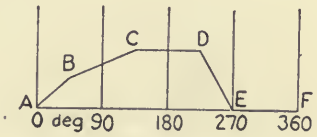

FIG. 74.—Motion diagram.

Fig. 72 hardly counting as a kinematic element—see Sect. 28 (*l*).

The powerful shear outlined in Fig. 73 shows a cam in heavy service and introduces nonsymmetry of action: the cutting stroke, produced by side AB of the cam and made at a slow, uniform speed, is followed by a quick return along BC and rest from C to A.

In order to get an idea of methods and results in the laying out of cams, consider the problem outlined in Fig. 74, where the base is angular position of cam and the ordinate up to broken line ABCDEF is displacement or "travel" of follower.

The inclined straight lines of uniform rise (AB, BC) and fall (DE), when transferred to the polar diagram, Fig. 75, are easily laid out as arcs of the simple spiral of Archimedes, in which radius vector varies directly as angle. A knife-edge follower can use the transferred movement curve as plotted, but the introduction of a roller complicates the problem somewhat. Thus in Fig. 76 the dotted curve with primed letters is first laid down (larger by the radius of roller), then the cam profile is drawn equidistant to this, as the envelope of a series of roller arcs struck from points on the primary curve.

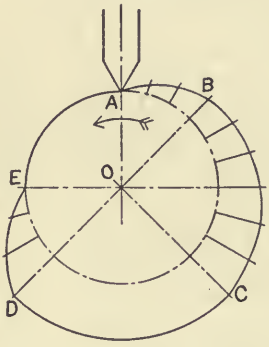

FIG. 75.—Knife-edge follower.

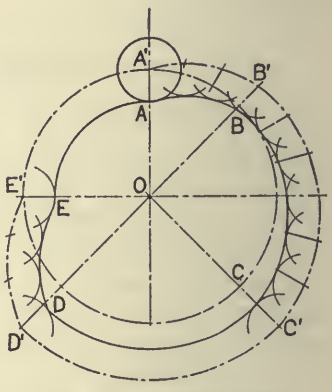

FIG. 76.—Roller follower.

An important item to be considered when the matter of cam design is taken up in detail is the action at transition points A, B, etc., where motion here changes abruptly. For rapid operation such corners or cusps should be rounded or "eased off." In this connection note that the cam shown in Fig. 71 is not of correct shape for use with a flat-foot follower: its straight "side" works very well with a roller, but in the position sketched it is just about to start the flat follower with a severe jolt: Fig. 81 suggests the proper shape of profile.

(*b*) **Positive Cams.**—In the mechanisms thus far shown contact between cam and follower is maintained by an external force action, or the pair is force-closed—in Figs. 71 and 72 by a stiff spring, in Fig. 73 by the weight of the lever. By adding an

equidistant outer profile surface, or by cutting a cam groove or slot in the side of a plate as shown in Fig. 77, the action is made mechanically positive. The same effect is secured when the groove is cut in a cylinder, after the manner of Fig. 78. The developed slot profile at (*b*) is a diagram just like Fig. 74, so that the primary motion curve can be directly applied to the cylindric cam, as against a radial replotting for the disc cam as shown in

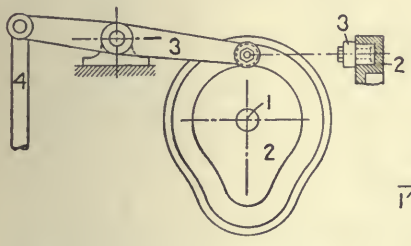

FIG. 77.—Grooved positive cam.

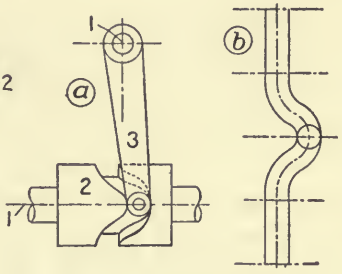

FIG. 78.—Cylinder cam.

Figs. 75 and 76. Conditions of rolling and the question of securing line contact are, however, less simple here than with parallel axes and coplanar motion.

(*c*) **Tripping and Oscillating Cams.**—Tripping cams, which alternately drive and release the follower, are shown in Figs. 79 and 80. The trip hammer, with its cam shaft driven by a water wheel, belonged to the early days and small beginnings of

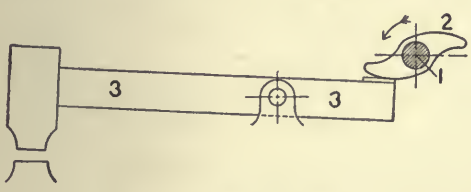

FIG. 79.—Trip hammer.

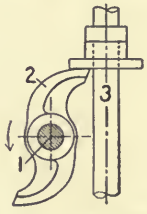

FIG. 80.—Ore-stamp cam.

the iron industry. The ore stamp partially outlined is a standard machine for crushing hard ores such as gold-bearing quartz.

All the cams so far shown have complete and continuous rotation, but sometimes the cam oscillates or reciprocates as well as the follower. The examples in Figs. 81 and 82 are from steam-engine valve gears, for valves of the lift or "poppet" type. In the first case the two extreme or limiting positions are drawn, to

show the range of motion: this oscillating disc cam with flat-foot follower is called a wiping cam. Figure 82 shows a heel cam, actuated by crank 5 and rod 4. The equivalent straight sliding cam in Fig. 83 is rarely used—but note the modification of it in Fig. 85.

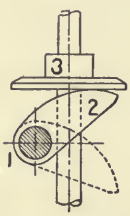

FIG. 81

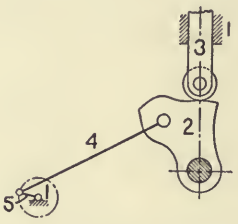

FIG. 82.

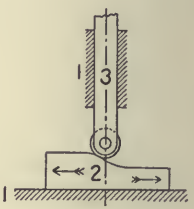

FIG. 83.

Oscillating or reciprocating cams.

(*d*) **Cams with Double Contact.**—In the last two examples, which by no means exhaust the possibilities of this type of machine element, there are distinctly two higher-pair contacts. The rotating equidistant or constant-breadth cam in Fig. 84 works between parallel follower faces and gives to slide 3 a reciprocating motion with a pause at each end of its stroke. In a general way this represents a large class of possible arrangements, of which a few are practically useful. Figure 85 shows a common form of belt shifter for metal planers—see Fig. 198 for the place

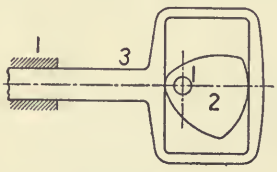

FIG. 84.

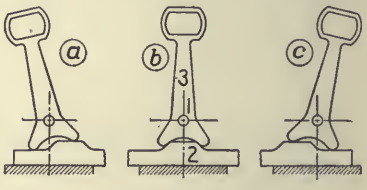

FIG. 85.

Double-contact positive cams.

of its application. Slide 2 is pushed by the planer table at each end of its stroke, and there are two of these arms with double toe cams, the action being that one belt is shifted clear off its tight pulley before the other starts on.

The following descriptive definition covers the ground pretty thoroughly:

A cam is a machine piece with a profiled edge of variable or irregular curvature. This working surface is formed either to a

plane curve when on the rim of a disc or sector or in the side of a bar, or to a curve of helical type when wrapped on a cylinder. Driving through line contact and itself receiving either continuous rotation or a simple oscillation derived from a linkage, the cam gives to its mating piece or follower some desired periodic motion. In developing the form and timing of this motion there is far greater freedom and flexibility than can be had with the linkage type of mechanism.

SECTION 6. MACHINE FORCES

(*a*) **Force Determination.**—A few simple examples of the graphical representation and determination of machine forces will now be given, in order to develop certain ideas which will be needed in advance of the fuller treatment of the subject in Part II. Figure 86 shows a straight lever with equal arms AO and BO and with parallel working or moving forces P and Q.

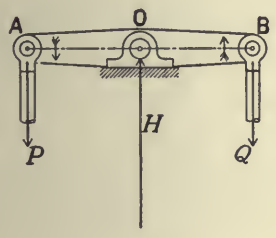

FIG. 86.—Parallel forces.

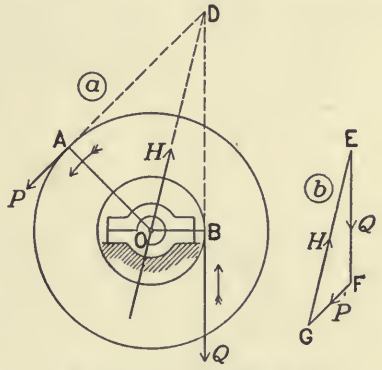

FIG. 87.—Intersecting forces.

By "moving forces" are meant those which act upon moving bodies or are applied at moving points: only such forces enter into the performance and transmission of work. For equilibrium under P and Q there must be an equality

$$P \times \text{AO} = Q \times \text{BO} \qquad (11)$$

between the moments about axis O; and at this axis or bearing must act a third force H, parallel to P and Q, equal to their sum, and opposing them in direction.

In Fig. 87 is presented the more usual case of three nonparallel forces in equilibrium. They must meet in a common point of

intersection D and their values are then found, when one is known, by means of the force triangle at (*b*). To construct this, lay out EF equal and parallel to known force Q, then draw FG parallel to P or AD and EG parallel to H or OD: intersection G fixes the length FG for P and GE for H. Note that the arrowheads point in circular order around the triangle.

The third example, Fig. 88, is more comprehensive in that it shows the forces acting upon several members of a machine. Force P corresponds with P_e in Fig. 3 and the action of the connecting rod is simplified by considering only the forces along its center line, ignoring its inertia. Triangle ABD at (*b*) shows the forces that are in equilibrium on the crosshead and meet at pin center W. Line AD, taking it in the direction DA for the pres-

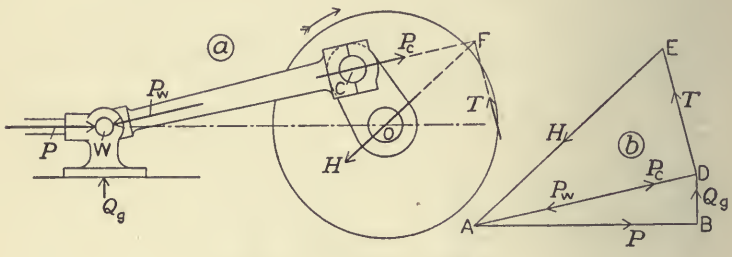

FIG. 88.—Forces in engine.

sure of rod on wrist pin, then reversing to AD for the pressure on the crank pin, is in itself a complete force diagram for the rod. Triangle ADE, with AD or P_c as known side, determines the forces on the crankshaft.

(*b*) **Working Forces and Holding Forces.**—There is a clear enough distinction between the various moving forces in these diagrams and the purely holding forces which are marked H in each case. But very often a moving force is not wholly a working force, and this point is more fully illustrated by Figs. 89 and 90. The gear teeth in Fig. 89 are of the involute form (see Sect. 32), with the consequence that the line of tooth pressure has a constant position, at a certain angle with the line of centers BC. As drawn, gear B is driving C; then pressure P is a driving force exerted by B on C while the equal and opposite reaction Q is a load force impressed by C on B.

For convenience force P is resolved at the pitch point A rather than at the changing point of actual tooth contact. Its tangen-

tial component T, tangent to the pitch circle, is the real working force, like P and Q in Fig. 87. The normal or radial component N, since there is no motion of a point of contact in its direction, does not enter into work performance.

The force P applied to wheel AC at or through A is opposed by a parallel and equal holding force at the bearing. This H may be resolved into two components H_t and H_n, corresponding with T and N. Then either P and H or T and H_t form a turning couple (of the same value), while N and H_n balance each other completely.

The other example, Fig. 90, is a detail from Fig. 88, giving a fuller showing of the force action upon the crank arm. This is a more general case than Fig. 89 because force P is constantly

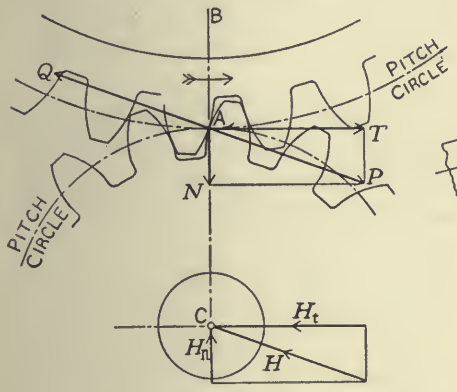

FIG. 89.—Gear-tooth action.

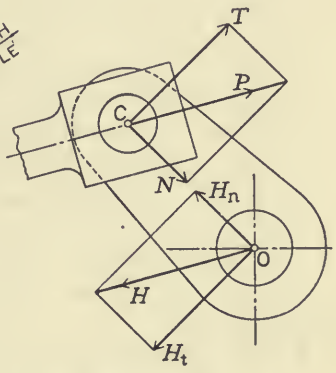

FIG. 90.—Force action on crank.

changing in its value and in its direction with respect to radius OC. Otherwise the force resolutions and relations are the same as in Fig. 89.

The guide-bar pressure Q_g, Fig. 88, looks like component N, Figs. 89 and 90, but is really in a class with bearing pressure H, being a constraining force of frame on moving member. It is like the N component, however, in that its point of application moves along a working path to which it is perpendicular. Either guiding force or normal component is a good name for this kind or portion of contact pressure, which moves sidewise but not at all in its own direction.

A chief reason for thus distinguishing working forces is found in the fact that such forces are connected by work and velocity

relations which can be applied independently of the more complete graphical method of Figs. 86 to 88.

(*c*) **Kinds of Machines Forces.**—The various kinds of forces that enter into machine actions may be systematically described as follows:

1. The contact pressure of solid bodies upon each other: this type of force belongs particularly to the machine itself, in the interaction of its parts. Occasionally prime-moving forces are of this kind, when machinery is driven by men or animals; and in all manufacturing operations which modify the form of bodies the useful resistance is of the nature of contact force.

2. The static pressure of confined fluids: this serves as driving force in all piston engines or equivalent volume-displacing machines and as useful resistance in pumps and compressors. Sometimes a confined body of fluid is used as a transmissive element and then its pressure has the effect of a transmissive force.

3. The kinetic pressure of fluids: in all kinds of turbines, centrifugal pumps and the like the forces which perform work are really the inertia forces of jets or streams of fluid which are in the act of being compelled to change either the direction or the speed of their flow.

4. The attractive and repulsive electric or magnetic forces in motors and generators, which act without material contact or across space.

5. Weight or gravity, or the attraction of the earth exerted upon the mass of bodies: in water powers this is the real driving force, even though applied indirectly in the form of fluid pressure. It is the useful resistance in most pumping operations and in all kinds of hoisting machines and elevating operations. Also, the weight of machine members enters into the transmissive, contact pressures between them and sometimes into the detail of work performance.

6. Inertia manifests itself as a force resisting all changes in the velocities of machine parts and of load bodies. It may have large effects in modifying work performance and in developing stresses within machine members. In transportation service it forms a part of the useful load when a vehicle, train, ship, etc., is being brought up to speed; and it acts in the direction of the driving effort when that is removed and a body of this sort is being slowed down to rest.

7. Friction, or the resistance to relative motion of material bodies in contact with each other, forms a part of all machine actions. It is a useful force only when its purpose and function is to prevent relative motion; otherwise it is a wasteful, work-absorbing resistance. Only in transportation can the overcoming of friction be accounted a part of the useful effect.

The preceding classification does not include the internal forces or stresses within bodies by virtue of which they resist external forces tending to change their form. Despite the fact that the common machine forces get at each other and come into equilibrium only through the medium of these stresses, it is entirely feasible and very convenient to separate the two kinds of force in study and determination. As has been fully stated in Sect. 1, the subject of internal stress is taken to belong to the design of machine parts for strength and stiffness.

(*d*) **Force Concentration.**—In general the forces which are represented by straight lines in diagrams like Figs. 86 to 88, and commonly conceived as acting along such single lines, are not really thus concentrated, but are distributed over surfaces or over the mass bulk of bodies. In joints belonging to the lower order of kinematic pairs there is always a considerable surface of contact. In higher pairs the surface may be very small, but because of elastic compression and the interposition of a film of lubricant it is never quite the mere geometric line or point. Fluid pressure must of course be distributed, as represented in Fig. 3. An action like that of gravity or inertia has really the form of a little force on each little bit of mass. The replacement of actual distributed force by equivalent concentrated force is, however, a very great convenience and need involve no lack of clearness of thought.

SECTION 7. WORK AND POWER RELATIONS

(*a*) **Measurement of Work and Power.**—When a force P drives a body through a distance S against an equilibrant resistance Q it performs work of which the quantity is measured by the product $P \times S$ or $Q \times S$. The unit of this work is the foot-pound, so that P must be in pounds and S in feet. In the metric system the corresponding units are the meterkilogram, the kilogram and the meter.

In connection with machinery, rate of work is commonly a more important quantity than absolute amount irrespective of

time. If t is the time of movement S the work per unit of time is evidently

$$U = P \times \frac{S}{t}, \tag{12}$$

the quotient S/t being equal to velocity V. This rate U will be in footpounds per second or per minute according to the terms of V.

The rate of work performance by a machine is called its power and is usually measured in terms of the horsepower unit. The horsepower is an arbitrarily established rate of 550 fp. per second or 33,000 fp. per minute or 1,980,000 fp. per hour, these numbers all expressing the same ratio of work to time. The metric horsepower, *force de cheval* or *pferdestarke*, is 75 mkg. per second or 4,500 mkg. per minute. With P in pounds or kilograms and V in feet or meters per minute, the horsepower developed or applied is then,

$$H_{\text{eng}} = \frac{PV}{33{,}000} \text{ or } H_{\text{met}} = \frac{PV}{4{,}500}. \tag{13}$$

If V is a circumferential velocity of rotation, as defined in Eq. (8), and if a tangential driving force or resistance T is in action, the horsepower is, with English units,

$$H = \frac{2\pi RNT}{33{,}000} = \frac{RNT}{5{,}252.1}. \tag{14}$$

Prob. 9. If a horse is walking at the rate of 2.5 miles per hour and is developing one horsepower, what pull is he exerting?

Prob. 10. Find the value of the meterkilogram in footpounds and of the footpound in meterkilograms, having given 1 m. = 39.37 in. and 1 kg. = 2.2046 lb. 1 mkg. = 7.233 fp. 1 fp. = 0.13825 mkg.

Prob. 11. Find the value of the metric horsepower in footpounds per minute and in terms of the English horsepower.

32,548.5, say 32,550 fp. 0.9863 hp.

Prob. 12. (*a*) If a locomotive draws a train at 60 miles per hour and is developing 1,000 horsepower at the drawbar, what pull is it exerting? (*b*) If a locomotive at 95 km. per hour is exerting a pull of 3,000 kg., what horsepower (metric) is it developing?

Prob. 13. What horsepower is being usefully applied by a pump which in one minute raises 500 cu. ft. of water to a height of 200 ft.? One cubic foot of water weighs 62.4 lb.

Prob. 14. If a pump delivers water to a height of 120 ft. at a rate equivalent to 20,000,000 gal. in 24 hours, what is its useful horsepower? One gallon of water weighs 8.33 lb. Find weight of water per minute, then horsepower.

Prob. 15. On the rim of a pulley 60 in. in diameter and running at 250 r.p.m. a belt exerts an effective pull P of 2,000 lb.: what is the linear speed of the belt and what the horsepower transmitted?

It is well to work out a problem of this sort step by step, somewhat as follows although not always with quite so short steps:

Circumference of 60-in. or 5-ft. pulley...	15.71 ft.
Speed of belt.....	$15.71 \times 250 = 3{,}927$ ft. per min.
Work per minute.....	$3{,}927 \times 2{,}000 = 7{,}854{,}000$ fp.
Horsepower.....	$7{,}854{,}000 \div 33{,}000 = 238.0$ hp.

Prob. 16. A belt is to transmit 60 hp. from a pulley of 48 in. diameter running at 300 r.p.m. (*a*) What working pull must be transmitted? (*b*) And if each inch of width of belt is good for 60 lb. of this pull, how wide shall the belt be? (*a*) 525 lb.

Prob. 17. For N in r.p.m. and R in inches, prove that

$$TR = 63{,}025\frac{H}{N}. \tag{15}$$

This very convenient and useful formula should be memorized. It bears, of course, a close relation to Eq. (14).

Prob. 18. For R in millimeters, T in kilograms and H in metric horsepower, prove that the constant in a formula corresponding with Eq. (15) is 716,200.

(*b*) **Angular Velocity and Torque.**—In mechanical calculations and deductions rate of turning is often called angular velocity. In measuring this the unit of angle is not the degree but the radian, which is the angle whose subtending arc is equal to the radius, so that one turn or revolution equals 2π radians. Angular velocity, for which the Greek letter omega is the accepted symbol, is regularly expressed in radians per second and its value is

$$\omega = 2\pi\frac{N}{60} = 2\pi n, \tag{16}$$

small n standing for revolutions per second.

Multiplying both sides of Eq. (16) by R in feet we get

$$\omega R = 2\pi nR = V \text{ ft. per sec.}, \tag{17}$$

which is in accord with Eq. (8). Physically, if a radius of length R turns through one radian its outer end will necessarily traverse R feet. Again, from Eq. (17),

$$\omega = \frac{V}{R} \tag{18}$$

or angular velocity equals linear velocity divided by radius, provided that both are measured in the same linear unit.

In terms of Eqs. (12) and (8), using tangential force T instead of general force P, the expression for work rate would be

$$U = T \times V = T \times 2\pi RN \text{ ft. per min.}$$

A slight rearrangement changes this to

$$U = 2\pi N \times TR = 60\omega \times M, \tag{19}$$

symbol M standing for turning moment or torque TR. Work rate per second is then angular velocity $\times$ torque in pound-feet and the expression for horsepower becomes

$$H = \frac{2\pi n}{550} \times TR = \frac{\omega M}{550}. \tag{20}$$

Torque has the same factors as work, force and length, so that the compound units of measurement are much alike in form and naming. To distinguish them the work unit is called the foot-pound or sometimes the inch-pound, the unit of moment the pound-foot or pound-inch. Machine dimensions being commonly given in inches, moments are generally stated in pound-inches; and for this unit the divisor in Eq. (20) changes to 6,600.

Prob. 19. A toothed gear with an effective or pitch-circle diameter of 90 in. is to transmit 200 hp. at 40 r.p.m. What will be the tangential tooth pressure and what the torque on the shaft? 7,003 lb. 315,125 lb. in.

Prob. 20. If a gear 36 in. in diameter is good for a tangential tooth pressure of 1,600 lb., what horsepower will it transmit at 180 r.p.m.?

82.25 hp.

Prob. 21. A belt running on a pulley 16 ft. in diameter has a linear speed of 4,800 ft. per min. and transmits 425 hp. Find (*a*) r.p.m. of wheel, (*b*) effective belt pull, (*c*) torque on shaft. (*b*) 2,922 lb.

Prob. 22. If an engine expends 575 hp. in driving a direct-connected electric generator at 100 r.p.m., what is (*a*) the torque transmitted by the shaft and (*b*) the total tangential resistance (magnetic force) distributed around the rim of a rotor 15 ft. in diameter? (*b*) 4,027 lb.

Prob. 23. An engine with crank 21 in. long makes 100 r.p.m. What is (*a*) the angular velocity of the crank and (*b*) the linear velocity of the crank pin in feet per second? (*c*) What average tangential force T must act on the crank pin in order to develop 580 hp.?

(*a*) 10.472 rad. (*b*) 18.33 f.p.s. (*c*) 17,407 lb.

Prob. 24. An engine with crank 7.5 in. long makes 240 r.p.m. and its shaft carries an armature of 48-in. diameter. Find angular velocity of crank, linear velocity of crank pin, mean turning force T on crank pin for 120 hp., also torque and tangential resistance on armature for same power. (*c*) 4,202 lb.

Prob. 25. An engine with crank 600 mm. long makes 96 r.p.m. Find angular velocity of crank and linear velocity of crank pin. What must be the value of average T on crank pin (kilograms) for the development of 600 metric horsepower? Equation (15) with the constant in Prob. 18 will be useful here. (*b*) 6.032 m.p.s.

(*c*) **Pressure Work.**—In piston engines or pumps work is done or absorbed by a fluid pressure. Commonly this pressure varies almost continually throughout the cycle of piston movement; but for purposes of power calculation the actual, variant pressure can be replaced by an average, uniform pressure of equivalent overall effect. The typical cycle for one face of an engine piston consists of a forward or working stroke under the push of a high mean forward pressure p_{mf} and a return stroke against a low mean back pressure p_{mb}. In a pump or compressor the forward stroke is against the high p_{mf}, while p_{mb} helps to push the piston back to its starting point. In either case the mean effective pressure or m.e.p. is

$$p_m = (p_{mf} - p_{mb}); \tag{21}$$

and work is calculated as if done by this mean pressure in acting through the one forward or working stroke of the cycle. Note that there is here an idea slightly different from the one represented in Fig. 3, where the effective force P_s (at any instant) is the difference between simultaneous pressures on the two sides of the piston.

The common units of pressure in power engineering are the pound per square inch and the kilogram per square centimeter. With area A of the piston expressed in square inches or square centimeters, the whole effective force P is then $p_m A$ pounds or kilograms. And with N working strokes per minute, of length S in feet or meters, the piston moves a distance NS with or against P. The horsepower is therefore,

$$H = \frac{p_m ANS}{33{,}000} \text{ (English) or } H = \frac{p_m ANS}{4{,}500} \text{ (metric).} \tag{22}$$

In the ordinary double-acting steam engine there will be a value of p_m and of A for each piston face; then partial values of H will be calculated and combined (added together) for the whole machine. Or average values of m.e.p. and area may be used, with a factor representing the number of piston faces or cylinder ends—which factor will be 2 for the single cylinder, 4 for the double (two-cylinder) locomotive, and so on.

(*d*) **The Pressure-volume Measure of Work.**—Let a pressure p push a piston of area A through a distance S. Under the definition *work* = *force* × *distance* we have

$$U = pA \times S, \tag{23}$$

letting U here stand for work quantity rather than work rate. A rearrangement of factors gives

$$U = p \times AS = pV, \tag{24}$$

where AS or V is the volume displaced or swept through by the piston. This pressure-volume measure of work is a very useful concept in some lines of investigation. Do not confuse work quantity pV (V = volume) with work rate PV (V = velocity).

In English-unit practice piston areas are commonly expressed in square inches and are so used in calculations by Eq. (22). But in Eq. (24) volume AS should be in cubic feet or A in square feet; to compensate for this change the measured pressure p must be multiplied by 144, changing it to pounds per square foot. The general statement covering consistency of units is as follows: in the pressure-volume measure of work pressures must be given as on the square of the linear dimension of the work unit (foot-pound or meter-kilogram) and volumes must be in terms of the cube of that dimension. This calls for pounds per square foot with cubic feet and kilograms per square meter with cubic meters.

This method of calculating work or work rate can be used for other volume displacements beside that made by a piston. Thus if a fluid under pressure p (lb. per sq. in.) flows through a pipe at the rate of V cu. ft. per min., the current may be said to carry or transmit the horsepower

$$H = \frac{144pV}{33{,}000}. \tag{25}$$

Prob. 26. A steam engine of 14-in. diameter and 15-in. stroke, with 2¼-in. piston rod, runs at 240 r.p.m. With m.e.p.'s of 36.8 lb. in the head end and 37.5 lb. in the crank end, what is the work done on each piston face per revolution and what the horsepower developed?

Note. The head end of the cylinder is the one away from the shaft, at the left in Fig. 3, the crank end is the one toward the shaft, containing the piston rod.

Solution:

Area of 14-in. circle, or of head-end piston face	153.9	sq. in.
Area of 2¼-in. circle, or of piston rod	4.0	sq. in.
Net area of crank-end piston face	149.9	sq. in.
Stroke, 15 in. =	1.25	ft.

Work per revolution,

Head end	$36.8 \times 153.9 \times 1.25 =$	7,079 fp.
Crank end	$37.5 \times 149.9 \times 1.25 =$	7,027 fp.
Total		14,106 fp.

Horsepower

$$H = \frac{14{,}106 \times 240}{33{,}000} = 102.59 \text{ hp.}$$

Or, using the horsepower formula, Eq. (22),

$$H_h = \frac{36.8 \times 153.9 \times 240 \times 15}{33{,}000 \times 12} = 51.49 \text{ hp.}$$

$$H_c = \frac{37.5 \times 149.9 \times 240 \times 15}{33{,}000 \times 12} = 51.10 \text{ hp.}$$

Total power H = 102.59 hp.

Prob. 27. A 24 by 48-in. engine, 4-in. piston rod, 80 r.p.m., m.e.p. 38.5 head and 39.2 crank, is developing what horsepower, in each end and total? 336.4 total.

Prob. 28. A steam engine of 450-mm. diameter and 600-mm. stroke, with 80-mm. piston rod, running at 176 r.p.m. with m.e.p.'s of 2.86 in head end and 2.84 in crank end (kg. per sq. cm.) is developing what horsepower, in each end and total? 209.4 met. hp. total.

Prob. 29. A common two-cylinder locomotive has pistons 20 in. in diameter by 26-in. stroke, with 4-in. piston rods. Running at 60 miles per hour with 72-in. driving wheels and with an average m.e.p. of 55 lb., what horsepower is it developing in the cylinders?

Solution:

Full area of head-end piston face..... 314.2 sq. in.
Net area of crank-end piston face..... 301.6 sq. in.
Average piston area..... 307.9 sq. in.

Find rotary speed of engine by Eq. (10), Prob. 8,

$$N = 336.13 \times 60/72 = 280.1 \text{ r.p.m.}$$

Then by Eq. (22), with factor 4,

$$H = \frac{4 \times 55 \times 307.9 \times 280.1 \times 26}{33{,}000 \times 12} = 1{,}245.7 \text{ hp.}$$

Prob. 30. Two-cylinder freight locomotive, cylinders 22 by 28 in., 4-in. piston rods, 56-in. drivers: at 25 miles per hour what is r.p.m. of wheels; and with 62 lb. m.e.p. what horsepower is being developed in the cylinders?

Prob. 31. Four-cylinder compound locomotive, two high-pressure cylinders 15.5-in. diameter, two low-pressure cylinders 26-in. diameter, all with 26-in. stroke and 3-in. piston rods; wheels 79-in. diameter.

(*a*) When running at 240 r.p.m., what is the speed in miles per hour?

(*b*) With average m.e.p.'s of 67.5 lb. in the high-pressure cylinders and 25.7 lb. in the low-pressure cylinders, what horsepower is being developed in each pair and in the whole engine?

Treat each pair, high and low, like the plain duplex engine in the last two problems.

(*a*) 56.41 m.p.h. (*b*) High 787.7, low 853.1, total 1,640.8 hp.

Prob. 32. A two-cylinder, single-acting, four-stroke cycle gas engine, with cylinders 12-in. diameter by 18-in. stroke, is developing what horsepower at 224 r.p.m. on an m.e.p. of 75 lb.? 86.37 hp.

A single-acting engine has pressure work performed on but one face of the piston or in but one end of the cylinder. With a four-stroke cycle there is one working stroke out of four, or one in each two revolutions of the crankshaft. The double engine will therefore have a working stroke to each revolution. The piston is of full area, since the cylinder end used corresponds with the head end of a common steam engine.

Prob. 33. A duplex double-acting pump, with plungers of 10-in. diameter by 24-in. stroke, is making 45 r.p.m. against a water m.e.p. of 160 lb. per sq. in. What horsepower is being applied by its plungers?

This pump is similar to the locomotive in having four working plunger faces and a factor 4 in Eq. (22). It is so arranged that no piston rods enter into the problem. 137.1 hp.

Prob. 34. A pumping engine has three single-acting plungers of 40-in. diameter and 60-in. stroke, running at 24 r.p.m. against an m.e.p. of 80 lb. What horsepower is the pump applying? Here there are three working strokes, as of one plunger face, per revolution. 1,096.6 hp.

Prob. 35. What is the plunger displacement or nominal discharge of the pump in the last problem, expressed (*a*) in cubic feet per minute, (*b*) in gallons per minute, (*c*) in gallons per 24 hours. The U. S. gallon of 231 cu. in. is intended. (*c*) 33,840,000 gal.

Prob. 36. If a pump is delivering water at the rate of 30,000,000 gallons in 24 hours against an effective pressure of 75 lb. per sq. in., what horsepower is it applying? Use here the method of Eq. (25). 911.5 hp.

Prob. 37. If a pump is delivering water at the rate of 10,000 litres per minute against a pressure of 5.25 kg. per sq. cm., what is its useful horsepower? 116.7 met. hp.

Prob. 38. Given, 1 kg. = 2.2046 lb., 1 sq. in. = 6.4515 sq. cm., prove that 1 kg. per sq. cm. = 14.223 lb. per sq. in.

(*e*) **Transmission of Work and Power.**—The last four articles have shown very fully how to calculate the amount of rate of work done, but have gone no farther than the calculation of this quantity at one place in the machine, whether at input, at output, or at some intermediate place of transmission by belt pulley or toothed wheel. The next matter to be taken up is the transmission of work or power through the machine. Using the definition *work rate* = *force* $\times$ *velocity* or $U = PV$, it may be said that the usual effect of a machine is to change the factors P and V in amount and in direction. Also, there is a progressive decrease in the value of product PV from input to output, because of harmful or wasteful resistances within the machine. In many discussions, however, absence of this loss or dissipation of work is assumed, or relations are found as for the ideal, frictionless machine in which PV remains undiminished.

A glance at Figs. 86 to 88 will yield all needed illustration for the following general statements:

Let V_p be the velocity of driving force P and V_q that of load force Q; then under the assumption of frictionless machine action the work-rate relation is an equality,

$$PV_p = QV_q. \tag{26}$$

From the form or "configuration" of the machine the velocity ratio

$$r_v = \frac{V_p}{V_q} \tag{27}$$

can be found by purely geometric or kinematic methods; and when this ratio is known there result the simple relations,

$$Q = P \times r_v, \qquad P = Q \div r_v. \tag{28}$$

The term "velocity of a moving force" is an enlargement of the idea of the "moving force" as set forth in Sect. 6 (*a*). Strictly it means the velocity of the point of application of a force or component that is acting along the path of the point. When the point of application is continually changing on the machine piece, as does the point of tangency on the rim of each wheel in Fig. 87, the velocity of the point instantaneously in action is what is understood. Applying the concept stated at the end of Sect. 3 (*d*), it may be helpful to think of the force lines as tangents which continually wind on or off the wheels.

Velocity ratio r_v might be taken in either direction; but the convention will now be adopted of taking it always as the ratio of velocity of driving force to velocity of load force, in the normal direction of running. This proviso covers the case of hoisting machines and perhaps some others in which the ordinary load becomes driving force when lowering or running backward. Since forces are inversely as velocities this makes r_v (exact or modified) the ratio of load force to driving force.

(*f*) **Friction and Efficiency.**—As has been stated, work-rate Eq. (26) is for the ideal case of the perfect machine. With a part of the input PV_p absorbed in friction work U_f the equation becomes

$$PV_p = QV_q + U_f. \tag{29}$$

And the ratio of output to input, or the efficiency of the machine, is

$$e = \frac{QV_q}{PV_p}. \tag{30}$$

An important practical use of Eq. (28) is to help make efficiency a matter of force ratio, which is often easier to determine or simpler to express than the work ratio in Eq. (30). Without friction driving force P would just overcome the ideal load force

$$Q_o = P \times r_v, \tag{31}$$

which is greater than actual Q; then the efficiency is

$$e = Q \div Q_o. \tag{32}$$

Or, considering the machine in the other direction, to carry a load Q would require, without friction, the ideal driving force

$$P_o = Q \div r_v, \tag{33}$$

which is less than actual force P; and the efficiency is now

$$e = P_o \div P. \tag{34}$$

Always, the efficiency of a machine is a number less than one.

Another useful relation is got by combining velocity ratio r_v with efficiency e. Instead of Eq. (29) for the actual machine we may modify Eq. (26) to the form

$$ePV_p = QV_q. \tag{35}$$

Whence, in analogy to Eq. (28),

$$Q = P \times er_v, \qquad P = Q \div er_v. \tag{36}$$

Remember that r_v, in direction, is the factor by which to get Q from P. It is appropriate to call r_v the ideal or nominal and er_v the real or effective working-force ratio of the machine.

Prob. 39. In a certain hoisting machine the driving force moves 200 times as fast as the load: with an efficiency of 0.80 what force must be applied in order to lift 20,000 lb.?

Solution:

Without friction, P_o will be $20{,}000 \div 200 = 100$ lb. This is 0.8 of P, which therefore equals $100 \div 0.8 = 125$ lb.

Or, using Eq. (36), $er_v = 0.8 \times 200 = 160$, and $P = 20{,}000 \div 160 = 125$ lb.

Prob. 40. In a hoisting machine the driving force is applied at the rim of a wheel of 40-in. diameter which makes 10 revolutions while the load rises 1 ft. With an efficiency of 75 per cent what force P is needed to lift a load of 25 tons? Here, and elsewhere, is meant the "short" ton of 2,000 lb.

636.6 lb.

Prob. 41. If it be assumed that one man can exert an effective force of 40 lb. on a crank handle all around its circle of travel, and that an efficiency of 0.75 may be expected in a winch or hoisting gear, what velocity ratio should the machine have in order that two men may comfortably lift 5,000 lb.?

Prob. 42. In a hydraulic elevator the piston, driven by the water pressure, moves 1 ft. while the load rises 10 ft. If the mechanical efficiency of

the whole gear is 0.76, what total pressure must be exerted upon the piston in order to lift a load of 2,000 lb.? If the water pressure available is 240 lb. per square inch what diameter shall the piston have for this load as rated maximum? 11.82, say 12 in.

Prob. 43. A steam pump is to deliver 2,000 gallons of water per minute against a pressure of 120 lb. per sq. in. If the efficiency from steam pistons to discharge pipe be 0.85 (this including loss by leakage in pump as well as by friction in the whole machine), what horsepower must be developed in the steam cylinders? 164.7 hp.

Prob. 44. A locomotive with cylinders 20 by 26 in., 4-in. piston rods, and wheels of 72-in. diameter, is running at 56 miles per hour and overcoming a total train resistance (including its own) of 5,400 lb. With an efficiency of 0.87 from pistons to rails, what horsepower must be developed in the cylinders and what m.e.p. will be needed?

Solution:

At 56 m.p.h. (compare Prob. 6, page 36), the linear velocity of the locomotive is $V = 88 \times 56 = 4{,}928$ ft. per minute.

The effective or delivered horsepower is then

$$H_1 = \frac{5{,}400 \times 4{,}928}{33{,}000} = 806.4 \text{ hp.}$$

And the cylinder power must be

$$H_2 = 806.4 \div 0.87 = 926.9 \text{ hp.}$$

The m.e.p. might be found from this hp. by means of Eq. (22), but another way will also be shown, as follows:

In one revolution of the engine the locomotive advances, or the load resistance is overcome, through a distance equal to the circumference of the drivers, which for 72-in. wheels is 18.850 ft.

At the same time each of the four piston faces moves 26 in. under the working push of the m.e.p., which is the same as if one face moved 4 times 26 in. or 8 ft. 8 in. Then the velocity ratio from force $P = p_m A$ on the piston to load Q at the wheel rim or drawbar is $r_v = 8.67 \div 18.85 = 0.4598$.

The real ratio is now $0.87 \times 0.4598 = 0.4000$

Then $p_m A = 5{,}400 \div 0.4 = 13{,}500$ lb.

And with average $A = 307.9$ (see Prob. 29), the value of m.e.p. is $p_m = 13{,}500 \div 307.9 = 43.85$ lb.

Check this by calculating p_m by Eq. (22) from the horsepower 926.9, as suggested above.

Prob. 45. Two-cylinder locomotive, 22 by 26 with 4-in. rods, 56-in. wheels; at 81.6 r.p.m. average m.e.p. = 68.1 lb., drawbar pull = 13,300 lb.; find speed M, indicated horsepower, drawbar or dynamometer horsepower, and efficiency.

Prob. 46. Same locomotive, 26.67 m.p.h., 852 i.h.p., efficiency 0.819; find r.p.m., average m.e.p. and drawbar pull.

Prob. 47. Two-cylinder locomotive, 21 by 30 with 3¾-in. rods, 63-in. wheels; 22.3 m.p.h., drawbar pull 16,100 lb., efficiency 0.872; find r.p.m., dynamometer and indicated horsepower, average m.e.p.

CHAPTER II

MACHINERY FOR UNIFORM MOTION

This chapter is concerned with the transmission and production of uniform motion, commonly rotary, sometimes rectilinear. Mechanisms for this purpose are made up of wheel pairs of the kinds presented in Sect. 4, which combine into trains of belt and tooth gearing, with an occasional friction pair. Machine action may be continuous over long periods as in transmission machinery or rotary machine tools, or it may be intermittent as in hoists and yet more in reciprocating machine tools such as the planer. For the present the actions of starting up and slowing down are recognized but not discussed. A few devices for starting and stopping are shown incidentally, but all detailed study of such apparatus is reserved for the following chapter. Advancing from the general relations developed in Sect. 7, the discussion will at first deal about equally with forces and with motions, but will tend toward the side of kinematics as the subject is developed.

SECTION 8. RELATIONS IN TRAINS OF GEARING

(*a*) **Change of Speed.**—A very usual and important function of wheel pairs and trains is to change speed, with correlative change in transmitted force. The single pair is limited in the ratio of change that it can produce efficiently, so that for large ratios a series of pairs, cumulative in effect, becomes necessary. The general principle is to get a change in the linear velocity transmitted, by means of compound wheels such as combination B-C on axis 2 in Figs. 91 and 92. Taking a pair at a time, relating speed N to shaft rather than to wheel, and using the designating letter of the pulley or gear to represent its radius or other determining dimension, we have from Eq. (9), page 35,

$$\frac{N_1}{N_2} = \frac{b}{a}, \qquad \frac{N_2}{N_3} = \frac{d}{c},$$

If now the left-hand and right-hand members respectively of

these equations are multiplied together the intermediate speed N_2 will cancel out, with the result

$$\frac{N_1}{N_3} = \frac{\omega_1}{\omega_3} = r_n = \frac{b}{a}\frac{d}{c}. \qquad (37)$$

This important conclusion may be put into the following general statement, good for all such trains:

The rotary speed of the first driving wheel or shaft is to the speed of the last driven shaft as the continued product of (the radii of) all the followers is to the product of all the drivers.

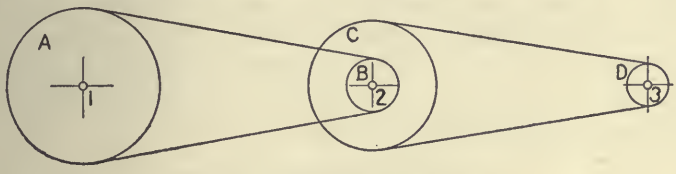

FIG. 91.—Three-shaft belt train.

The overall ratio of angular velocities r_n is taken in the same direction as the ratio of linear velocities in Sect. 7 (*e*). With emphasis upon the function of changing speed it would be more logical to reverse these ratios, getting factors by which to multiply initial in order to get final speed; and this is done in the purely kinematic study of Chap. IV, where the reciprocal of r_n (the ratio of last to first speed) is called the "value of the train." As here taken r_v and r_n are more directly force factors, as appears from their use in Eqs. (28) and (39). If the effect of

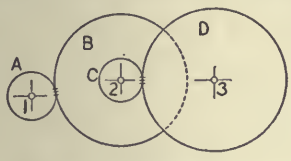

FIG. 92.—Simple gear train.

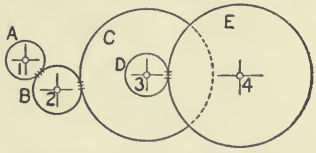

FIG. 93.—Train with idler.

the train is to decrease velocity factor V and increase force factor P in the work-rate product PV, the ratios r_v and r_n will be greater than one; but in speeding-up trains they will be fractions.

(*b*) **Relative Directions: Idle Wheels.**—If an extra wheel like B in Fig. 93 is interposed between the two wheels A and C of a pair, or if three or more simple wheels act upon each other in series, the one or more intermediates are called idle wheels or idlers. They do not cause any change in the linear rim velocity transmitted, hence do not affect the value of speed ratio r_n.

In applying the rule under Eq. (37) to the train in Fig. 93, wheel B can be made to appear as both follower and driver: but the immediate cancellation of b shows that it might just as well have been left out of account. A single idle wheel reverses motion; and sometimes such wheels are used to extend a train, so that it will connect distant axes.

The general statement regarding relative direction of turning is best made in mathematical terms, easily confirmed by putting arrows on the wheels of any train. Call a turning ratio positive for like rotation, negative for unlike, so that ordinary external-contact gears and pulleys with crossed belt will be minus pairs. Then the sign of r_n, Eq. (37), will follow the rule of algebraic multiplication, being positive with an even number of reversing pairs, negative with an odd number.

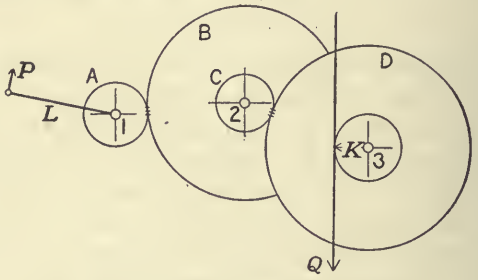

FIG. 94.—Gear train used for hoisting.

(*c*) **Transmission of Power.**—In Fig. 94 the gear train of Fig. 92 is sketched as it might be used in a hand-power winch or hoist. A driving force P, taken to be tangential all around its circle of action, is applied at the end of crank arm L, while K is the effective radius (out to middle of rope) of a winding drum on axis 3. The linear velocities of P and Q are $V_p = L\omega_1$ and $V_q = K\omega_2$, whence the velocity ratio of P to Q is,

$$r_v = \frac{L}{K} r_n = \frac{L}{K} \frac{b}{a} \frac{d}{c}. \qquad (38)$$

In Fig. 95 the belt train of Fig. 91 has its first axis 1 made the shaft of an engine, while the last one 3 is that of a dynamo, with a jack shaft 2 between them. The magnetic resistance opposed to motion of the armature is best represented by the moment or torque M_q and the average turning moment on the engine shaft may be called M_p instead of $T_m L$. Expressing work rate by product $M\omega$, as in Eqs. (19) and (20), and covering

both ideal and actual conditions, we get formulas analogous to those in Sect. 7 (*e*) and (*f*), namely:

Without friction, like Eq. (28),

$$M_p\omega_1 = M_q\omega_3, \text{ whence } M_p \times r_n = M_q; \tag{39}$$

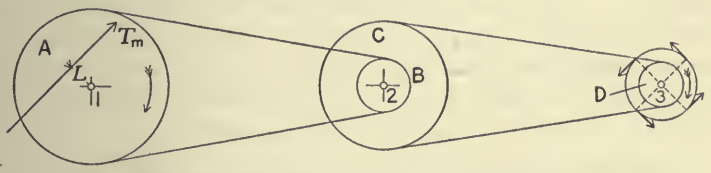

FIG. 95.—Belt-gear train, engine to dynamo.

With friction, like Eq. (36),

$$M_p\omega_1 = M_q\omega_3 + U_f \text{ or } eM_p\omega_1 = M_q\omega_3,$$

whence

$$M_p \times er_n = M_q, \tag{40}$$

e standing for machine efficiency, as before.

(*d*) **Forces by Moments.**—Figure 96 shows the gear train of Fig. 94 with the intermediate tangential working forces drawn in. Each T is laid off in two directions and double subscripts

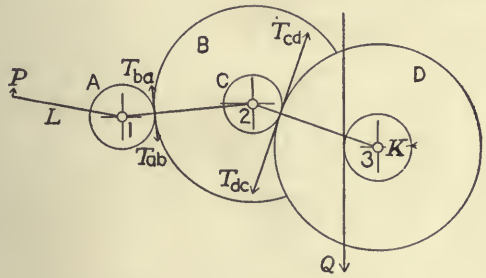

FIG. 96.—Tooth pressures in hoisting train.

are used to indicate direction of impression: thus T_{ab} is the driving tooth pressure exerted by A on B, T_{ba} the reaction of B on A, and so on. Taking the three axes 1, 2 and 3 as origins of moment and continuing the assumption of no friction we have the several moment equations,

$$PL = T_{ba}a,\ T_{ab}b = T_{dc}c,\ T_{cd}d = QK.$$

Whence

$$\frac{P}{T_{ba}} = \frac{a}{L},\quad \frac{T_{ab}}{T_{dc}} = \frac{c}{b},\quad \frac{T_{cd}}{Q} = \frac{K}{d};$$

Then by multiplication and in agreement with Eq. (38),

$$\frac{P}{Q} = \frac{a}{L}\frac{c}{b}\frac{K}{d} \text{ or } P \times \frac{L}{K}\frac{b}{a}\frac{d}{c} = Q \tag{41}$$

These equations show a general relation which may be put into words as follows:

Load force Q is to driving force P as the product of all the driving arms is to the product of all the resisting arms—a driving arm, such as L or b being the arm of a driving force. This is very similar to the statement in Art. (*a*) under Eq. (37), but the naming of the arms or radii is now reversed: thus in counting for r_n radius a belongs to a driving wheel but when considering forces it is the arm of a resistance.

(*e*) **Hoist Gearing.**—The gear trains used for lifting in cranes and other hoists are of the general type of Fig. 94. Two rather

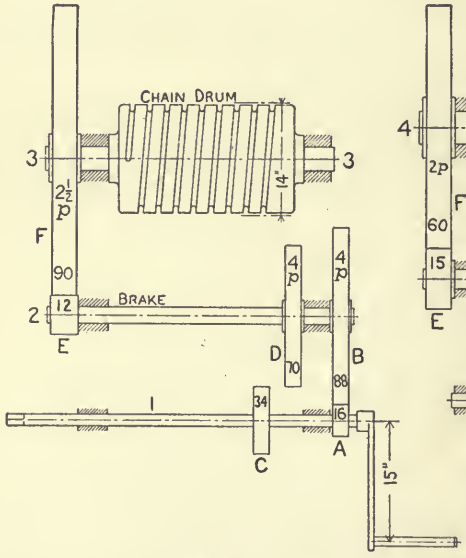

FIG. 97.—Double-geared Teo-speed hand winch.

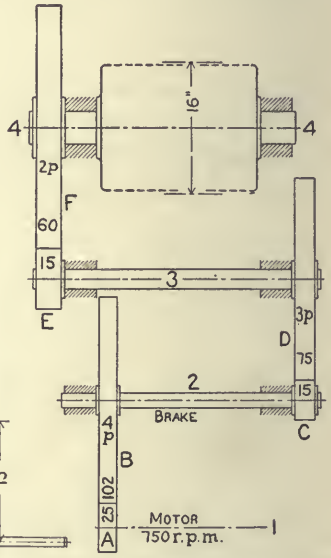

FIG. 98.—Triple-geared motor-driven winch.

simple examples are given in Figs. 97 and 98, drawn in true proportion but with the whole train laid out flat. Tooth numbers are marked on the gears and with them the diametral pitch, for later reference. The number or degree of "gearing" in any such train is determined by counting the number of pairs of gears which change speed between driving shaft and drum shaft. In Fig. 97 there are two such pairs, A-B (or C-D) and E-F, while Fig. 98 has three pairs, A-B, C-D, E-F.

Figure 97 carries our first example of a much used device for changing speed, in the shifting gears on shaft 1. Note the spacing which insures A that will slide out of engagement or "mesh" with B before C enters mesh with D; if it were not for this clearing the train would be locked while the change was being made. Each winch has a brake on an intermediate shaft, of which no more than the place is here indicated. Both drums are grooved for common oval-link or round-bar chain and the diameter given is the effective length, to middle of chain.

The hoist gears in Figs. 97 and 98 belong to rotary cranes of the "foundry" type, with horizontal jib and facilities for moving the load in and out along the jib. The tackle on the jib of such a crane is sketched in Fig. 99. At A the lifting chain comes up from the drum of the winch; it passes over fixed guide pulley B, then over pulleys C and E on the trolley and under the load

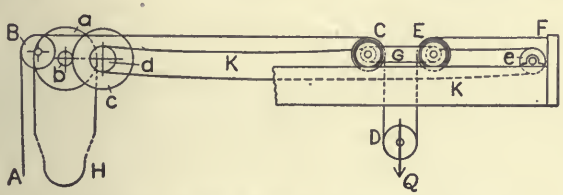

FIG. 99.—Rigging on jib of foundry crane.

pulley D, and its end is fastened to the outer end of the jib at F. With the winch held stationary, so that there is a constant length of chain from B to F, the load moves in and out on a constant level. Being hung in a two-ply tackle it rises only half as fast as chain is wound on drum.

The mechanism for "racking" in and out consists of a single-geared winch train, with chain wheel a (actuated by hand chain H) as driver, pinion b working on gear c, and chain wheel d in place of a drum. Chain K, with its ends fast to the frame of trolley G, is in effect an endless chain around wheels d and e, and can pull G either way along its track.

The high-ratio winch in Fig. 100 requires two views for its complete showing, since the train cannot be laid out flat without breaking some of the wheel contacts. This example combines some of the features of Figs. 97 and 98, in that it provides for either double or triple gearing and slides the crank shaft (with its two driving pinions) in order to make the change. Idler

G is put into the double-geared train to keep unchanged the relative directions of turning between shafts 4 and 1. The respective active trains are, triple-geared A-B, C-D, E-F, double geared, A′-G-D, E-F.

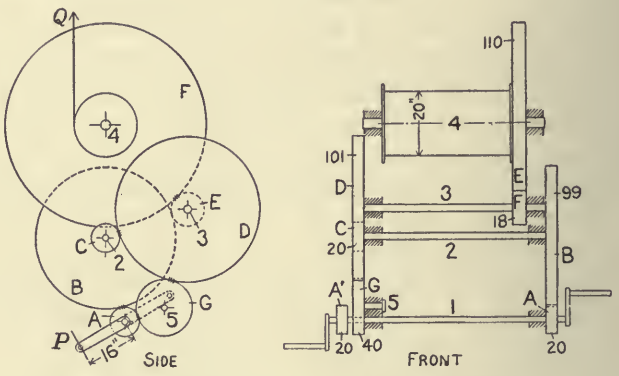

FIG. 100.—Double- and triple-geared hand winch.

Prob. 48. If shaft 1 in Fig. 101 makes 1,200 r.p.m. find the following: (*a*) Rotary speed of 2, 3 and 4 in r.p.m.; (*b*) overall angular ratio r_n; (*c*) linear velocity of rim, feet per minute, in each of the pairs A-B, C-D, E-F.

$$N_3 = 60,\ r_n = 100,\ V_{cd} = 392.7$$

Prob. 49. Given that shaft 1 in Fig. 102 runs at 40 r.p.m., make the same calculations for this train as in the last problem for Fig. 101.

$$r_n = \tfrac{1}{14.4},\ V_{ef} = 1{,}508$$

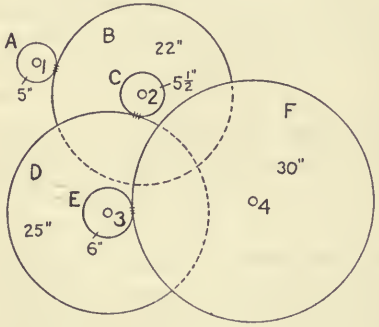

FIG. 101.—Speeding down.

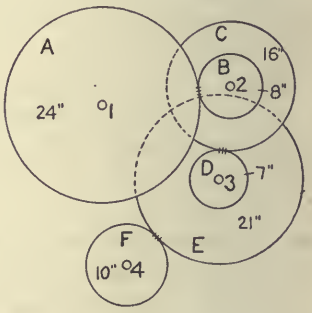

FIG. 102.—Speeding up.

Prob. 50. In a train like Fig. 94 let $L = 16$ in., $K = 7$ in., and the numbers of teeth be $a = 20$, $b = 112$, $c = 16$, $d = 100$. Find angular ratio r_n for the gear train and linear ratio r_v for P and Q. With an efficiency of 85 per cent what P will be needed to lift 3,000 lb.?

Solution:

$$r_n = \frac{112 \times 100}{20 \times 16} = 35 \qquad r_v = \frac{16}{7} \times 35 = 80$$

$$er_v = 0.75 \times 80 = 60 \qquad P = 3{,}000 \div 60 = 50 \text{ lb.}$$

Prob. 51. Windlass gear like Fig. 94: dimensions in inches, crank arm 14, wheel diameters 5, 27, 6, 30, drum diameter 12; efficiency 0.80, load 5,000 lb. Calculate same quantities as in last problem.

Prob. 52. For Fig. 2 the tooth numbers are $a = 16$, $b = 96$, $c = 16$, $d = 84$, $e = 13$, $f = 52$; with drum diameter 24 in. and motor speed 500 r.p.m., what is the speed of lift? Find torque at motor to lift 20 tons (of 2,000 lb.), first with no friction, then with efficiency of 82 per cent.

Prob. 53. In Fig. 95 let crank arm $L = 8$ in. and the pulley diameters in inches be $a = 72$, $b = 24$, $c = 36$, $d = 10$. The generator runs at 1,800 r.p.m. and absorbs 96 horsepower. Calculate (*a*) r.p.m. of shaft 2 and of engine shaft, (*b*) linear velocity of each belt, feet per minute, (*c*) effective pull transmitted by each belt and value of mean turning force T_m, assuming no machine friction.

Solution:

$$N_2 = 1{,}800 \times \frac{10}{36} = 500 \qquad N_1 = 500 \times \frac{24}{72} = 166.7$$

Circumference of 10-in. pulley D..... 2.618 ft.
Velocity of belt C-D, 2.618 × 1,800 =..... 4,712 f.p.m.
Circumference of 72-in. pulley A..... 18.85 ft.
Velocity of belt A-B, 18.85 × 166.7 =..... 3,142 f.p.m.

By Eq. (15) the torque on shafts 3 and 1, in pound-inches, are

$$M_q = 63{,}025 \times \frac{96}{1{,}800} = 3{,}361 \qquad M_p = 63{,}025 \times \frac{96}{166.67} = 36{,}303.$$

Then with radius $d = 5$ in. the effective pull of belt C-D is

$$T_{cd} = 3{,}361 \div 5 = 672 \text{ lb.}$$

With radius $a = 36$ in. the effective pull of belt A-B is

$$T_{ab} = 36{,}303 \div 36 = 1{,}008 \text{ lb.}$$

And with the 8-in. radius of the crank the mean turning force is

$$T_m = 36{,}303 \div 8 = 4{,}538 \text{ lb.}$$

As a check on the moments M_p and M_q get the ratio

$$r_n = \frac{10}{36}\frac{24}{72} = \frac{1}{10.8}$$

Then the relation $M_p = 10.8M_q$ must be satisfied, according to Eq. (39).

Prob. 54. Arrangement similar to Fig. 95: engine with 15-in. crank, wheel 12 ft. in diameter, speed 120 r.p.m.; belt from engine to 40-in. pulley on jack shaft and from 50-in. pulley on jack shaft to 12-in. pulley on generator; horsepower transmitted 250, friction neglected. Find (*a*) r.p.m. of jack shaft and of generator, (*b*) linear velocity of each belt, (*c*) mean turning force on crank pin and torque on generator shaft, (*d*) effective pull transmitted by each belt.

(*a*) $N_3 = 1{,}800$; (*b*) $V_{cd} = 5{,}655$; (*c*) $M_q = 8{,}753$, (*d*) $T_{ab} = 1{,}824$.

Prob. 55. For each gear setting in Fig. 97 find angular ratio r_n from crank shaft to drum, also linear ratio from P to Q through a tackle like that in Fig. 99. (*a*) With an overall efficiency of 0.78 what force P will be needed to lift a load of 10,000 lb., pinion A driving? (*b*) With $P = 80$ and the same efficiency what load can be lifted through gears C-D?

(*a*) $P = 72.5$ lb.

Prob. 56. In Fig. 98 find r_n from 1 to 4. With an efficiency of 0.75 what torque must the motor exert in order to lift 10,000 lb. through the tackle of Fig. 99 and what horsepower will it develop at 750 r.p.m.? How fast will the load rise?

Suggestion: Disregard friction from load Q to chain drum, bringing no more than a chain pull of 5,000 lb. to the drum. Find M_q on the drum, then apply whole efficiency between drum and motor, using er_n as in Eq. (40). H = 7.78.

Prob. 57. If shaft 1 in Fig. 97 is turned at the rate of 32 r.p.m., how fast will the load rise with each gear setting? If shaft 1 turns clockwise when viewed from the right, will the chain come down at the front (near side) or back of the drum and at which end of the drum (right or left) will it be fastened?

Prob. 58. For the hoist gear in Fig. 100, with tackle as in Fig. 99, determine the following: (*a*) Ratios r_v and r_n for both drives. (*b*) With efficiency of 0.65 for the whole machine what force P will be needed to lift 20 tons (2,000 lb.), triple-geared? (*c*) With the same driving force and efficiency what load will be lifted in double gearing? (*d*) Which way does the crank turn in lifting, with or against the clock in the side view? (*c*) 8,090 lb.

Prob. 59. For one of the following assignments, under the assumption of no friction, compute velocity and transmitted pressure at each place of gear contact throughout the train:

(*a*) Fig. 2 as in Prob. 52 with the diametral pitches as added data, pair A-B 4, C-D 2.5, E-F 1.5.

(*b*) Fig. 97 as in Prob. 55.

(*c*) Fig. 98 as in Prob. 56.

SECTION 9. BELT AND ROPE GEARING

(*a*) **Belt Transmission.**—In a factory built before the general use of electric motors, say a textile mill, the system of power transmission from engine to machines was about as follows:

1. **Main belt,** from engine to main line shaft. If several floors were to be served this main transmission would expand into a rather elaborate system, with one or more intermediate **jack** shafts; these might change belt velocity or direction, or might be used to divide the power current or simply to put two shorter belts in place of a very long one.

2. **A main lineshaft** to carry power to the whole floor, possibly also serving some individual machines.

3. For each room or group of machines a **lineshaft** from which power is taken at short intervals.

4. For each machine a **countershaft.**

With electrical transmission the largest unit is likely to be the room or group, so that the mechanical system runs no higher than the lineshaft. Rope transmission was a rival of the main belting system, with the advantages of longer distance reach and greater flexibility for change of direction; and it is a rival of electrical distribution under conditions of steady, heavy load, being lower in first cost but less flexible for cutting down the amount of machinery in motion at any time.

In the present section we view belt and rope gearing from what may be called the geometrical side, considering form and arrangement for motion but not going into the question of power transmitting capacity.

(*b*) **Belt Drive for a Machine.**—The typical arrangement of countershaft and belt drive for a machine tool—specifically, for

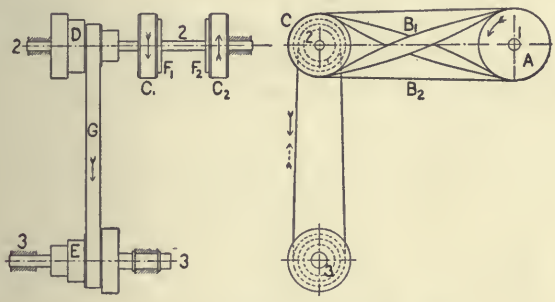

FIG. 103.—Countershaft drive for lathe.

a lathe—is outlined in Fig. 103. Power is taken from continually running lineshaft 1: and provision is made for starting and stopping the machine, by means of friction clutches F_1 and F_2 in the pulleys on countershaft 2; for running it in either direction, by the use of open and crossed belts B_1 and B_2; and for changing speed, by shifting belt G on cone pulleys D and E. The operating gear for the clutches is not even suggested in this sketch; detailed examples of clutches and their accessories will be found in Sect. 13.

Tight and loose pulleys are a much used means of starting and stopping the countershaft, especially in the older installations. Commonly the two pulleys are of the same diameter, as in Figs.

106, 130 and 131, but the improved design shown in Fig. 104 has the advantage that tension is much diminished when the belt is on the smaller loose pulley. Consequently there is much less tendency to wear the bearing of loose pulley on shaft, which is harder to lubricate effectively then the fixed shaft bearings. The belt climbs easily enough the conical edge of the rim of the loose pulley, even at a 45-deg. slant; but this device is most

FIG. 104.—Tight and loose pulleys with tension relief.

appropriate when the shiftings of belt are not too frequent and when the machine is likely to stand idle for considerable periods.

The elements of a two-speed countershaft for a lathe are shown in Fig. 105. Clutch F_1 gives high speed, F_2 low speed and F_3 reversal. For a milling machine, which does not need backward motion, pulleys A_3 and B_3 could be omitted, or they might be proportioned for a third speed, with open belt.

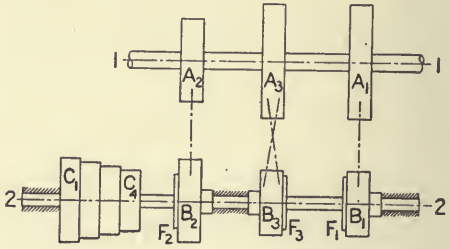

FIG. 105.—Two speeds and reversal.

(*c*) **An Elaborate Belt Drive.**—A full belt drive for a grinding machine of older style is diagrammed in Fig. 106. This is a top or plan view of shafts 1, 2 and 3, which are in about their true relative positions and are, of course, suspended from the ceiling of the shop. Axes 4 to 7, all on the machine, are here swung into the horizontal plane of 1 to 3 and crowded up close to them to save space. Shaft 1 is the countershaft proper, with tight and loose pulleys at A. Through cones B and C, drum F and small pulley G the grinding wheel is driven at a very high speed:

drum F maintains the drive as the saddle carrying the wheel shifts along the bed of the machine. Cones D and E, drum H and pulley J give the work piece a comparatively slow rotary motion. Cones K and M drive the feed gearing, which slowly traverses the wheel saddle back and forth. Finally, pulleys

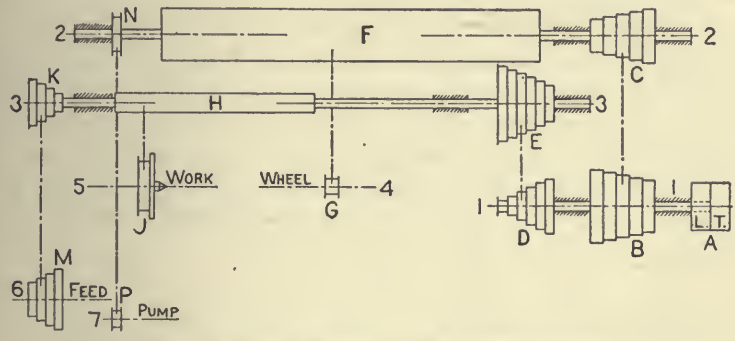

FIG. 106.—Belt gearing for a grinding machine.

N and P drive a small pump which throws a liberal stream of water on wheel and work at their place of contact.

(*d*) **Belt Guiding.**—In a usual flat or coplanar belt pair the belt is running correctly when its middle or median line keeps continually in the common middle plane of the two pulleys. A general principle, governing not only this simple case but also

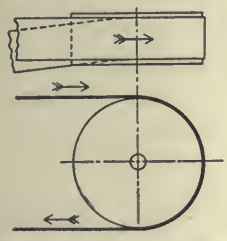

FIG. 107.—Belt guiding.

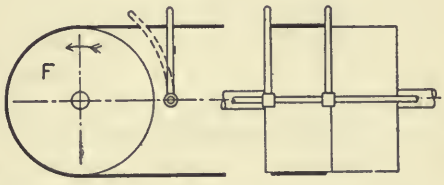

FIG. 108.—Shifting a belt.

the more complex biplanar arrangements, is that the belt is very sensitive to outside influences where it is approaching a pulley and must be led on squarely. Once wrapped, however, it is insensitive and can even be led off at a considerable angle without disturbing its grip on the pulley.

These facts are illustrated by Fig. 107, while Fig. 108 goes on to show how the problem of shifting the belt is met in a tight-

and-loose pair. The shifter or shipper is close to the countershaft pulley, its two fingers enclosing the slack approaching side of the belt; and only a very moderate pressure is required to push the (running) belt sidewise. If there is an extra resistance, as in Fig. 104, the fingers may be curved so as to act closer to the point of tangency.

(*e*) **Crown on Pulleys.**—On plain cylindrical pulleys there is nothing to hold the belt in its proper running plane, as against incidental or accidental influences. Occasionally one pulley is flanged for special security, but the usual way to get sufficient stability is to "crown" the pulley, making its diameter greater at the middle than at the edges. The profile may be curved or of a very flat V form, the pulley surface being then either spherical or double-conical, (*a*) and (*b*) in Fig. 109.

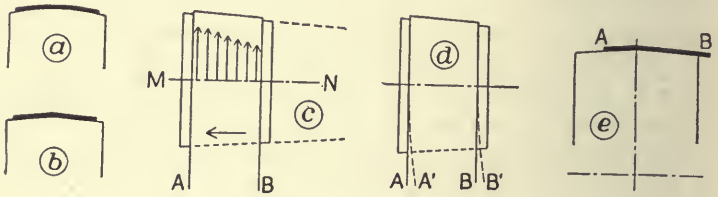

FIG. 109.—Pulley crown and its effects.

To see the principle of crown action consider Fig. 109 (*c*), which represents a belt that, so far as concerns external guiding, is being led squarely onto a slightly conical pulley—at right angles to the axis, that is. Because of greater diameter there is also greater belt tension at edge A, with a resulting internal tendency to swing the approaching belt toward the left and make it climb the cone. Another argument, illustrated at (*d*), is that if a segment of the cone surface is "developed" or laid out flat it will be a curved strip (dotted lines), therefore the straight belt tends to wrap toward the base (larger end) of the cone.

On the double cone of the crowned pulley there are opposing tendencies of the two halves of the belt to climb toward the middle. If the belt is off center, as at (*e*), the preponderant tendency on long side B will bring it back.

Consider the case, exaggerated in Fig. 110 (*a*), of pulleys with axes just a little out of parallel. It is easily seen that each side of the belt is now being led on obliquely, and that it will run off

the near side of the pulleys. This can be prevented by using the slightly conical pulleys at (*b*), larger on the far side, so as to balance external guiding by the internal action shown in Fig. 109. Of course all the angles involved in this case must be very small.

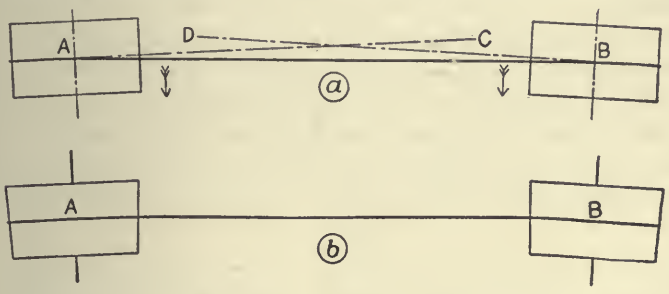

FIG. 110.—Axes not parallel.

(*f*) **Biplanar Belt Pairs.**—The simplest case of axes not parallel is that in Fig. 111, which calls for no added guide pulleys. End view (*a*), projected on a plane parallel to both axes, is the key diagram. To meet the requirement of straight-on guiding each pulley must have the midpoint of one edge in the midplane of the other pulley, so that in this view the two points B and D

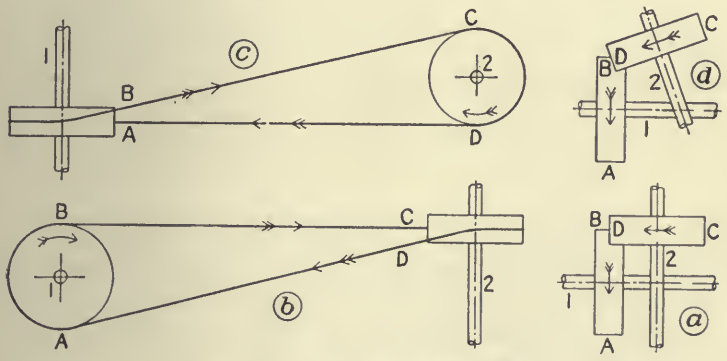

FIG. 111.—The quarter-turn belt.

have a common projection. Since these are running-off or delivery points, rotation must be such that along the line BD of common tangency the two rims run toward each other: trace this out in the other two views. At (*d*) is shown the general case of any angle. Starting with parallel axes and open belt, we see that the crossed belt is then the limit of this general case, with a

twist angle of 180 deg. Note that if there are quarter-turn belts in a system, accidental reversal of engine or motor will run these belts off their pulleys. How could such a pair be made reversible?

Beside the twisting of the belt pair there is the possibility of bending it out of its single plane, over guide pulleys. In Fig. 112 the angle of turn is 90 deg., resulting in an arrangement frequently used for connecting lineshafts at right angles. One of the regularly manufactured transmission appliances is a stand for the two "mule" pulleys A and B, with supports adjustable as to angle and height. If the belt is to run either way the middle circle of each guide pulley must be tangent to both main mid-

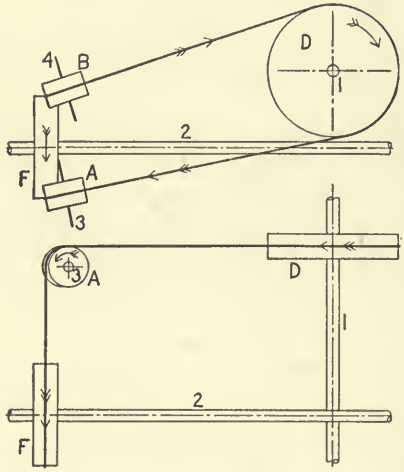

FIG. 112.—Axes crossing close together.

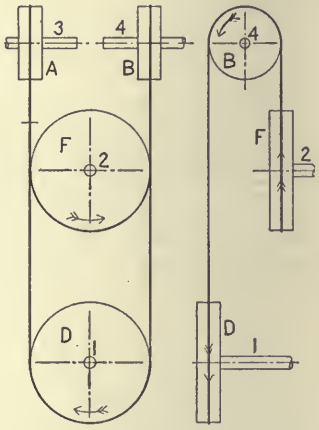

FIG. 113.—To get a long belt with close axes.

planes. With only the one direction indicated, pulley A could be raised or B shifted to the left in order to adjust tension of belt.

A complete half-turn gives the arrangement of Fig. 113, sometimes used when the shafts are too close together for a direct belt: note its reversing effect.

Just as Figs. 112 and 113 are related to or derived from the planar pair, so Fig. 114 is derived from the quarter-turn pair by bending it over guide pulleys. By shifting pulley A with slight change of angular position, as suggested at (*c*), it can be made to serve effectively as a belt tightener.

(*g*) **Belt Length.**—Figure 115 outlines a type of drive sometimes used on the radial drill. Driver D turns on the axis about which

the radial arm swings, follower F is on the drill spindle, guide A on the arm and guide B on the saddle with F: this saddle can move in and out (right and left) without varying length of belt course or tension of belt.

The belt tightener, Fig. 116, is a commonly used device for preserving constant tension as against stretch of belt in service. Tightener pulley A is correctly located on the slack side of belt and near to the smaller main pulley, so as to increase the arc of contact upon it. This pulley A is on a slide or arm, either adjustable mechanically or held against the belt by force of weight or spring.

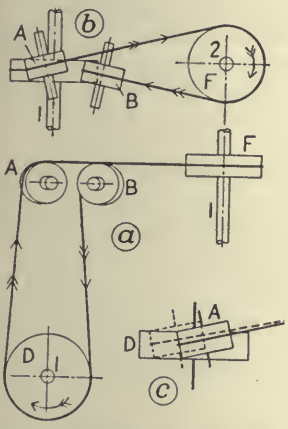

FIG. 114.—The bent quarter-turn drive.

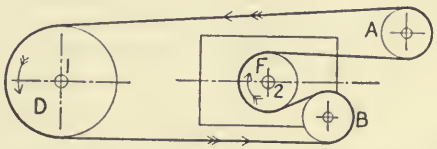

FIG. 115.—Constant-length drive for drill.

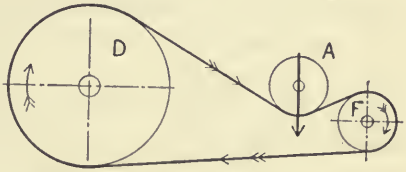

FIG. 116.—Use of belt tightener.

The question of belt length is of interest in estimating quantities for a layout; but its exact determination is of importance chiefly in connection with the matter of getting the same tension on the different steps of a pair of cone pulleys, or of securing *the same geometrical length around the successive wheel pairs that make up such stepped cones.* Solution of this particular problem will bring out the general relations.

(*h*) **The Cone-pulley Problem.**—Start with the equal pair of common diameter D_o, dotted lines in Fig. 117, letting axis distance $S = nD_o$ or expressing distance in terms of diameter: then very simply the length of belt for this case is

$$L_\varphi = (\pi + 2n)D_o. \qquad (42)$$

For the unequal pair, full lines, let r and mr be the radii, where m is also the velocity ratio and is always so taken as to be greater than one; now the length of belt is

$$\begin{aligned} L &= \text{arc AB} + \text{arc CD} + 2\text{BC} \\ &= (\pi - 2\beta)r + (\pi + 2\beta)mr + 2S\cos\beta \\ &= \pi(m+1)r + 2\beta(m-1)r + 2nD_o\cos\beta. \end{aligned} \tag{43}$$

It is a fact, much more easily proved by numerical or graphical trial than by algebraic deduction, that if the mean diameter D_m or $(r + mr)$ is equal to D_o length L will be greater than L_o.

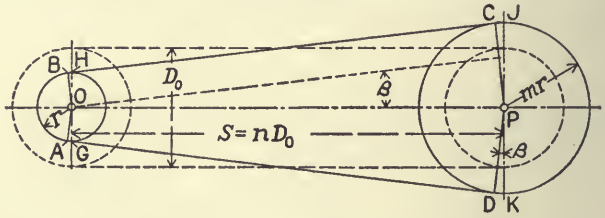

FIG. 117.—Diagram for open belt pair.

For equality of lengths D_m must be less than D_o by a small fraction q, and we express this relation by writing

$$(m+1)r = (1-q)D_o. \tag{44}$$

Now $r = \left(\frac{1-q}{m+1}\right)D_o$ and $(m-1)r = (1-q)\frac{m-1}{m+1}D_o$, so that for one determining relation we have

$$\sin\beta = \frac{(m-1)r}{S} = \frac{1-q}{n}\,\frac{m-1}{m+1}. \tag{45}$$

To get the other equate L with L_o, then

$$\pi(1-q)D_o - 2\beta(1-q)\frac{m-1}{m+1}D_o + 2n\cos\beta\, D_o = (\pi + 2n)D_o.$$

Solving this for q we have

$$q = \left[2\beta\frac{m-1}{m+1} - 2n(1-\cos\beta)\right] \div \left[2\beta\frac{m-1}{m+1} + \pi\right]. \tag{46}$$

The evaluation of this expression, which cannot be performed directly but must be done by a trial method, can best be shown by an example.

Example 1. Find q for $n = 4$, $m = 2$.

From (45) $\sin\beta = (1-q) \times \frac{1}{4} \times \frac{1}{3} = 0.08333(1-q)$.

In the table of five-place natural trigonometrical functions, making some allowance for q, select as trial value $\beta = 4°\ 45'$, for which

$$\sin\beta = 0.08281, \quad \cos\beta = 0.99656.$$

And from the general relation

$$\beta^{\text{rad}} = \frac{2\pi}{360}\beta^{\text{deg}} = 0.017453\beta^{\text{deg}} \tag{47}$$

we have for 4° 45′, $\beta = 0.08290$.

Now work out (46),

$$2\beta\frac{m-1}{m+1} = \frac{2}{3} \times 0.08290 = 0.05527 \qquad 0.05527$$

$$2n(1 - \cos\beta) = 8 \times 0.00344 = 0.02752 \qquad \pi = 3.1416$$

Difference 0.02775 Sum 3.1969

$$q = 0.02775 \div 3.1969 = 0.008680$$

With this in (45) $\sin\beta = 0.99132 \times \frac{1}{12} = 0.08261$, as against the 0.08281 assumed, a discrepancy of −0.00020.

Solving again for $\beta = 4° 44'$, tabular $\sin\beta = 0.08252$, we get $q = 0.008693$ and computed $\sin\beta = 0.08261$, a discrepancy of +0.00009. Then by proportional interpolation, going $\frac{9}{29}$ of the way from 0.008693 to 0.008680, the exact value of q is found to be 0.008689.

(*i*) **A Method for Cone Pulleys.**—Relation (44) is simple and covers the ground completely, but expression (46) does not put it into shape for practical use. When, however, the results of a large number of calculations like Example 1 are charted, in Fig. 118, an effective and convenient method is established. In this diagram the base is pair ratio m on a logarithmic scale, beginning at the left edge with $m = 1$ or $\log m = 0$ and running to $m = 10$, which is beyond the range of practice. The ordinate is q to a uniform scale, measured down from the top line because q is a subtractive quantity. Each curve is drawn for the particular value of shaft-distance factor n with which it is numbered. The lower left-hand section of the diagram is merely a vertical enlargement of the closely spaced group of curves above it, on the same base but with the scale of q made 2.5 times as large. The use and full meaning of the diagram can better be shown by numerical examples than by a general explanation.

Example 2. In Fig. 117 let $D_o = 12$ in. and $S = 48$ in., so that n is 4. Find diameters for a pair of pulleys that will fit the same belt and have a speed ratio of 3 to 1.

The vertical line for $m = 3$ is crossed by the curve for $n = 4$ at $q = 0.0192$. This 1.92 per cent of 12 in. is 0.230 in. Subtracting it from 12 we have $D_m = 11.77$ in.

Since $D_m = r(1 + m)$ and m is 3, the radius r of the smaller pulley is $11.77 \div 4 = 2.9425$; then either by subtraction as $D_m - r$ or by multiplication as mr the other radius is 8.8275.

The diameters are then $D_1 = 5.885$ and $D_2 = 17.655$ in.

There is no practical demand for accuracy beyond 0.01 in. and even on this reduced diagram and with interpolation for both m and n that degree of accuracy can easily be attained.

With D_0 thus given we have the simplest problem that can arise: it covers the design of like cone pulleys and of those that

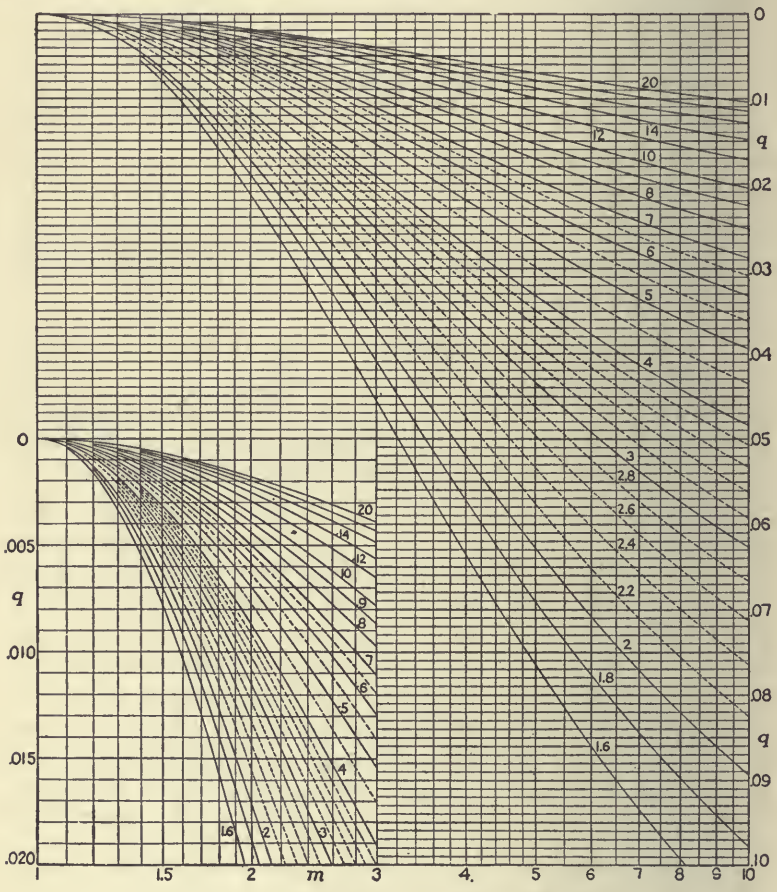

FIG. 118.—Chart for cone-pulley belt length.

have an equal pair within their range of variation. But when the diameters of an unequal pair are the primary data conditions are not so simple, and a solution by trial and correction may be necessary.

Example 3. With no allowance for sag, find the length of belt around a 12- and 60-in. pulley pair at 20 ft. between axes.

Ratio m is 5, and with $D_m = 3$ ft. on $S = 20$ ft. the approximate value of n is 6.67. Taking these to the chart q is found to be 0.0205, whence $D_o = 36 \div 0.9795 = 36.75$ in. $= 3.063$ ft. Comparing this with $S = 20$ the more exact value of n is 6.53, to which corresponds $q = 0.0208$. This changes D_o to 36.76+ in., so that the first approximation was quite good enough.

To be exact length should be measured at middle of belt, not at surface of pulleys, or in Eq. (42) the circumference term should be $\pi(D_o + t)$ instead of πD_o. Since t is about 0.2 in., this correction will add approximately $\frac{5}{8}$ in. to a length computed from D_o.

On the question of considering or disregarding belt thickness it may be remarked that this is of much more importance as affecting belt length and velocity ratio (where its influence is small enough) than in the matter of design for constant length around different steps. As between $D_m = (1 - q)D_o$ and $D_m + t = (1 - q)(D_o + t)$ the difference in D_m is only qt; and for any possible value of q this is practically negligible.

Example 4. A foot-power lathe has steps of 3-, 4- and 5-in. diameter on the headstock cone, while the diameter of the largest step on the driving cone is 20 in. and the distance between axes is 30 in. Find the other two diameters of the large cone.

With these extreme dimensions the element of trial becomes large.

For the given pair $m = 20 \div 3 = 6.67$ and $D_m = 11.5$ in. If this were D_o the value of n would be $30 \div 11.5 = 2.61$. Knowing that D_o will be larger, try $n = 2.4$ and get $q = 0.064$. Then $D_o = 11.5 \div 0.936 = 12.29$ and $n = 30 \div 12.29 = 2.44$.

A closer value of q is now 0.0633, which makes $D_o = 11.5 \div 0.9367 = 12.28$ and gives n exactly 2.443, a good enough check.

Having thus got D_o the procedure for the unknown steps must be to assume a trial value of q, get D_m and m, then check back to q.

Take the step for which $r = 2$ in. As a guess try $q = 0.058$ (moderately less than for the given step), $D_m = 12.28 \times 0.942 = 11.57$, $mr = 9.57$, $m = 9.57 \div 2 = 4.79$. For $n = 2.44$ and $m = 4.79$ the chart gives $q = 0.0507$, showing that the guess was too high.

Try again with $q = 0.051$: $D_m = 12.28 \times 0.949 = 11.65$, $m = 9.65 \div 2 = 4.83$, $q = 0.510$. This is a close check, and with $D_m = 11.65$, we have $D_1 = 4.00$, $D_2 = 19.30$, $m = 4.825$.

From first to second step q has changed from 0.0633 to 0.0510, or by 0.0123. Since we are working toward equality of D's the next change will be smaller, perhaps about 0.011. Try $q = 0.040$, $D_m = 12.28 \div 0.96 = 11.79$; with $r = 2.50$, $mr = 9.29$ and $m = 3.72$, for which $q = 0.040$. The diameter sought is then 18.58, and for the two pulleys we have

$D_1 =$	3	4.0	5.0
$D_2 =$	20	19.30	18.58

The step differences on the larger cone are now 0.70 and 0.72, as against 1.00 on the smaller.

Obviously and very decidedly this is a case where it would be well to get an approximate graphical solution—method will be found in any hand book—then correct to a surely accurate result by means of Fig. 148.

(*j*) **The Case of Crossed Belt.**—Here no special interest attaches to the equal pair, and from Fig. 119 we write

$$\begin{aligned} L &= (\pi + 2\beta)r + (\pi + 2\beta)mr + 2\,S\cos\beta \\ &= (\pi + 2\beta)(m + 1)r + 2nD_m\cos\beta \\ &= [(\pi + 2\beta)\sin\beta + 2n\cos\beta]D_m; \end{aligned} \tag{48}$$

and since $\sin\beta = D_m/S$ it is evident that to keep D_m constant—by using the same step differences on both cones—will also keep L constant. The calculation of L involves the same numerical

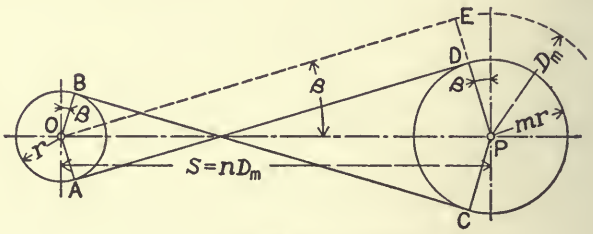

FIG. 119.—Diagram for crossed belt.

methods as in Example 1, especially the use of Eq. (47), but is a simple, straightforward operation.

(*k*) **Rope Gearing.**—In general principle and method of power transmission this is like belt gearing; but it has two distinctive features, namely, the multiplication of grooves or rope turns and the use of V grooves to increase friction grip—see Figs. 53 (*b*) and 125. For long-distance cross-country transmission single ropes have been used, in the form of rather light wire cables at high speed, bedding (not wedging) in grooves cushioned with leather and rubber. What we now consider, however, is the system for fiber rope, longer in its reach than belting but keeping within factory range.

There are two types of rope drive, the "English" system with a number of single ropes in parallel and the "American" system with one continuous rope winding round and round the wheels. The first is simple in detail and less liable to a disabling breakdown, but requires many splices and cannot well be adjusted to take up stretch. Into the continuous system is regularly

introduced a tension regulating device, as outlined in Figs. 120 and 121: this maintains constant driving conditions and by keeping all rope turns equally stretched facilitates the use of guide pulleys for change of direction.

The schematic end views in Figs. 122 to 124 illustrate important details of wheel arrangement. Figure 122 brings out the

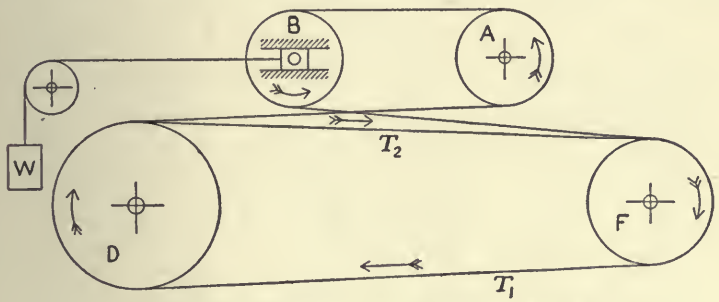

FIG. 120.—Common type of tension regulator.

fact that each rope of the multiple system is a simple coplanar unit. With a continuous rope there must be a coiling action (about the two wheels) so that the grooves are offset by half a pitch, as seen in Figs. 123 and 124. Even without tension regulation it would be necessary to have auxiliary guiding sheaves, to transfer the rope from one side of a first main wheel to the opposite side of the other.

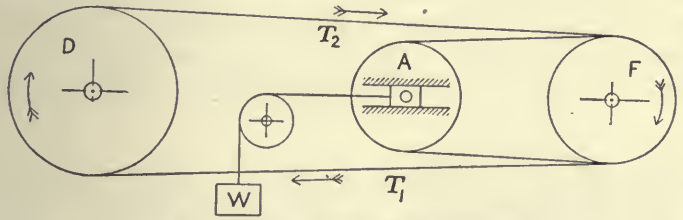

FIG. 121.—Self-contained arrangement.

Commonly the auxiliary system is outside the main drive, as in Figs. 120 and 123, requiring at least two separate sheaves A and B, one on a tension car: more guide wheels may be used, to lead the rope to a more convenient location of the tension wheel. Note how these guides are inclined so that they make the necessary sidewise transfer of the rope. The most compact arrangement possible, Figs. 121 and 124, puts guide A on the same shaft with driven wheel F—but as a separate one-groove *loose*

wheel. This serves to carry a slack-side ply around axis F to tension wheel B, for it is highly essential that the tension regulator be on the slack side, not the tight side, of the drive.

Section (*a*) of Fig. 125, reproducing Fig. 53 (*b*), shows typical grooves for the condition of Fig. 122: the high dividing ridges insure that a slackened rope will keep in its place and the sharp

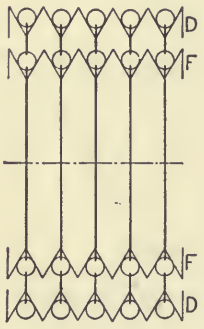

FIG. 122.—Multiple ropes.

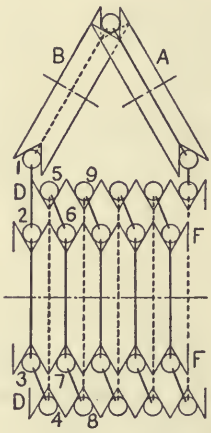

FIG. 123.—Scheme of FIG. 120.

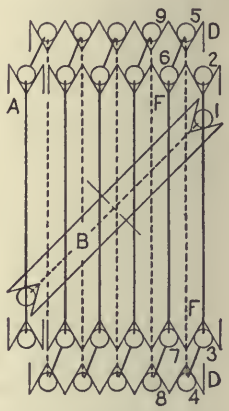

FIG. 124.—Scheme of Fig. 121.

wedge angle maintains driving power even with relaxed tension. Because the continuous system requires that the ropes run out of plane the grooves in section (*b*) are not nearly so deep; and because tension is maintained in spite of stretch the angle is not so sharp, with the result that the rope lets go a little easier at runoff. For guiding sheave the rounded groove at (*c*) is standard.

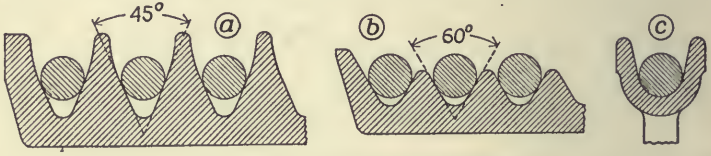

FIG. 125.—Grooves for rope wheels.

Prob. 60. Find exact length of belt for a pulley pair of diameters D_1 and D_2 at axis distance S, by evaluating Eq. (43) directly. Then compute by the method of Example 3, checking against first result. Use $D_1 = 15$ in., $D_2 = 35$ in., $S = 12$ ft. 6 in., or other data as assigned.

Prob. 61. Find length of crossed belt for the same pair, by means of Eq. (48). What is the effect of belt thickness in this case?

Prob. 62. Choosing m and n get q for some point of Fig. 118, as in Example 1.

Prob. 63. Open belt cone pulleys, mean diameter 10 in. for equal step, distance 54 in.; get diameters of a step pair for the turning ratio 4 to 1.

Prob. 64. Open-belt cone pulleys, diameters of one step pair 6 and 24 in., find exact diameters for a pair having the ratio 2 to 1.

Prob. 65. Small cone-pulley of 4-, 5- and 6-in. step diameters, smallest step on large cone 24 in., find diameters of other two steps, with axis distance 36 in.

The next four problems involve ideas as to speed series that are developed in Chap. IV—see Sect. 15.

Prob. 66. Design a pair of five-step equal belt cones with a middle diameter of 10 in., to be placed 54 in. apart and to give speeds in a geometric series having the term ratio 1.4.

Prob. 67. Design a five-step equal cone pair for the shaft distance 6 ft., with middle diameter 9 in. and step difference 1 in. on radius. The approximate series of diameters will be 13, 11, 9, 7, 5, but this is to be corrected for exactly constant belt length.

Prob. 68. Design a four-step equal cone pair to give speeds in a geometric series with the term ratio 1.2, the largest diameter being 16 in. and the shaft distance 60 in.

Prob. 69. Design a four-step unequal cone pair with greatest turning ratio 3, speeds to be in a geometric series of term ratio 1.25, greatest step diameter 15 in. and shaft distance 60 in.

SECTION 10. VARIOUS GEAR TRAINS

(*a*) **Gear Trains from the Lathe.**—The whole mechanism of a simple but complete "engine" lathe is outlined in Fig. 126.

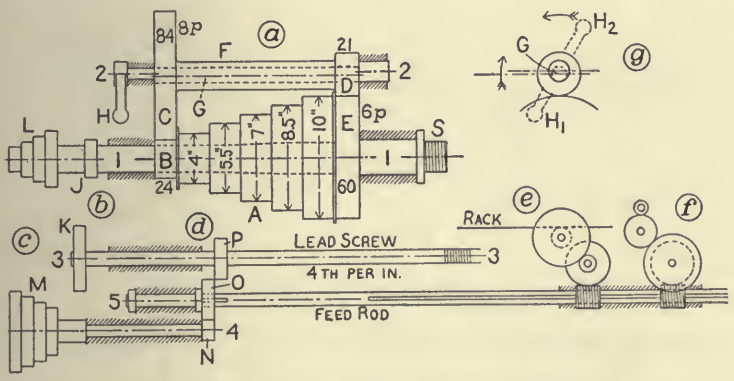

FIG. 126.— Gearing of old-style 16-inch lathe.

The main train at (*a*) drives the spindle S, which rotates the work piece against the cutting resistance of the tool. The first secondary train (*b*), more fully shown in Fig. 127, turns the lead screw: this screw engages a split nut on the tool carriage or saddle, so as to give the latter a smooth and steady motion along

the ways or shears (parallel to the rotation axis) for screw cutting. Belt cones L and M at (*c*), with the gears at (*d*), drive the feed rod, which by means of gear trains (*e*) and (*f*) can give a longitudinal feed to the whole saddle or a cross feed to the slide rest on the saddle.

Cone pulley A, with pinion B fast to it, is the first member of the main train; this combined pulley and gear turns freely on the

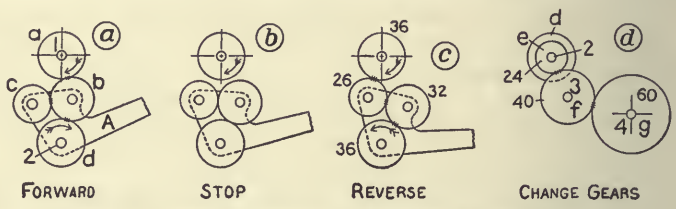

FIG. 127.—Gearing to drive lead screw.

spindle, to which gear E is securely keyed. Through back-gear shaft F or the back-gear train B-C-D-E a slow and powerful movement is given to the spindle. For faster cutting the back bearing is thrown out and gear E is latched or keyed to cone A, so that the belt drives the spindle directly. Gears C and D are on a hollow quill shaft and this turns on the eccentric shaft G, which has eccentric bearings in the frame. A half turn or less

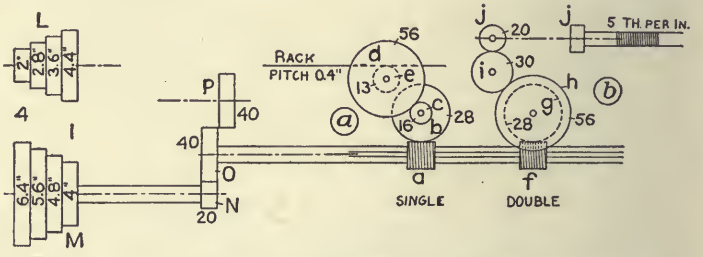

FIG. 128.—Gearing at tool carriage.

of handle H, say from H_1 to H_2 at (*g*), draws C and D out of mesh with B and E.

In Fig. 127 gear a is the one marked J in Fig. 126 and g is the same as K. Concerning the reversing gearing, shown in positions (*a*), (*b*) and (*c*), it need be remarked only that the plate carrying intermediates b and c is pivoted on axis 2 and that arm A can be latched in any one of these three positions. The change gears at (*d*) complete the train from spindle to lead screw:

different gears can be slipped onto studs 2, 3 and 4, so as to give different ratios of turning of screw relative to spindle—stud 3 being adjustable to any position required. The lead screw is generally reserved for the operation of screw cutting, since its use for ordinary longitudinal feeding will cause unevenly distributed wear. In this lathe, however, by sliding gear O, Fig. 126, to the right, the change-gear train can be made to drive the feed rod, with a greater range of speeds than is given by the little belt train and with possibility of reversal.

The belt drive for feed rod is an old fashioned arrangement, modern lathes having some form of all-gear drive; but it has the advantage that the belt can slip if the tool strikes an obstacle or if the carriage is carelessly allowed to run up against the headstock. The tool-carriage feed trains, (*e*) and (*f*) in Fig. 126, (*a*) and (*b*) in Fig. 128, are carried in the "apron" which hangs down in front of the carriage. In each there is a friction clutch (of the type in Fig. 173) between the worm wheel *b* or *g* and the spur gear *c* or *h* on its axis, so that either feed can be thrown in or out as desired. Pinion e engages a rack which is fastened to the bed, pinion j is on the cross-feed screw.

(*b*) **Headstock Gearing.**—Sometimes a lathe built for restricted service such as the routine turning of shafting with but a small

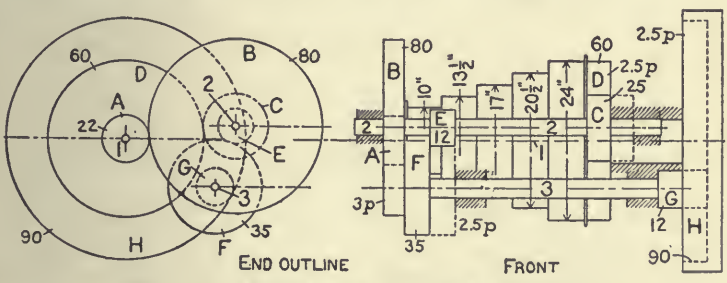

FIG. 129.—Main gearing of triple-geared lathe.

range of diameter is made single geared, as if the cone pulley were on back-gear shaft F in Fig. 126 and there were but one pair of gears, like D-E. The double-geared arrangement of Fig. 126 is common for all moderate sizes, up to 24 in. or a little more of diameter capacity or "swing." Larger lathes have higher gearing, of which one example is given in Fig. 129. As drawn it is set in double gear, with train A-B-C-D in action and second shaft 3 idle. For triple gear shaft 3 is slid to the right, engaging F with

E and pinion G with internal gear H on the face plate; at the same time C is slid to the right until clear of D. With an internal gear in pair G-H the addition of axis 3 to the train does not cause a reversal of spindle rotation.

The common form of lathe drive is what is called a retracted train, or one that folds back on itself so that the first and last axes are coincident: thus in Fig. 126 there are really three axes, for the cone pulley, for the back-gear shaft and for the spindle. In very large lathes the spindle must be of so great diameter, to meet requirements of stiffness, that the cone pulley is put on a side shaft; and four- or five-pair gear trains are used.

Example 5. Find the back-gear ratio in Fig. 126, then with 200 r.p.m. of countershaft and equal cone pulleys (as in Fig. 103) make out a table of all the speeds that the spindle may have.

The angular ratio of the gear train is

$$r_n = \frac{84}{24} \times \frac{60}{21} = 10;$$

the back-gear ratio is commonly stated thus, as 10 to 1, but sometimes is reversed to the value of the train, 1 to 10 or 0.1.

Beginning with the highest spindle speed, call the smallest step of the lathe cone No. 1, the largest one No. 5; then arrange work as follows:

STEP	DIRECT	BACK-GEAR
1	$200 \times \frac{10}{4} = 500 \quad \div 10 =$	50 r.p.m.
2	$200 \times \frac{8.5}{5.5} = 309.1 \div 10 =$	30.9 r.p.m.
3	$200 \times \frac{7}{7} = 200 \quad \div 10 =$	20 r.p.m.
4	$200 \times \frac{5.5}{8.5} = 129.4 \div 10 =$	12.9 r.p.m.
5	$200 \times \frac{4}{10} = 80 \quad \div 10 =$	8 r.p.m.

Prob. 70. Make the calculations of this example for one of the following lathes:

(*a*) Equal cone pulleys, steps 6, 8.5, 11, 13.5, 16 in., countershaft speed 180 r.p.m., back-gear train (in order of drive), 25, 55, 14, 66 teeth.

(*b*) Equal cone pulleys, steps 5, 7, 9, 11, 13 in., countershaft 150 r.p.m., tooth numbers 24, 76, 17, 58.

Prob. 71. Make similar calculations for Fig. 129, getting the two gear ratios and applying them both, so as to find the fifteen spindle speeds. Countershaft runs at 200 r.p.m., with same cone as on lathe.

Prob. 72. Take the spindle speeds found in the last two problems, write them in a sequence, and find the ratio of each to the next lower, in order to see how nearly they come in a geometric series.

Thus from Example 5,

Speed.....	500.0	309.1	200.0	129.4	80.0	50
Ratio.....		1.618	1.545	1.545	1.618	1.600 etc.

A vertical row will generally be more convenient.

Example 6. Applying the method of Eq. (40) to Fig. 126, find the relation between effective belt pull and torque resistance to cut; that is, at a certain "speed" or setting of the mechanism, with a given belt pull and assumed efficiency, find the torque that can be overcome.

Take the first speed, back-geared, with belt on 4-in. pulley, assuming belt pull (effective) of 80 lb. and efficiency of 0.85. The radius being 2 in., input torque is $80 \times 2 = 160$ lb. in. $= M_p$.

With $r_n = 10$ and $e = 0.85$, the ratio of torques is $er_n = 8.5$; and the delivered torque is then $M_q = 160 \times 8.5 = 1{,}260$ in. lb.

Prob. 73. By the method of this example find the whole series of load torques for one of the following lathes, under the assumption that pull and efficiency remain unchanged as the belt is shifted on cone pulleys:

(*a*) Prob. 70 (*a*) , with $P = 100$ lb., $e = 0.85$.

(*b*) Prob. 70 (*b*), with $P = 75$ lb., $e = 0.87$.

(*c*) Fig. 129, $P = 300$, efficiency 0.84 double-geared, 0.80 triple-geared.

Example 7. From the data on Figs. 126 and 127 find what screw will be cut by that lathe.

This is a question of how far the tool carriage advances while the spindle makes one turn, or of how many turns the spindle will make while the tool moves 1 in.

Train a-b-c-d, Fig. 127, drives stud 2 at the same speed as the spindle; then the turning of the lead screw is determined by gears e and g, with the ratio 24 to 60 or 1 to 2.5. The screw having four threads per inch, the tool traverse for one spindle turn is the pitch

$$p = \frac{1}{2.5} \times \frac{1}{4} = \frac{1}{10} \text{ in.},$$

or the lathe will cut a screw of ten threads per inch.

Prob. 74. For this same lathe find gears for positions e and g that will produce screws of 12, 9, 8, 6 and 5 threads per inch.

Prob. 75. Find what feeds will be given by the gearing in Fig. 128, expressed in tool movement per spindle turn and in spindle turns (or cuts) per inch of feed movement.

Get first feed per turn of feed rod, both longitudinal and across and in both the terms asked. Then apply the ratios given by belt-cone pair L-M with gear pair N-O, finding the four rates of cut (per inch) in each direction. Find also what feeds will be got by sliding O into mesh with P, the change gears being as in Fig. 127.

Prob. 76. Run over the drawings of the gear trains in Figs. 126 to 128 and check up the following matters:

(*a*) With the reversing gear at "Forward" in Fig. 127 the screw cut by the lathe should be right-hand.

(*b*) The longitudinal feed by belt drive should be from right to left along the lathe bed, or toward the face plate.

(*c*) The cross feed should be from front toward back of lathe: shall the cross-feed screw (with nut moving) be right-hand or left-hand?

Prob. 77. Measure up some simple lathe to which you have access and make all of these speed calculations for which you can find data—surely the spindle speeds, perhaps all the feeds.

(*c*) **Planer Drives.**—Figure 130 shows the driving mechanism of a metal planer, from countershaft 1 to the rack which is fastened beneath the sliding table. The driving movement or velocity is really produced at the rim of pinion J, but a large idler

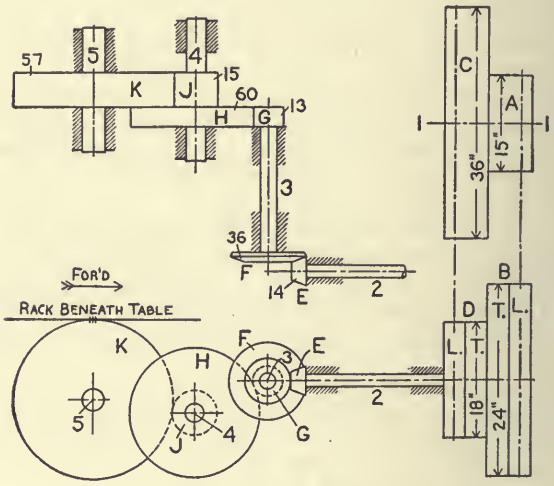

FIG. 130.—Two-belt planer drive.

K, called the bull wheel, is needed to carry this movement up to the rack. There are two belts, one open and the other crossed, with tight and loose pulleys on driving shaft 2. These belts are automatically shifted by a mechanism like Fig. 198, which is actuated by the table at each end of its stroke; and this is so designed that the belt which has been driving is shifted clear off its tight pulley before the other is started on. For the slow forward or cutting stroke smaller pulley A drives larger pulley B, while the quick return is got by belt pair C-D.

A planer drive having but one belt is shown in Fig. 131. Pulley A and gear B are fast upon a sleeve which turns freely on shaft 1, and the gear train for forward motion is B-C-D-E-F-G. Pulley A′ is keyed to shaft 1, so that pinion H drives wheel E for the return movement by the two-pair train H-E-F-G. The belt shifter is much simpler than with Fig. 130.

(*d*) **Trains for Different Speeds.**—The term "uniform motion" in the title of this chapter does not mean that the same ratio of velocities shall always be produced. In Figs. 97, 100 and 129 we have already had examples of trains in which change of gear engagement causes change of velocity effect. Two further examples showing the more systematic development of this

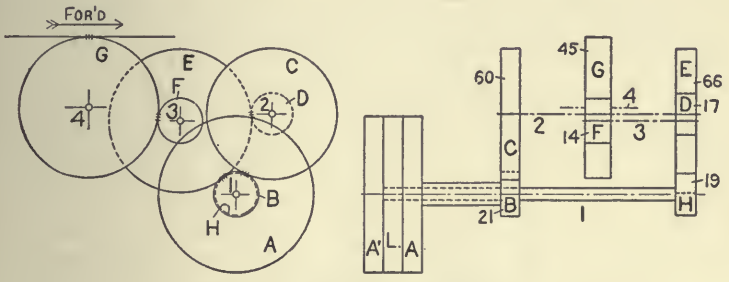

FIG. 131.—One-belt planer drive.

principle will now be given, introducing a class of mechanisms that is of sufficient interest and importance to form the separate subject of Chap. IV.

The driving mechanism of a machine for testing the strength of materials is outlined in Fig. 132. Pulleys A_1 and A_2 are driven by open and crossed belts from an overhead countershaft, and

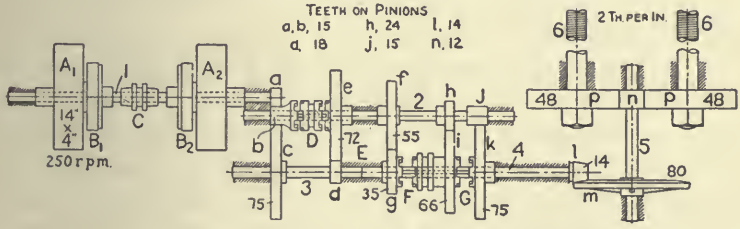

FIG. 132.—Gear train of a metal testing machine.

either one can be coupled to the shaft by friction clutches B_1 and B_2. Pinion a drives b directly and c through b. When jaw clutch D is slid to the left it engages b and shaft 2 is driven at high speed; with D to the right and engaging e there is slow drive through the back-gear train b-c-d-e. Change gears f, h and j are keyed to shaft 2, while g and k are loose on shaft 4 and i is on a sleeve splined to this shaft. The three speeds got by this arrangement are, high with clutch F engaged, middle

when i is in mesh with h, low with clutch G engaged. The final motion is given to a yoke or crosshead on screws 6, 6, which is drawn downward to apply a load to the specimen.

The outline drawing in Fig. 133 shows the gear system of an automobile, without the devices for securing flexibility or even a suggestion of the ball bearings. Shaft 1 is driven by the engine, through a friction clutch. By means of sliding members D-E and G-J four speeds in forward running are obtained. Shifting D-E to the right so as to engage tooth clutch E gives direct drive or full speed forward. Side shaft or back-gear shaft 2 is always running, and successively slower speeds are got by pairs C-D, F-G and H-J. For reversal compound wheel K-L is slid to the left

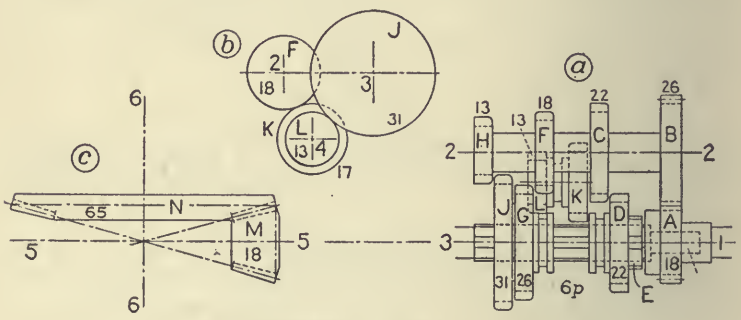

FIG. 133.—Transmission gearing of a motor car.

until K meshes with F and L with J: the system of shifting rods is, of course, so arranged and interlocked that but one of the three sliding members can be moved out of neutral position at one time. Between shafts 3 and 5 there is a pair of universal joints, for the flexibility that will allow the spring-controlled movement of the rear axle. Bevel gear N is on the axis of the rear axle, the halves of which it drives through the "differential" gearing shown in Fig. 330.

Prob. 78. For the planer drive in Fig. 130 find the following:

(*a*) Angular ratio of the gear train, from pulley shaft 2 to compound wheel H-J on shaft 4.

(*b*) The pitch of the teeth of wheels J and K and of the rack being $1\frac{3}{8}$ in., what r.p.m. must countershaft 1 have in order that the cutting or forward speed of the table shall be 24 ft. per minute.?

(*c*) With this r.p.m. of countershaft, what will be the return-stroke speed of the table and what its ratio to the cutting speed?

(*d*) If the forward-driving belt A-B is the one that is crossed, which way does the countershaft turn when viewed from the right?

Requirement (*c*) can be met by direct calculation in full; but a short cut is to consider the belt train from loose D through C and A to loose B. Its angular ratio is

$$r_n = \frac{36}{18} \times \frac{24}{15} = 3.2;$$

consequently, with belt on tight D shaft 2 will turn 3.2 times as fast as with belt on tight B. (*b*) 265.1 r.p.m.

Prob. 79. For the planer drive in Fig. 131 find the following:

(*a*) The pitch of gears F and G and of the rack being 1.5 in., what r.p.m. of the belt pulley will give a forward table speed of 30 f.p.m.? Which way does the belt run, or pulley A turn?

(*b*) With this pulley speed, what will be the velocity of the table in its return stroke?

(*c*) How fast does wheel A′ turn when the belt is on A and how fast does A turn when the belt is on A′?

Again, the simplest way of answering (*b*) and (*c*) is to get the angular ratio from A′ to A, through the train B-C-D-E-H.

Prob. 80. With the pulleys A running at 250 r.p.m., find the six speeds of the testing machine of Fig. 132; get the r.p.m.'s of shaft 4 and the rates of movement of the crosshead in inches per minute. Which way must the driving pulley turn in order to lower the crosshead?

Suggestion: With D to the left, or shaft 2 at 250 r.p.m., find the three speeds of shaft 4, then apply the back-gear ratio of train b-c-d-e and get the three speeds with D to the right. Calculate the crosshead movement for one turn of shaft 4 and use this with each r.p.m. of 4 in order to get the pulling speeds of the machine.

Prob. 81. With the engine making 1,800 r.p.m., what will be the five speeds of shaft 3-5 in Fig. 133 and what the running speeds of the car in miles per hour if the wheels have an effective diameter of 35 in.?

Highest speed of car, 51.9 m.p.h.

SECTION 11. SOME ELEMENTS OF GEAR TRAINS

In this section are collected a number of gear-train elements, some belonging to trains in Sect. 10, others to those of Chap. IV.

(*a*) **Sliding Gears.**—In many machines it is necessary that a gear shall shift along a shaft while yet remaining constrained to turn with it. The first example in Fig. 134 represents a drilling-machine spindle and has gear 2 held at a fixed height while shaft 3 can move up and down within it. Bevel gear 2 in (*b*) slides along its shaft, the bearing 1 which contains its extended sleeve hub being part of a sliding base such as the apron of a lathe, the saddle of a radial drill or the tool head of a planer. The key-way in either shaft has a sliding fit on a "spline" or "feather" (key) 4 which is held in the gear hub as shown by drawing (*b*).

Figure 135, from an automobile gear box like Fig. 133, shows a double gear 3-4 which is slid along shaft 2 by shifting rod 7 so as to engage either of the two gears 5 and 6. The conditions of this service are very severe, and instead of depending upon one key the shaft is made to a special cross section which gives four keys or splines solid with it—being formed by milling away the metal between keys. Sometimes the shaft has been made square for

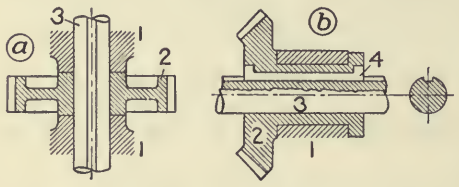

FIG. 134.—Splined gears, machine-tool service.

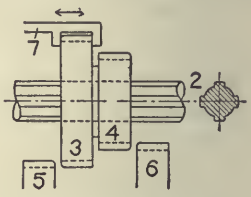

FIG. 135.—Sliding gears, automobile service.

an automobile, and in the days of long-distance mechanical transmission, as to a shop travelling crane before the advent of electric motors, long square shafts were used with sliding gears. The shaft had journals turned on it at intervals and the bearings were carried by linkages that folded them out of the way as the crane approached, the gear hub being considerably longer than the journal.

(*b*) **Gear Cones.**—One of the standard devices for speed changing is the pair of closely spaced gear cones represented by

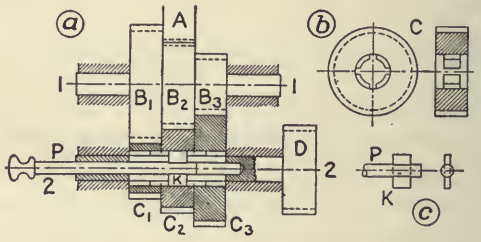

FIG. 136.—Old-type internal-key gear cones.

Fig. 136, where solid upper cone B is driven by gear A while lower cone C has an internal keying device which will engage any one of its loose wheels with shaft 2. This shaft is slotted for key K and drilled for push rod P, these parts connecting as seen at (*c*). Detail (*b*) shows how gears C are recessed or counterbored on the sides so as to leave intermediate free spaces for the key, which slides sidewise into the slots cut in the bore of the gear.

Later designs of this general type show a universal change to the scheme of pushing the key out radially into full-length engagement. For light feed-train service it is usual to have but one engagement, as by the single-end plunger keys of Fig. 137. Here the separate gears are to be mounted on steel bushings 2, spaced by rings 3 which confine rod 5 so that it will not scrape oil from the bushings that are running free. Plungers 4 fit into pockets drilled in the shaft and are held from turning by rod 5,

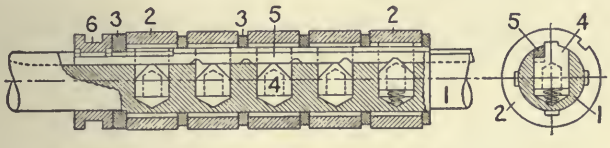

FIG. 137.—Plunger keys for light gear cone.

and any one of them is pushed up by its spring when the notch in this rod is brought to proper position by shifting collar ring 6. The notches are so spaced that a key is just fully lowered before the next one begins to rise: this close spacing eliminates definite neutral positions between adjacent engagements, which do not happen to be needed in the particular service.

The catching of a gear-cone key in its slot is much like the catching of a jaw clutch, but because of the contracted diameter

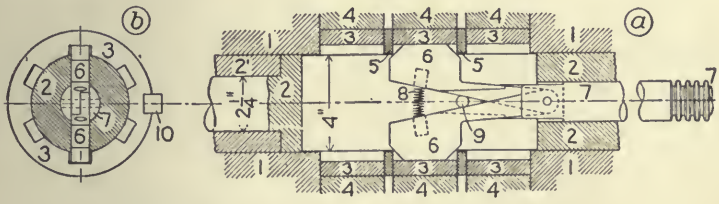

FIG. 138.—Internal snap keys, medium service.

and narrow pressure surfaces the capacity for power transmission is relatively small. Although this pair is therefore best adapted to feed-train service it has been used in the main train of a few machines absorbing a fair amount of power. The example in Fig. 138 has a number of features that fit it for severe conditions. First of all, shaft 2 is enlarged from a normal 2¼-in.—at the left end of view (*a*), inside the extended sleeve hub of gear E, Fig. 211 — to a 4-in. diameter at the gear cone, thus increasing the lever arm of the key pressures. The hardened steel bushing 3 fitted into

each gear 4 has three pairs of key-ways, to give quick engagement. Keys 6, likewise hardened at the working edges, are shifted by rod 7 and pushed apart by spring 8. Light steel rings 5 between the gears serve to compress the double key while it is being shifted. Rod 7 is moved by a small pinion which gears with the "cylindrical rack" at its free end, as is also rod 7 of Fig. 187 which belongs to the same machine. The hinge pin can be inserted or removed through hole 9 when the keys are over against the left-hand end of the slot.

The final example, Fig. 139, is so designed as to realize all the possibilities of the general scheme: maximum diameter for key

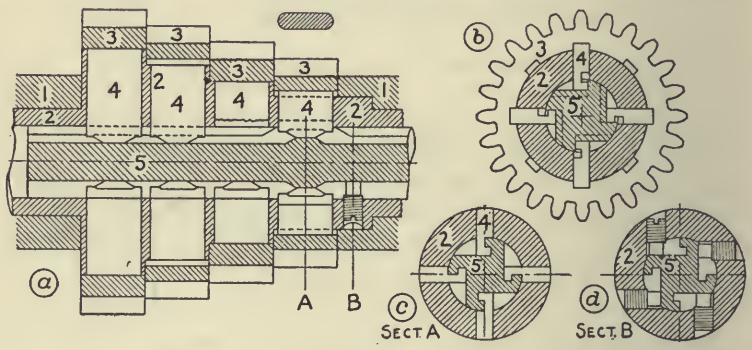

FIG. 139.—Multiple keys with positive control.

engagement is secured by enlarging shaft 2 to a cone on which the gears turn as mere rims; and by using four keys the needed intensity of pressure is further diminished. The device for operating keys is unique in that it moves them positively instead of depending on spring action. Round rod 5 is machined to the section seen at (*c*), with quartered slots which are undercut on the short side so as to engage little hooks formed on the inner ends of the keys, these resting on the longer side or flat. The keys are pushed out, as seen in the smallest gear, view (*a*), and in cross section (*b*), by a hump on each flat; and with this goes a gap in the restraining tongue or ridge. Section (*d*) shows how the rod is kept from turning away from the keys, by four pivot-point set screws.

(*c*) **Cone-and-tumbler Gearing.**—As illustrated by Figs. 140 and 141 the simple cone-and-tumbler unit consists of a solid gear cone C and a tumbler gear B which is carried on a movable arm F

and serves to bridge the gap between any step of the cone and a shifting pinion A on the other shaft. Figure 140 has the tumbler arm swung away back, as it must be represented in flat diagrams of gear trains, but Fig. 141 shows it in working position, where it

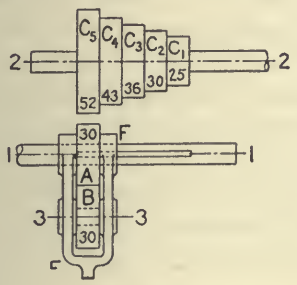

FIG. 140.

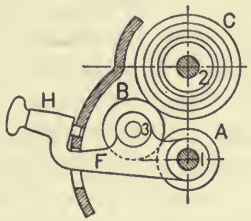

FIG. 141.

Arrangement of cone and tumbler.

is held by a latch pin in handle H. This combination is extensively used for feed trains and enters into the main train of a number of machines of moderate power requirement, notably milling machines. Various examples, running into more complex gear arrangements, will be found in Chap. IV, Figs. 232 to 236.

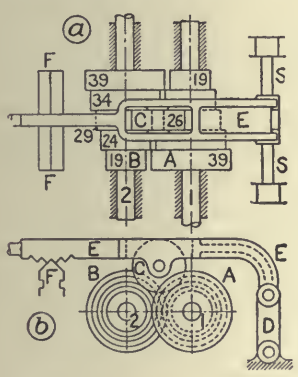

FIG. 142.—Gear cones with intermediate.

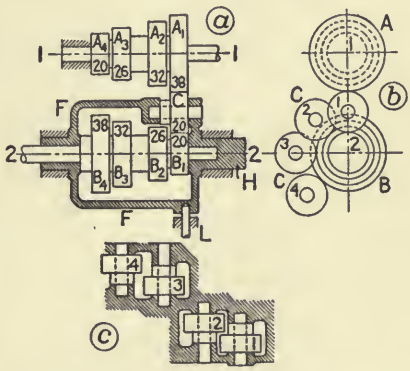

FIG. 143.—Gear cones with individual intermediates.

The crowded internal keying arrangement of Art. (*b*) can be avoided by using a shifting intermediate, much like a tumbler, to connect two close solid gear cones as seen in Fig. 142. The hinged framework carrying gear C slides along bar SS and is clamped down to bar FF by a locking device not drawn. This

example, from the main drive for a boring mill of medium size, belongs to the early days of the development of the multichange gear train for machine tools. The big, clumsy holding rig and the large space occupied soon caused its abandonment in favor of more rigid and compact construction. A better form for cones with idlers, very effective in a feed train, is represented by Fig. 143. Here four intermediates C_1 to C_4, one for each wheel pair of the cones, are held in a frame or cage F which can turn or oscillate on the axis of cone B. End view (*b*) diagrams the angular positions of these wheels, while the developed (flattened cylindrical) section at (*c*) shows how they are placed in the cage casting. A handle fastened at H turns the cage, which is locked in its four working positions by latch pin L.

CHAPTER III

INTERMITTENT MOTION

In this chapter we first take up devices for producing the kind of continuous motion by steps that is represented by the wavy curve in Fig. 29. Then follows a rather full presentation of the various clutches used for stopping and starting rotary motion of the uniform type. In the final section is given a selection of representative examples of devices for reversing and for accurate stopping.

SECTION 12. CONTINUOUS ADVANCE

(*a*) **Ratchet Drives.**—A ratchet may be defined as a holding or engaging device acting between two members that turn on the same axis (or sometimes between parallel sliding members), to the end that one may impart its own motion to the other, but in one direction only. Considering rest as the particular case of zero motion, this will cover even the holding function of the fixed pawl, piece 5 in Fig. 144. Dependent upon whether the driver rotates or oscillates, the ratchet will serve as a one-way clutch—to be considered elsewhere—or will produce the type of motion described by Fig. 29. As exemplified by Fig. 144, the ratchet mechanism comprises driver 2 (here supposed to be in regular oscillation) and follower 4, with click or pawl 3 as connector. This little intermediate piece is the essential and characteristic member of the combination. It is permanently jointed to one larger member, while the other is so formed as to offer a number of suitable engagement surfaces.

If the arrangement of Fig. 144 were used in a hand-power hoist to be operated by lever instead of crank, holding pawl 5 would be necessary; but moving ratchets are not likely to be employed against an active resistance of the kind that tends to drive the machine backward.

The pawl generally pushes upon the tooth of the ratchet wheel, althought it may be given the hook or pulling form 3′. In any case the contact surfaces should be so inclined that they will not tend to disengage under pressure: as shown in Fig. 145, this means

that the line of pressure (perpendicular to face of tooth) should pass inside of pin A for pushing pawl, outside for a pulling pawl. The arrowheads show direction of force exerted by tooth on pawl, and in either case the tendency is to turn the latter inward toward the wheel, not outward.

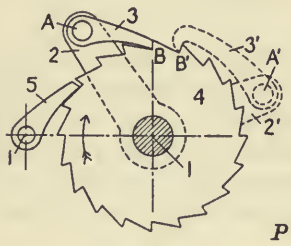

FIG. 144.

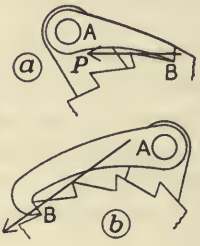

FIG. 145.

Ratchets and pawls.

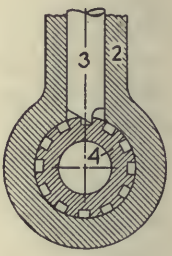

FIG. 146.

A reversible ratchet is shown in Fig. 146. Pawl 3 is now a plunger rod, inside of the drill handle and pushed down by a light spring, which can be turned so as to drive in either direction. There is an interchange or inversion of form, for the teeth must have engagement surfaces on both sides and the slanted disengaging surface has to be transferred to the pawl. A form

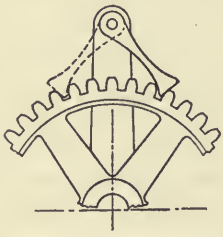

FIG. 147.—Pawl for gear teeth.

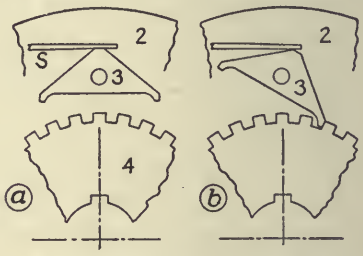

FIG. 148.—Ratchet for feed screw.

of reversible pawl that can be used on a common spur gear is shown in Fig. 147.

(*b*) **Ratchet Feed Motions.**—These are used on metal-planing machines, including the shaper and slotter, where the feed movement takes place at one end of the stroke, no matter whether the latter be long or short. Pawls for that service vary a good deal in form, but must have the characteristics of the example in Fig. 148. This pawl is double and need be given but a small

angular movement in order to reverse feed; it is spring-actuated, not only because it is balanced on the pin but also because it must operate in any angular position of the driver; and it has a definitely held neutral position, view (*a*).

A single type of device is used for driving the feed mechanisms of planers, differing only in details from the example in Fig. 149. On the end of the second shaft in the main gear train—No. 4 in Fig. 130, here piece 1—is keyed a flat disc or head 2. This is surrounded by a friction clutch consisting of plates 3 and 4, held together by bolts and springs and faced with leather washers L, L. Every time the machine reverses clutch 3-4 swings through an angle determined by arm 6 and stops S, S, then disc 2 turns

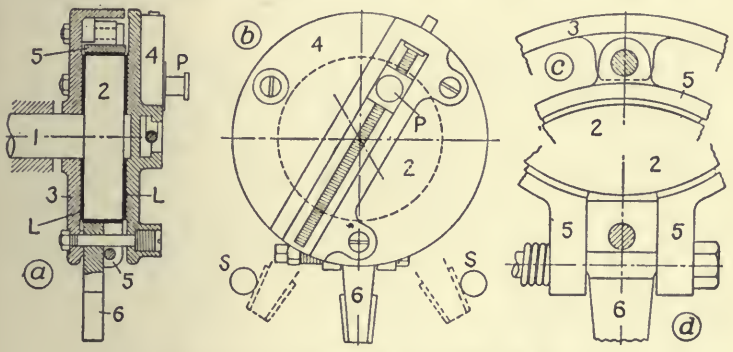

FIG. 149.—Friction drive for planer feed.

within the clutch during the rest of the stroke. Drive pin P can be adjusted for amount of feed and should be so located (with respect to axis 1) as to cause ratchet action at the beginning, not the end, of cutting stroke.

In older practice (especially on smaller planers) the whole driving function was performed by such a simple disc clutch; but for heavier and faster machines it is better to have a device that will relax a part of the friction grip after the useful motion has been made. Here this requirement is met by friction band 5, anchored to back plate 3 as shown at (*c*) and gripped by a stout spring. Lever 6 is formed as seen in view (*d*); and as the result of disc friction it is pressed hard enough against either stop to pry apart the ends of band 5 and relieve its pressure on 2. Some designs use only the band, but there is then an element of uncertainty in the balancing of relaxing effort against residual friction, with some tendency to jerky action.

The cams in Figs. 77 and 78 are used for actuating ratchet feed motions on crank-driven shapers and slotters, where stroke of tool bears a constant time relation to rotation of crankshaft.

(*c*) **Lifting Ratchets.**—The combination of elements in Fig. 144 may take the particular form in Fig. 150. Such a ratchet train might find practical use as a rig for lifting a water gate, but it is given here chiefly because of the interesting analogy (drawn by Reuleaux) between this mechanism and the common suction-and-lift pump, Fig. 151. Here the fluid ratchet train parallels the mechanical one very closely, water column 2 and notched bar 2 having essentially the same motion; but when we go to the

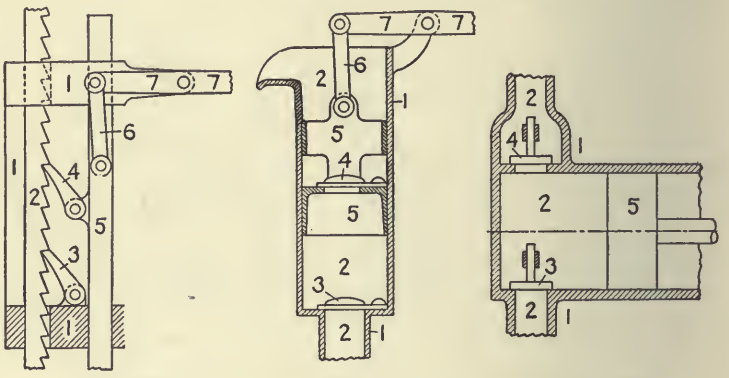

Fig. 150. Fig. 151. Fig. 152.

Lifting ratchet and pumps.

more usual force pump, outlined in Fig. 152, the similarity stops at overall effect and does not run to detail. With both valves seated on the pump body and movement of the fluid column produced by varying the cross section of a shut-off portion of its channel, we have a type of action which is beyond the capabilities of a solid machine member.

(*d*) **Double-acting Ratchets.**—By using two pawls on a double-armed lever, Figs. 153 and 154, a ratchet motion can be made nearly continuous, in the sense that the pause between advances is practically eliminated. Note that when the fulcrum is thus located elsewhere than at the wheel center a small turning movement of pawl on tooth is introduced, combination 1235 or 1245 becoming in effect a four-member linkage.

In Fig. 155 the pawls are so proportioned that the distance between their faces contains an odd half-pitch. This gives

closer engagement, insuring that the average wheel advance shall be more nearly equal to swing of lever; it makes smaller the minimum oscillation that will cause advance of wheel; and it leads up neatly to the idea of the escapement.

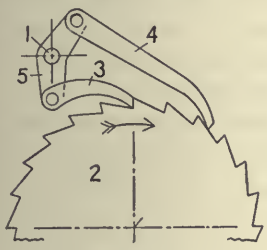

FIG. 153.

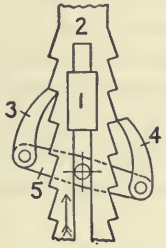

FIG. 154.

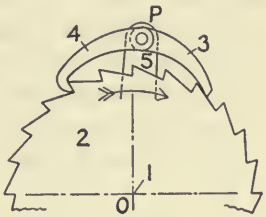

FIG. 155.

Double-acting ratchets.

(*e*) **An Escapement** is a device for permitting a controlled, intermittent advance of a rotating member which is under the steady propulsion of some driving force; it differs from a governor in that the latter regulates speed through a balancing of forces, whereas an escapement exerts a primarily kinematic control of motion, force actions being incidental and of secondary effect.

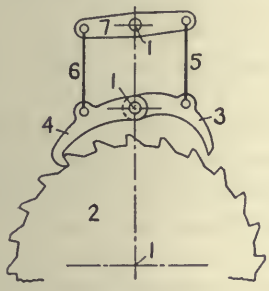

FIG. 156.

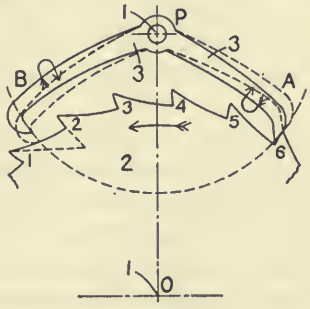

FIG. 157.

Evolution of the escapement.

In terms of Fig. 155, suppose that wheel 2 is under a constant effort that tends to turn it anticlockwise and that its movement is to be positively controlled by ratchet action. Pawls 3 and 4 are now to perform a holding function, hence their pivot P will be on the frame. If the two are alternately raised and lowered by some such mechanism as is suggested in Fig. 156 they will permit the wheel to advance by jumps of half a pitch at a time. Obviously the arrangement can be much simplified by combining

the two pawls into a single member, Fig. 157, to which the necessary oscillation can be given.

In Fig. 157 we have an escapement of clockwork type fully evolved as to parts, but with the practical form of contact surfaces yet to be developed. Wheel 2 is called the escape wheel and pawl ends A and B are pallets, but for the double-pawl 3 (which is oscillated by the pendulum) there does not seem to be any single short trade name: we shall call it the oscillator.

Pallets and teeth are so formed in Fig. 157 that the wheel has no motion during contact, all its advance being by the free motion of successive jumps, each over half a pitch. Since there are fric-

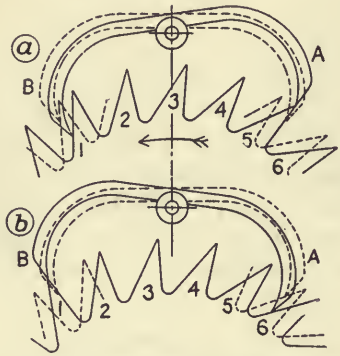

FIG. 158.—Whole motion with contact.

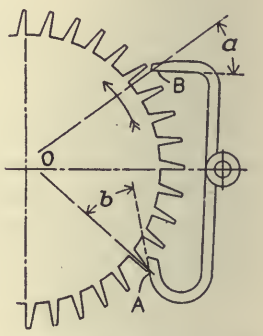

FIG. 159.—Example from clock.

tional resistances within this mechanism, especially to sliding of pallets on teeth, and since the impelling force on wheel 2 can exert no driving effect upon the pallets, this particular escapement will not keep going unless the pendulum or oscillator receives a periodic impulse from some outside source of energy.

Into Fig. 158 is introduced the new principle of slanting the active (left-side) faces of the teeth in such a way that the wheel advances as the pallets rise; through this action an impulse or driving effect is exerted upon the oscillator. As an extreme case, in fullest contrast with Fig. 157, the slant is here so great that the whole movement is thus controlled by contact; for just when either pallet reaches the releasing point—A at (*a*), B at (*b*), full lines—the other comes into contact with its tooth.

The layout in Fig. 158 is open to several objections, as follows:

1. There is too close a limitation of motion of the oscillator, each end of its swing being definitely stopped by mechanical

contact. The same condition exists in Fig. 157, but there it is a question merely of the back or clearance side of the tooth, and freedom can easily be gained by cutting the space deeper, as suggested by the dotted profile between teeth 1 and 2.

2. Excessive slant and too much contact-controlled motion means too great an impulse upon oscillator, with the result that the timing of the pendulum will be unduly affected by variation in strength of driving effort on escape wheel: in other words, a spring-driven clock with this escapement would run a good deal faster when freshly wound than when nearly run down.

3. In the tooth-and-pallet pair it is now apparent that one element will have a profile, the other a sharp or but slightly rounded point or edge. For a good distribution of wear the profile, not the edge, should be on the pallet, the points going to the many teeth with which it works.

An example from a common shelf clock is given in Fig. 159. There is plenty of freedom for enlarged swing of pendulum, but the tendency is to settle down to a swing just sufficient to make pallets clear teeth. With that range of oscillation the proportions of motion are, for each pallet, about one-quarter pitch of contact movement and one-quarter of free movement. Strength of impulse on pallet is dependent upon the angles marked *a* and *b*, between pallet faces and lines OA and OB that would be radial if the pallet was in middle position: here these angles are equal, giving a well balanced action.

The problem of escapement design, particularly for high-grade time pieces and chronometers, is an intricate one, of the kind to be worked out by specialists. A good collection of typical examples will be found in Robinson's "Principles of Mechanism," pages 225 to 235.

(*f*) **Intermittent Gearing.**—A pair of circular gears can be so formed that the follower will alternately roll with the driver and stand still while the driver advances. The simplest type of form and action, represented by Fig. 160, uses only the same kind of teeth as for continuous operation. This example is made up from what would be a pair of 20-tooth equal gears, to meet the requirement that the follower shall advance one-tenth turn for each turn of the driver. The unit of rolling action is the two-pitch angle *a*, as marked on both wheels, the single driver tooth working with one space on the follower over the range between dotted positions of its profile. During the rest of a driver turn

the follower is held fixed by a stop-plate combination, consisting of concentric circle S_1 on D and concave arc S_2 on F: whatever the conditions external to the pair, this definite positioning of F is necessary to the proper initiation of its next movement.

This type of pair can be varied a good deal; there may be more than one arc of action on the driver, and the several advances of follower may be of unequal length. Generally the cycle between like relative positions of the two wheels will be shorter than the ten driver turns in Fig. 160. The scheme involves abrupt starts and stops of follower, hence is good only for low and moderate

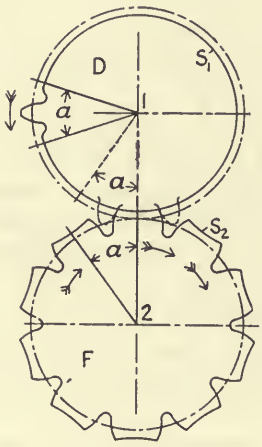

FIG. 160.—Pair for counter.

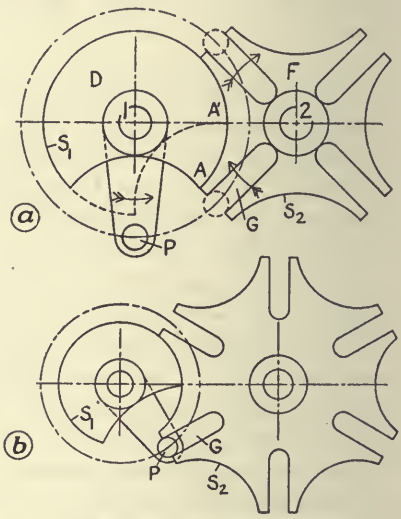

FIG. 161.—"Geneva Stop" pairs.

speeds. The example would be suitable for a revolutions counter, as used on engines and other machines, having from four to six dials in sequence. The first dial or wheel is commonly actuated by an oscillating lever through ratchet drive (with spring clicks, pushing and holding); and since the ratchet engages and releases when the lever is near end of swing, hence moving slowly, conditions are less severe than for continuous rotation of the first wheel.

Figure 161 shows a rather special device which gives accelerated and retarded movement in simple fashion. In case (*a*) pin (or roller) P enters groove G tangent to its sides, and thus can start the follower without shock. Note how stop plate S_1, by having its corner at A′ reach the line of wheel centers just as P enters

G, releases the follower for advance. Obviously recess action is like, or symmetrical with, approach action. In case (*b*) the follower carries more interval units and P enters G a little beyond tangency, sacrificing a very little of the smoothness of motion in (*a*).

A single example, Fig. 162, is given to show how special teeth or cams may be used to give smooth acceleration and retardation at the ends of a period of uniform rolling. In the position drawn wheel F is just beginning a uniform acceleration as the result of which points A and B will reach position C together. The layout

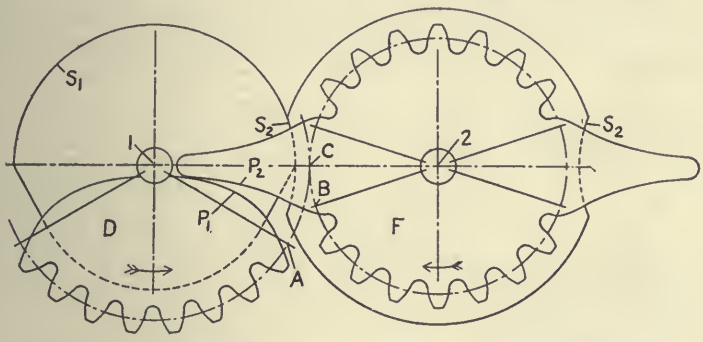

FIG. 162.—Accelerating-gear pair.

of these accelerating teeth is an interesting problem, to be taken up in connection with others of the same class. The stop plates now form a separate system, being fastened back of the wheels; really plate S_1 carries the mere sector to which wheel D is reduced.

(*g*) **Other Intermittent Motions.**—For introductory purposes the ground has been sufficiently covered by the examples given. Going farther we should run into problems of advanced kinematics or into intricate and extensive mechanisms requiring description too lengthy for this book. The best single reference for further information is Jones' "Mechanisms and Mechanical Movements," Chap. VI.

SECTION 13. CLUTCHES

(*a*) **Service and Types.**—The function of a clutch is to make a temporary and easily separable connection between two machine members which turn on the same axis. Sometimes it is a matter of the end-to-end coupling of two shaft sections, as suggested in Fig. 163 and as in the transmission rig of an auto-

mobile; but quite as often and in a wider variety of cases the connection is between a shaft and a wheel or pulley upon it. There are two types of clutches, the positive or jaw clutch and the friction clutch. The first is simpler and more powerful, but for its engagement without shock the two bodies must have either the same motion (commonly rest) or a relative motion that is smaller as the masses involved are larger. Since a friction clutch may be thrown in gradually and will slip while the driven member is being accelerated it can cope successfully with big differences of speed.

(*b*) **Positive Clutches.**—A simple shaft-coupling jaw clutch is shown in Fig. 163, externally and by a sectional view. Half 1 is keyed fast to its shaft half 2 is splined so that it can be

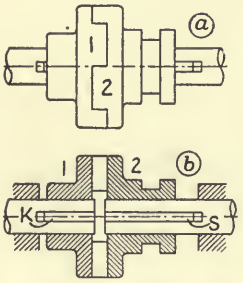

FIG. 163.—A simple jaw clutch.

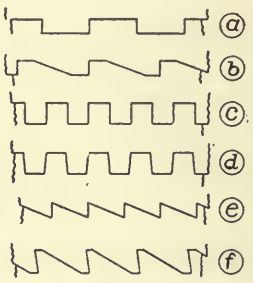

FIG. 164.—Clutch-tooth profiles.

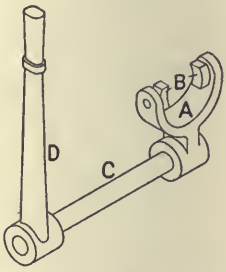

FIG. 165.—Clutch shifter.

moved longitudinally by some such shifting rig as is outlined in Fig. 165. Fairly good alignment must be maintained by bearings close to the clutch on both sides, and the jaws or teeth must fit loosely so that they can engage even if the two shafts are a little out of line. Even with this rough-and-ready form of clutch it is better to have the two rotating members enter into a reciprocal journal and bearing, such as can be seen in Fig. 175; and with the small clearances of most friction clutches this feature of self-alignment becomes necessary.

Typical tooth profiles, unrolled or "developed," are given in Fig. 164. Form (*a*), with the teeth wide and relatively shallow, is used for large clutches that are to transmit heavy forces in either direction. When but one direction of drive can possibly be required the more easily engaged form at (*b*) may be adopted. In light drives, as in the feed trains of machine tools, the teeth are given the narrow square shape at (*c*), or sometimes slightly

tapered as at (*d*), so as to facilitate entrance; while for single direction the saw-tooth profile (*e*) is common: with narrow teeth the angle between mating positions of the jaws is small and engagement can be quicker. Occasionally it is desirable to insure that the clutch shall not pull out while under load—typically on the hand-starting crank of an automobile—and this can be secured by slightly undercutting the teeth as at (*f*).

Figure 165 shows a simple shifting rig which takes the single function of throwing a jaw clutch in or out—suitable for such service as in Fig. 132. Commonly there is added some device for holding the lever either at neutral or at one or the other engaged position; while for big and heavy clutches (of any type) some more powerful mechanism than a simple lever will be used. Bearing in the collar slot may be by short blocks as here or by a complete ring (of halves bolted together like an eccentric strap) with trunnions.

(*c*) **Periodic Positive Clutches.**—The most prominent and important field of service in which jaw or tooth clutches have frequent action is found in the mechanism of punching and

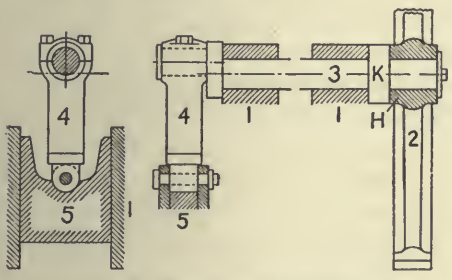

FIG. 166.—Mechanism of a punch press.

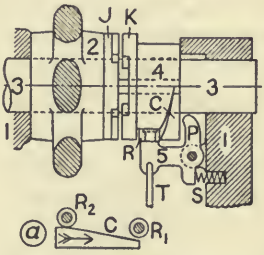

FIG. 167.—Use of jaw clutch.

shearing machines and of power presses. As shown by the outline in Fig. 166, there is a large driving wheel 2 which turns loosely on shaft 3 except when clutch K is engaged; and a short-arm crank or eccentric on the shaft drives ram 5 through rod 4. Motion is comparatively slow, the driven parts are relatively light and there is no load resistance at the time when the clutch catches or releases: these conditions all favor the positive type, while the very heavy resistance encountered when tool strikes work practically requires it. On extra large and powerful machines, however, the clutch is likely to be taken from the crank shaft and placed on a quicker-running shaft in the drive

train—say the one carrying the pinion that engages with big gear 2 of Fig. 166: in that case a strong friction clutch will be used, but its automatic release, in the manner now to be described, will be determined and effected by the crank shaft.

An essential feature of this service is that the clutch is required to release when the crank shaft is at a particular angular position, or when the ram is at the top of its stroke. We shall now view a series of examples selected to show different forms of the clutch itself and of the automatic releasing device.

In Fig. 167 a regular jaw clutch is used, type (*a*) of Fig. 164, with its one half in the form of a steel ring J fastened to the hub of wheel 2. Sliding jaw K, solid with sleeve 4, is controlled by trip lever 5, which is actuated by rod T from the treadle. When the operator presses down this treadle roller R drops away from cam C and arm P pushes 4 to the left: with three teeth the longest wait for engagement is just short of one-third of a turn of wheel 2. When the foot is raised lever 5 rises, and as the turn is completed the working of cam C with roller R opens the clutch: sketch (*a*) shows the relative movement of cam and roller.

(*d*) **Bolt or Key Clutches.**—The large mass of piece 4 in Fig. 167 makes the clutch rather sluggish in movement. For lighter and quicker service the moving member takes the form of a mere latch-bolt or sliding key, as in Fig. 168, which represents a number of designs that are essentially equivalent while differing widely in detail. Here part K is an enlargement of the shaft into which is cut a slot for slide 4, whose right-hand end serves as bolt B to enter one of four slots J in the wheel hub. The action of release cam C should be clear from the drawing: note the interchange of parts as compared with Fig. 168, the cam being now on the trip lever, its follower on slide 4.

A development of this type into the field of heavy service is shown by Fig. 169. One feature that enters into all advanced designs is the use of small detachable parts for striking pieces, as here the stout pins P. These are made of hard steel and can be replaced if they wear out. Lever 5 is part of the clutch mechanism proper, remaining always in connection with slide 4; and trip lever 6 is only a light piece mounted on 5, as seen in view (*c*). Release cam C is now fast upon the shaft and acts on roller R to swing lever 5.

An interesting variation of the key clutch appears in Fig. 170, where engagement is made by means of an oscillating key.

In wheel 2 there is a heavy bronze bushing 3, turning on shaft 4 between collar 5 and cap 6 which are fast upon the shaft. Half-

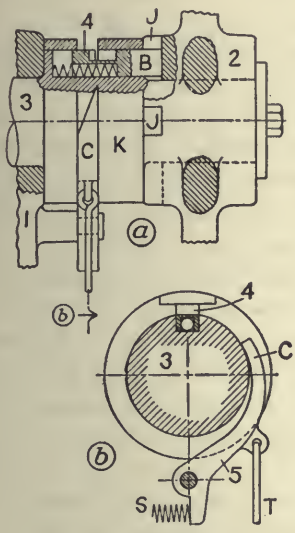

FIG. 168.—Sliding-key clutch.

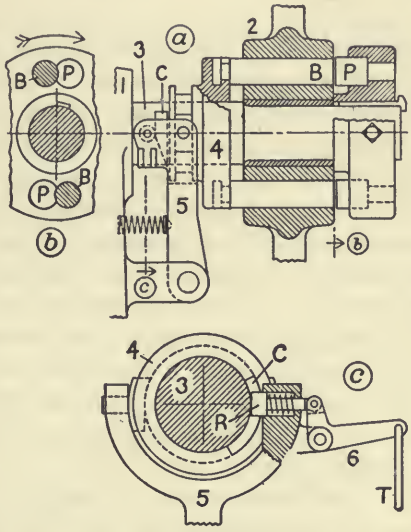

FIG. 169.—Mossberg bolt clutch.

round steel key 7, fully cylindrical at the ends, is seated in the shaft and two key-ways for it are cut in bushing 3: these may

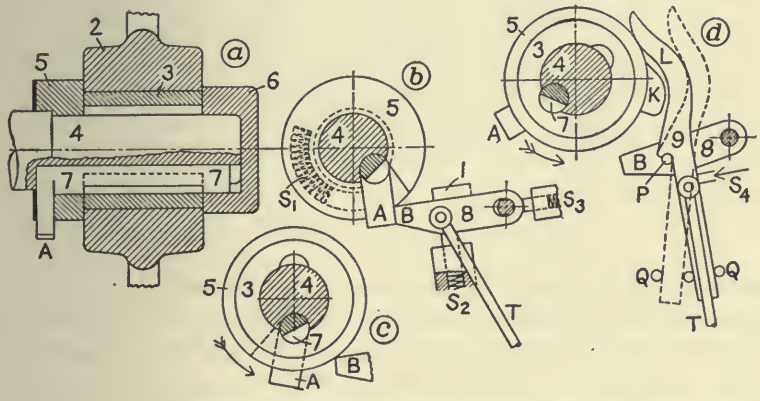

FIG. 170.—Bliss clutch with oscillating key.

even have inserted striking pieces of hard steel on the active edges. View (*b*) shows the key in free position, stop B pushing

arm A to the left against spring S_1: in view (*c*) the clutch is engaged, with arm A swung to the right, but the clutch is approaching release. Note the cushion spring S_3, to ease the action of stop B.

In view (*d*) of Fig. 170 is given a representative example of a type of safety device that can be used with various clutches of this general class, to prevent the machine from repeating a stroke whether or not the operator releases the treadle. Instead of the plain treadle rod T directly to stop piece 8 as at (*b*), there is an added hook piece 9 which engages pin P. Somewhat before the shaft turns to its stop position knock-off cam K pushes lever 9 to the right (on a guide pin at Q as fulcrum) and lets stop piece 8 swing upward. Before another stroke can be made the operator must raise his foot so as to get a new hold on pin P. If desired this safety release can be thrown out of action, by a shift at guide pins Q which swings lever 9 to the dotted position but leaves it free to make the normal up-and-down movement that oscillates piece 8.

(*e*) **The Wedge Clutch.**—Because of its special use in presses—although not restricted to such service—the clutch type in Fig.

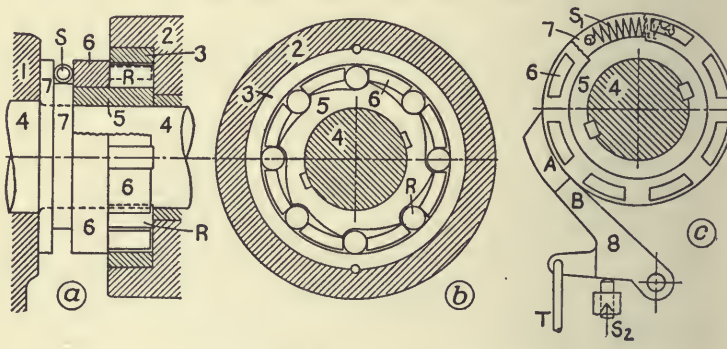

FIG. 171.—The Horton roll-wedge clutch.

171 may appropriately be introduced here. Further, it naturally comes between positive and friction clutches, for while acting by friction it has almost the surely positive hold of a tooth or key engagement. The active parts of this clutch are, a hardened steel outer ring 3 (fastened within the wheel hub) and cam ring 5 (keyed to the shaft), with a series of steel rollers R and a clutch ring 6. This ring has on its side projections which fill the spaces between rollers, serving as a cage much like that

used to space rollers in a bearing: it has also a small angle of oscillation on the shaft, controlled by spring S_1 of which one end is fast to collar 7, the other to ring 6.

In the free position of the clutch, view (*c*), the pressure of stop arm B upon shoulder A (fast on 6) pushes ring 6 backward, holding the rollers as in view (*b*). When arm B is pulled out the spring rotates 6 anticlockwise on the shaft, drawing the rollers up the cam slopes till they wedge inside ring 3. The action is distinctly one of wedging, but is a good deal sharper than it would be with wedges of the characteristic tapered form; for with rollers practically no frictional resistance is offered to the gripping movement. There is, of course, severe compression at the roller contacts and highly resistant materials must be used. Not only does the sharp wedging action inhibit initial slip but such slip (if produced by overloading) would be quickly destructive, by wearing flats on the rollers. Real tapered wedges have also been used in designs similar to this in general manner of action, giving a somewhat less positive engagement but being less liable to injury by overloading.

Because of its quick and strong engagement this may well be called a positive friction clutch; and in lighter service it is also known as a friction ratchet.

(*f*) **Friction Clutches.**—Into a study of friction clutches enter three main considerations, namely, form and disposal of contact surfaces, material and lubrication as affecting character of friction, and manner of producing the gripping pressure. These will all appear in the presentation of typical examples that follows, but the quantitative study of frictional action will be deferred to the chapter dealing with friction in general. Since form and arrangement are a good deal influenced by method of gripping, two contrasting examples illustrating in simple terms the two typical ways of producing contact pressure will first be taken up.

For this purpose we use, in Figs. 172 and 173, a plain cone clutch such as finds place in the feed train of a lathe, Fig. 128. The outer, hollow cone is formed in gear 3 which is free to turn on shaft 2 against a bearing or collar at the left, the inner, solid cone is on piece 4 which is splined to the shaft. In the first case the clutch is gripped or closed by means of forces coming from without and acting through continually moving contacts at side bearing A and collar B. In Fig. 173, on the contrary, there is no

movement at contacts A and B after the clutch is gripped: here shaft 2 is drilled for rod 6 and cotter 5 runs through a slot in the rod—see Fig. 136 for another view of such an arrangement—connecting it to piece 4 and forming a self-contained gripping device.

In order to get short defining names we shall say that a clutch with outside forces, like Fig. 172, has "open grip," while one with self-contained force action has "closed grip." The latter is, of course, decidedly the preferable type where large forces are involved, but with small forces (commonly at high speed) and for the sake of simplicity in moving parts the other scheme may sometimes have predominant advantage.

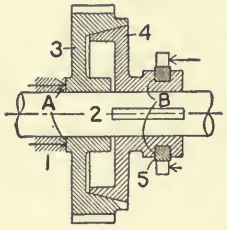

FIG. 172.—Simple friction clutch.

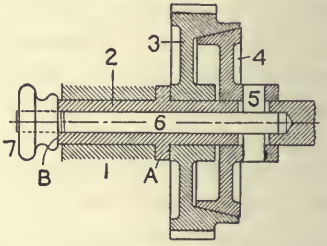

FIG. 173.—Self-contained clutch gear.

(*g*) **Forms of Clutch Elements.**—These are shown partly by the simple sketches in Figs. 174 to 180, partly by the succeeding drawings in which operating mechanism also appears. Certain matters of uniform notation and convention in these drawings are as follows: Piece 1 is the driving shaft, to which the driving member of the clutch is either keyed (mark K) or splined (mark S). Piece 2 is the driven member, in most cases shown as if intended to carry a gear, pulley or drum, Fig. 177, although it may equally well be the other half of a shaft-coupling clutch, Fig. 175. Higher numbers are used on other parts, whether movable or fast on 2 or 3. Rubbing or frictional surfaces are marked F and symbol *P* designates the gripping forces. The use of special material for the contacts is indicated, but will be discussed later.

Now as regards form and disposal of contact bodies, with the closely associated question as to how pressure is developed to produce friction, distinctions or variations may be noted along a number of lines.

1. *Geometrical Form of Contact Surfaces.*—Here we have the plane surfaces of discs, Figs. 174 to 176, 181 and 182; cylindrical surfaces of rings or drums, Figs. 183 to 190; conical surfaces in Figs. 177 to 180 and 191 to 194; with one mixed example in Fig. 179. Reason can be seen for saying that Figs. 178 to 180 are of cone-wedge rather than plain cone type.

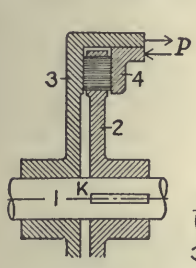

FIG. 174.

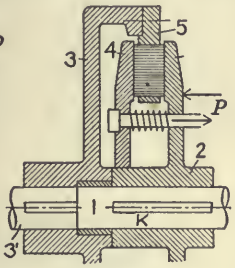

FIG. 175.

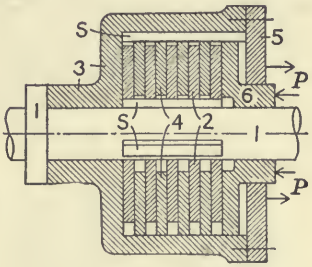

FIG. 176.

Types of disc clutch.

2. *Number of Pairs of Contact Surfaces.*—Single contact is found in the common form of cone clutch, Figs. 177 and 191 to 194, also in the ring clutch with pressure due to tangential force, Figs. 185 to 190. Double contact is almost necessary in the simple disc clutch of Figs. 174, 175, 181 and 182, and in the ring

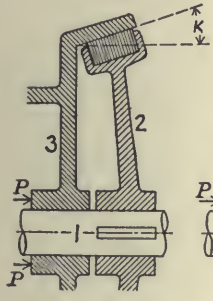

FIG. 177.

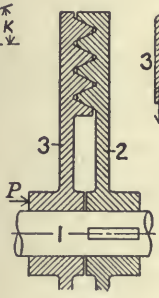

FIG. 178.

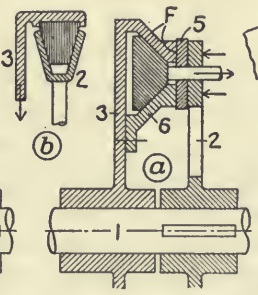

FIG. 179.

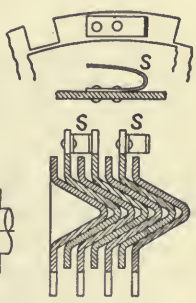

FIG. 180.

Cone and wedge clutches.

clutch with block grips, Figs. 183 and 184. The multidisc clutches, plain in Fig. 176, cone-wedge in Fig. 180, constitute a rather distinct class.

3. *Contact Continuous or Segmental.*—In all of the disc and cone arrangements each contact surface is formed on a single solid body, continuous around the circle: then contact is said to be

continuous, even though one surface may be made up of separate blocks (say of hard wood) inserted in pockets—see Figs. 174, 175, 177 and 181—or fast to a band, Fig. 188. By segmental contact is meant the kind in Figs. 183 and 184, where the blocks 4 and 5 really are short separate segments, the complete unit shown in either sectional drawing being repeated a number of times around the shaft. Each pair of block jaws must, of course, be strongly held against rotational resistance by their carrying frame or "spider" 2. In Fig. 185 the expanding ring 4 is in two or three segments, with a separating mechanism at each gap; but since they really form a single system contact will not be called discontinuous.

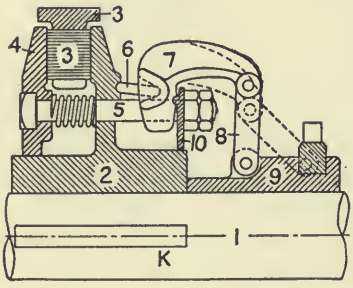

FIG. 181.—Wood disc clutch.

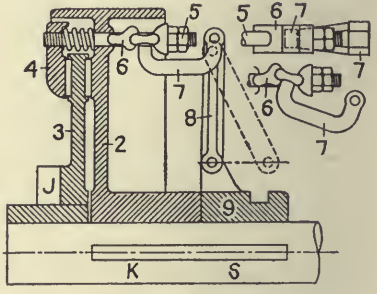

FIG. 182.—Link-Belt disc clutch.

4. *Force Normal or Circumferential.*—In all the disc clutches and in the ring-and-block form of Figs. 183 and 184 the gripping pressures act directly, normal to contact surfaces. In cone clutches the term "normal" is perhaps strained a little, but at least the effective component is of this same type. With an expanding or contracting ring, Figs. 185 to 190, the gripping force is applied tangentially, producing circumferential compression or tension, which in turn causes normal pressure between surfaces.

(*h*) **Clutch Mechanisms.**—Before going into this subject turn to Sect. 30 (*p*) and get the descriptive definition of a toggle mechanism. Requiring the development of heavy pressure just as they close up, clutches very largely employ the toggle principle, either in successive units or along with powerful levers. Thus in each of Figs. 181 and 182 there is a primary gripping toggle 67 and a secondary closing and holding toggle 789. Naming the parts, we have draw bolt 5, toggle strut 6, toggle lever 7, push link 8 and collar sleeve 9: lever 7 straddles the bolt in both

designs, strut 6 does so in the second. Four or six of these units enter into a complete clutch, spaced around the shaft and actuated by a common collar sleeve; in Fig. 181 there is also a common fulcrum ring 10 for levers 7. Springs are used to separate the gripping rings and thus eliminate any tendency to drag when the clutch is open. By means of lock nuts the tightness of grip can be regulated, when setting up a new clutch or when taking up wear. Note that in Fig. 181 the main toggle 67 does not quite come to its dead point, but link 8 is carried a little past perpendicularity to shaft. In Fig. 182 both units are just at dead point when the clutch is closed.

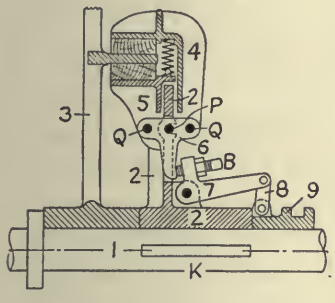

FIG. 183.—Hill ring clutch.

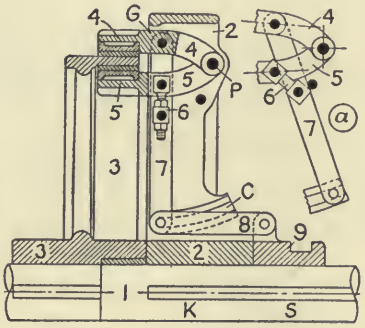

FIG. 184.—Ewart ring clutch.

In Fig. 183, which is typical of a number of designs for the class of clutch, primary unit 456 is a lever mechanism. As it is here laid out, middle pin P takes the thrust of set screw B but does not enter into the system of vertical forces on jaw pins Q, Q. With the entirely separate gripping units of such a clutch—as against the continuous rings in Figs. 181 and 182—uniformity of adjustment becomes highly essential.

Figure 184 shows several special features. Separate friction shoes are bolted fast to levers 4 and 5. The latter are pivoted to the frame at P, and the combination of these levers with the (double) link 7 must have a fairly close fit in the slot in frame 2 at its front edge G, where the driving force is transferred. The toggle mechanism is of mixed kind, with member 7 is tension, adjustable strut 6 in compression. The linkage itself does not position the jaws when open, leaving this function to be taken by cam slot C, as indicated in added view (*a*).

In Fig. 185 is seen a probably unique case of the use of right-and-left screws. With coarse-pitch double threads a small swing of arm 6 gives ample change of circumference. Note the counterweight on 6 for centrifugal balance, also the provision for screw adjustment by slight rotation and reclamping of nuts 5, 5. Each ring segment is anchored to frame 2 by a pin bolt, as seen at the bottom of view (*a*).

The tangential mechanism for an especially compact ring clutch is shown in Fig. 186. Here casing 3 and ring 4 are both of hard steel, the latter being strongly anchored to piece 2. Expanding levers 5, 5, bear upon each other through extra hard fulcrum blocks. These levers are operated by sliding cam 6, fast to sleeve

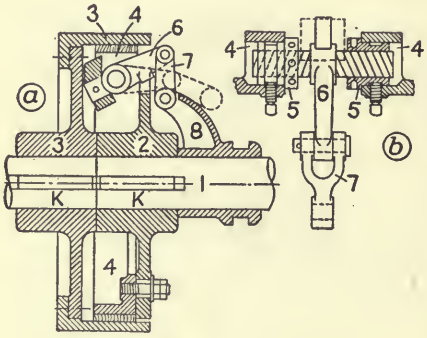

FIG. 185.—Brown ring clutch.

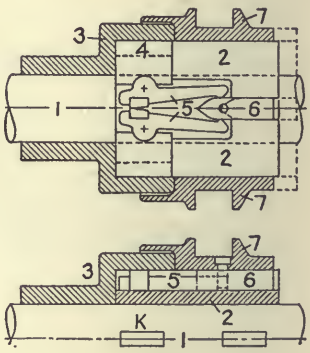

FIG. 186.—Johnson clutch.

7. Adjustment can be made by inserting thin shims back of one of the fulcrum blocks or by a screw-adjusted wedge back of a block. Other devices for the same kind of service use a pair of rollers pressed between flat wedges on the ring ends just as cam 6 is here pressed between the levers, or else have an elliptical dog in the gap, on a lever that can be swung so as to pry the ends apart; and the ring is often made in halves, with provision for adjustment at the other joint or gap.

Any further examples of operating mechanism will be considered in connection with the clutches to which they belong.

(*i*) **Disc Clutches** from transmission practice—that is, for use with shafting and pulleys—are well represented by Figs. 181 and 182. Like all clutches for heavy and continuous service, they operate with closed grip. The two designs offer an interesting example of inversion, in the interchange of outward- and inward-reaching parts. Single-disc clutches for automobiles commonly

have the general arrangement of Fig. 174, but with disc 2 a plain plate and the special facings on the two gripping rings. Figure 175 is drawn to represent coupling clutches, the end of shaft 1 extending into a bushed bearing in the hub of piece 3.

The multidisc or Weston clutch, merely outlined in Fig. 176, has wide use and varies greatly in design. It should be clear from the sketch that the discs are splined alternately to shaft 1 and to casing 3, outer group 4 including pressure plate 6. By thus using a large number of contact surfaces the effect of a given gripping effort is greatly multiplied. If conditions call for an open grip this cuts down the required collar-bearing pressures, while with the more powerful closed grip it yields high capacity. Further, the scheme is favorable to the use of full lubrication which, while it diminishes the intensity of friction, gives less abrupt action in starting the driven member, or makes the clutch "softer." A limit is set to the number of discs by the increasing difficulty of separating them cleanly and sharply on release. If separation is poor the clutch lets go sluggishly, tends to drag its standing member, and is more or less subject to wear while open. Release action can be accelerated and strengthened by the use of small intermediate springs.

One of the most highly developed clutches in existence is the Hele-Shaw modification of the multidisc principle, using cone-wedge contacts as shown by the elementary sketch in Fig. 180; the general arrangement is like Fig. 176. The plates are alternately of steel and bronze; with narrow contact there is less oil film to retard either engagement or release; and quick release is secured by means of the little U-shaped springs S, S, placed between the outward-reaching plates.

(*j*) **Cylindrical Clutches.**—The three examples in Figs. 183 to 185 complete a group from transmission practice, already described in connection with their mechanism. The compact form in Fig. 186 is a common machine-tool type, used for feed drive and at the high-speed end of all-gear main drives. The rather special design in Fig. 187 introduces an arrangement of mechanism in which an actuating member moves along the axis and makes a gripping lever swing in a plane perpendicular to the axis; other examples of this general scheme follow in Figs. 188 and 189.

Particular interest attaches to Fig. 187 because of the compactness required by the use of this clutch (in multiple) inside

the members of a "cone" of gears—see Fig. 211. Two out of the five units are partially shown by view (*b*), cam C in sliding rod 7 being able to raise any one of five plungers 6. View (*a*) shows the single clutch, with spring lever 5 to expand ring 4

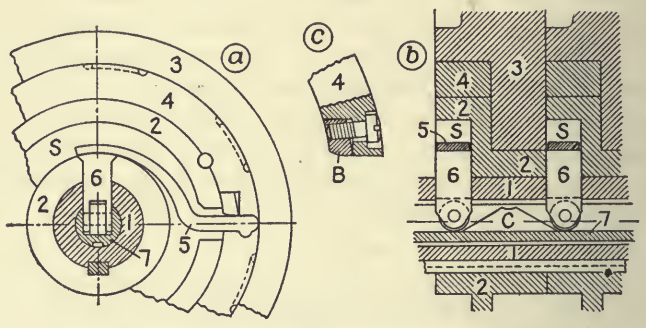

FIG. 187.—Bullard gear-cone clutch, for Fig. 211.

inside of gear 3. An enlarged detail of the adjusting fulcrum block on ring 4 is given at (*c*).

While having a certain kinematic resemblance to Fig. 187, the big hoist clutch in Fig. 188 presents a marked contrast in

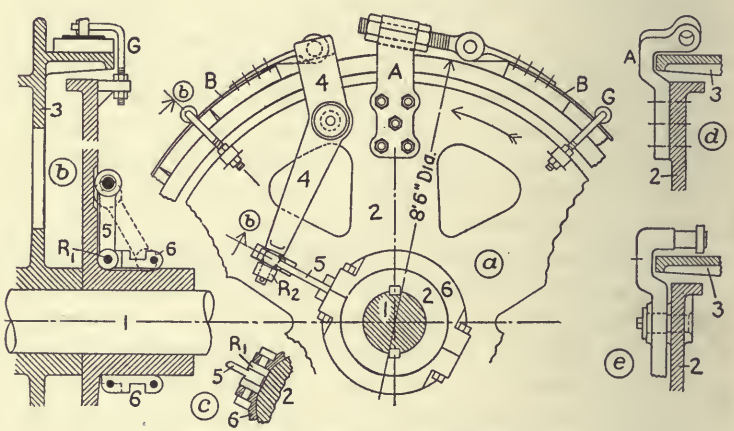

FIG. 188.—Clutch on mine hoist.

form, size and service. Band B, lined with wood blocks, is anchored to spider 2 at A and is gripped by lever 4: it bears on a ring projecting from the end of rope drum 3. Sliding ring 6 operates lever 4 through a single toggle strut 5, which has a

cylinder-on-sphere joint with the lever so as to permit motion in two planes. The ends of strut 5 are guided and their thrust is borne by rollers, R_1 upon the hub of 2, R_2 upon the face of this disc or spider. Note the guides G to hold the band to uniform clearance when relaxed. Direction of rotation must be such as to put the heavy, tight-end tension on anchor A, the lighter, slack-end tension on lever 4.

A powerful clutch of moderate diameter, used especially on wire-drawing machines, is obtained by the employment of a coil band, as shown in Figs. 180 and 190. The band is of spring steel, bored to a smooth fit on the drum but so coiled as to be relaxed by its own elasticity. The heavy-end tension comes on

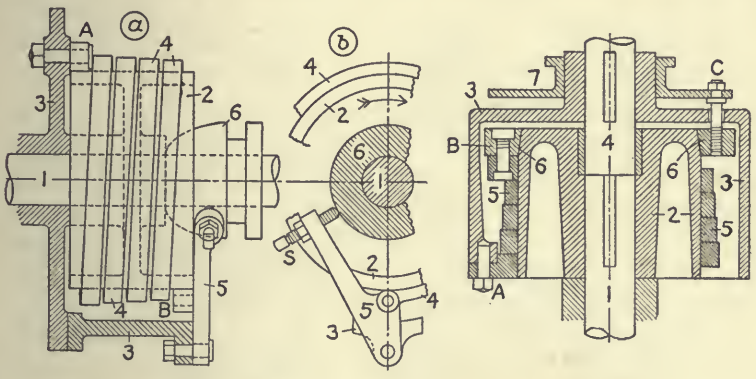

FIG. 189. FIG. 190.

Coil clutches.

anchor A which secures the coil to driven member 3, the light-end gripping tension is commonly applied by some such lever device as is most clearly shown by view (*b*) of Fig. 189.

Figure 190 introduces the relay principle of gripping action, by means of a pilot clutch. Here ring 6 has a cone grip on the upper end of drum 2, being sufficiently applied by its own weight plus that of control ring 7: bolts C pass through holes which are elongated so as to permit all the motion needed for tightening the coil. When ring 6 is dropped its friction imposes ample closing tension to the free end of the coil at B: at end A the spring bar is forged out into a flat shoe which can be fastened to outer drum 5 by several stud bolts in tandem.

This same principle can very well be applied to hoisting drums and the like, a light pilot clutch with open grip working upon a main lever like piece 4 of Fig. 188.

(*k*) **Cone Clutches.**—The conical clutch is widely used in all kinds of service. Of the geometrical forms it is best adapted to open grip, because of a natural wedging action: in the example of Fig. 177 it is so used on a small hoist drum, for the sake of simplicity and low first cost. Now, however, we shall consider some examples from more highly developed machinery, selected from machine-tool and automobile service.

The double clutch in Fig. 191 is used on a line of small high-speed automatic machines, to secure quick and frequent reversal by coupling spindle 1 to either of the pulleys 3, 3: Fig. 244 shows the general arrangement in which this clutch finds place. The

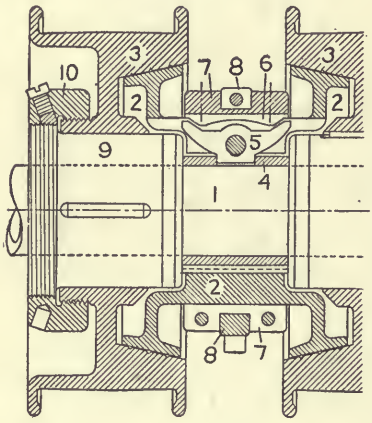

FIG. 191.—Clutch for quick automatic reverse, Brown & Sharpe screw machine.

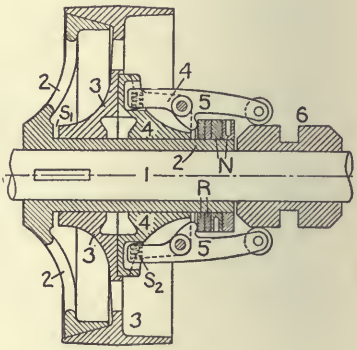

FIG. 192.—Light countershaft clutch, one of pair.

outer cones are formed in cast-iron pulleys 3, the inner cones on bronze casting 2, which is keyed to steel sleeve 4 and thus to spindle 1. Two double dogs 5—the second not appearing here because the halves of this sectional view are not taken in the same plane—are pivoted in piece 2 and engage slots in sleeve 4. The dogs are operated by steel shoes 6 fastened inside of collar sleeve 7, and act to shove cone member 2 to right or left. Grip is adjusted by means of right-and-left screw collar 10, which shifts the pulley bodily on bearing sleeve 9. Other schemes of adjustment used in the group of machines represented by this example are to thread member 3 directly on sleeve or bushing 9 at one end or to make the outer cone a separate ring threaded

into member 3. Here there is a roller bearing inside of bushing 9, together with a collar bearing to take the end thrust of clutch.

As compared with Fig. 191 the countershaft pulley clutch of Fig. 192 inverts or interchanges fixed and moving members, dog lever 5 now acting to move pulley 3 on the sleeve of cone member 2. Note the oil pocket formed within the loose pulley by bolting piece 4 to body 3. The levers are pivoted on 4 and push against bearing rings R and adjusting lock nuts N. A flat spring in the form of a crimped ring is inserted at S_1 to open the clutch, and is helped by the light coil spring S_2 in the end of each lever. An axial movement of a few hundredths of an inch is enough to release a clutch of cone form.

Various examples of the use of cone clutches will be found in Figs. 308 to 310.

(*l*) **Automobile Clutches.**—The friction clutch, placed right behind the engine, is an important member of the transmission system of the gasoline motor car: its function is to release the engine for starting, the transmission for gear changing. Practically all of the general types of gripping elements have been used in this service, but leading place has been taken by the leather-faced cone clutch and by the multidisc clutch, with predominance swinging from the former toward the latter. The only examples here given are of the cone type, outlined in order to illustrate certain general principles but not showing the full detail of highly developed practice.

Noting that the clutch is right in the engine fly wheel we see in Fig. 193 the early use of an open-grip arrangement, the clutch pressure reacting on thrust bearing B_1 and on a similar bearing somewhere along shaft 5. It is universal practice to have the clutch normally closed, by a spring, and open only when the clutch pedal acts on the shifting ring at pins P, P to produce further compression of the spring. Also, it has become general practice to employ a closed grip, as by extending shaft 1 to carry an abutment for the spring in Fig. 194. Now with the clutch open and the car coasting there is motion in spring-seat bearing B_1 as well as in ring bearing B_2.

One respect in which this example does not fully represent good practice is in showing too great rigidity from cone 3 to shaft 5. It is important that the cone have uniform contact all around its circumference, and here a lack of alignment of shaft 5 will not only spoil this contact but will also tend to cramp the hub of 3

on its guide. Shaft 5 should be connected to sleeve 4 by a universal joint and should have another flexible connection to the gear box at its rear end. With the gear box bolted right to the crank case of the engine this need for shaft flexibility disappears, but in that case some form of multidisc clutch is likely to be used.

The light cone 3 as in Figs. 193 and 194 has a leather facing, fastened by a number of small rivets. In order to secure smooth engagement, with enough initial slip to secure easy acceleration of the car, several designs have contained a series of light springs, spaced around the cone, to raise the leather in spots: when full engagement is reached these compress and there is contact over the whole surface. A similar effect is got with cork inserts in an otherwise metal-to-metal clutch: round plugs of specially prepared cork are pressed into shallow pockets in one face; when free they project above the metal surface, under full pressure they are squeezed down flush with that surface. Another plan is to have a double clutch spring, with a rigging that lets the lighter spring come into action first, then puts on the heavy one for steady holding.

In order to get more compact and yet powerful clutches the tendency has been to go to the multi-disc principle, using all metal surfaces in some cases but oftener having one set of discs faced (each on two sides) with asbestos fabric riveted fast. These are run dry so that they have a high coefficient of friction and only a moderate number of pairs, say six or seven, is needed. If separating springs are used (compare Fig. 180) the clutch opens up cleanly and drag is eliminated.

(*m*) **Operating Clutches.**—In most lines of service clutches are operated by hand or foot through some simple lever mechanism, with provision (when needed) for locking or latching the lever in position. On an automatic machine, Fig. 191, the clutch lever is actuated by a cam. Large power-transmission clutches may be moved by hand wheel and screw or by a piston with compressed-air or hydraulic drive: the latter type of device is appropriate for shifting ring 6 in a heavy hoist like Fig. 188.

Thus far we have seen friction clutches gripped or closed by linkage mechanisms or by the direct push of springs. Another scheme is to form a piston or plunger with its cylinder in the body of the clutch, then operate the gripping element by means of a pressure fluid admitted through an axial hole drilled in the

shaft. Maintenance of tight joints, at the shaft-end connection and at the plunger, is a troublesome feature of such devices.

Magnets can also be used, generally to actuate some lever mechanism within the clutch, but this scheme is much more appropriate on the brake of an electric crane hoist, where the magnet acts upon the standing member of the friction pair.

(*n*) **Clutch Materials.**—Summing up the allusions to rubbing-surface materials in preceding descriptions and very briefly rounding out the subject, the following skeleton outline is now given, under heading of the two contrasting types of arrangement.

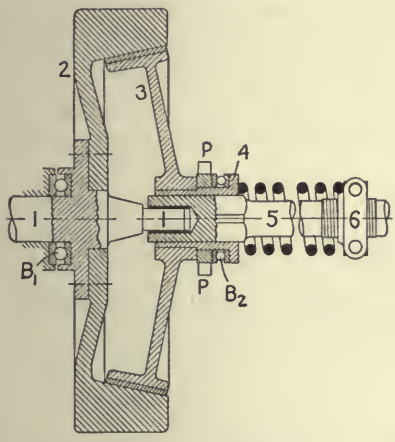

FIG. 193.—Open-grip.

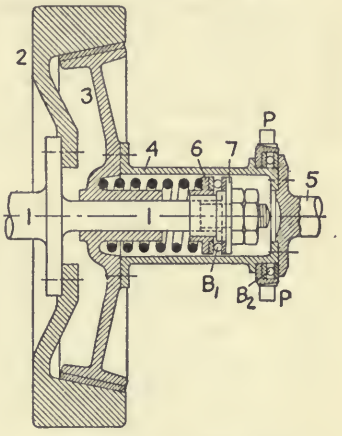

FIG. 194.—Closed-grip.

Automobile clutches, spring-closed.

1. *Metal on Metal.*—For small forces and low speeds, as in the feed trains of machine tools, cast iron on cast iron is satisfactory—see Figs. 172 and 173. This combination is also used in many light transmission clutches. The very powerful small clutch represented by Fig. 186 uses hardened steel for both casing and expansion ring.

Some small Weston clutches with a great many discs, low unit pressure and assured lubrication have all the plates of steel. But with fairly strong pressure, high speed and the possibility of a good deal of slip it is better to use unlike metals, as the steel and bronze named for Fig. 180.

2. *Non-metallic Facing on One Surface.*—For power transmission and for hoisting service wood on cast iron is the usual combination. The wood should be hard, dense and free from gum; maple, elm and beech are the kinds commonly used. When

blocks are set into slots or pockets, Figs. 174, 175, 177, the end-grain surface is exposed; but when flat shoes are riveted fast, Figs. 182, 183, 185, side grain has to be used. The earliest Weston clutches, in transmission service, were made with hard-wood discs between cast-iron plates.

In light and high-speed service, especially in cone clutches for automobiles, leather is a much used material for facing. It is fastened with light rivets and run nearly dry—generally with only enough of a special oil to keep the leather soft.

Artificial compositions such as highly compressed straw board or vulcanized "fiber" make effective facings, especially for light disc clutches: the thin liner in Fig. 184 is of some such material. In automobile service the woven asbestos-and-wire fabric first developed for brake bands has been carried over into clutch use, with the advantage of resistance to heat that may be generated by excessive slipping.

(*o*) **Brakes** are closely related to friction clutches in many ways; for example, the band clutch in Fig. 188 is a direct application of the brake commonly used in that same line of service. The similarity is greatest with those brakes that are like clutches in acting all around a wheel or drum so as to apply a simple torque. With a nonsymmetrical application of pressure and friction, as on the wheels of railway vehicles, force conditions are less simple and involve the analysis of linkage mechanisms. Because of the somewhat greater immediate importance of quantitative knowledge as to friction and of a greater tendency to run into questions of mechanism, it does not seem well to take up brakes in the descriptive manner of the present section. Some discussion of their action will be found in Part II.

REFERENCES ON CLUTCHES

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"Machinery's Encyclopedia," vol. II, pp. 132–144.
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HELDT, "The Gasoline Automobile," vol. II, chap. II.

SECTION 14. REVERSE AND STOP MECHANISMS

(*a*) **Reversing Mechanisms.**—Given a uniformly rotating source of motion, the following solutions have already been offered to the problem of providing for reversible running of a driven gear or wheel train:

1. By the use of open and crossed belts from the line shaft give to the train a choice of motion for its initial member: this covers countershafts with tight-and-loose pulleys, Fig. 103, or with friction clutches, Fig. 132, also the two-belt planer drive, Fig. 130.

2. Design the train so that to a continuously running driving element is given the choice of two members upon which to act: the planer drive in Fig. 131 is of this type, with the belt capable of acting on either pulley.

3. Provide for inserting an additional gear into the train, as in the lathe feed reverse of Fig. 127 and in the automobile gear box, Fig. 133.

The same kind of mechanism variation, namely, change in the number of train elements, really characterizes both of the last two methods, the distinction lying in whether or not the

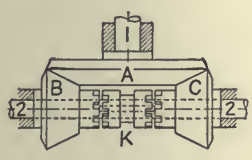

FIG. 195.

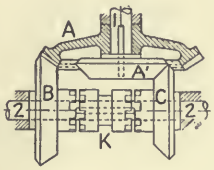

FIG. 196.

Bevel-gear reverse mechanisms.

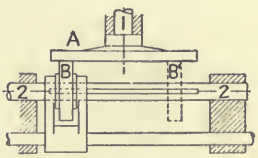

FIG. 197.—Friction-disc reverse.

arrangement is such that the shift can be made automatically: devices that can be operated by hand only fall into class (3).

Another form of reversing unit much used in feed trains and the like is the bevel-gear combination in Figs. 195 and 196. Either of shafts 1 and 2 may be driver, or pinion B, on a sleeve as in Fig. 261, may drive shaft 2 in either direction, with wheel A functioning as an idler. In any case, wheels B and C are loose on their shaft, to which clutch piece K is splined. Note that with shaft 1 driving this mechanism meets the specification of type (2) above, with shaft 2 as driver it conforms to type (1). Figure 196 shows how speed change as well as reversal can be secured if desired. The friction-disc arrangement outlined in Fig. 197 may be considered a kinematic generalization of this bevel-gear unit; it varies speed as the effective radius on A is changed, reverses when B is shifted past the center of A.

(*b*) **Automatic Reverse.**—The form and working of a belt shifting or "shipping" mechanism for the planer is illustrated by

Fig. 198. Two movable dogs F and R, acting at the end of forward and of return stroke respectively, are clamped to the edge of the table as seen in end view (*a*). These push against pin P_1 or P_2 on lever 2, so placed as to accommodate a much longer range of stopping movement for the quick return stroke than for the slow cutting stroke. An omitted detail is the provision for clearing dog R from contact with its pin, by hand, so that the table can be run back on occasion without disturbing the setting of that dog. The action of cam sector 4 with shipper arms 5 and 6 is clear from views (*b*) and (*c*), while view (*d*) shows neutral position, both belts being "off" and the table at rest. Note that the table follows the movement of handle H.

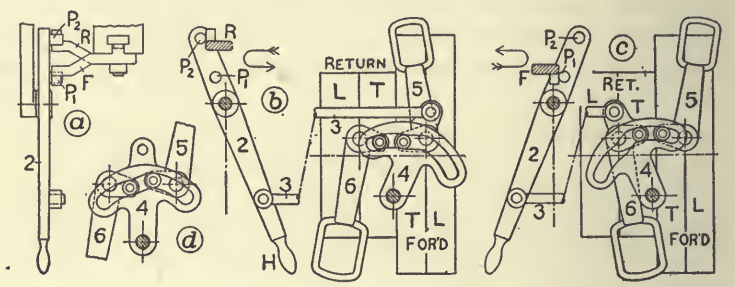

Fig. 198.—Reversing belt shipper for planer.

On a heavy machine such as the planer and with a fairly rapid motion there is sure to be plenty of momentum to carry the table through the range needed for a complete change of belt location; in fact the problem is rather one of braking or of stopping within a limited range. But when the moving mass is small or the motion is very slow a reciprocating table may throw out its active drive and then stop without going far enough to throw in the reverse drive. In order to obviate this possible failure a scheme of secondary or relay control is adopted, of which several types will now be illustrated by selected examples.

Prob. 82. Using a cam device like Fig. 85, sketch full arrangement like Fig. 198 for a planer belt shipper, showing proper relative position of arms for the two pairs of pulleys. Sketch setting for each drive and for neutral.

(*c*) **Reverse Mechanisms for Light Machines.**—Figure 199 shows the mechanism for control of table movement on a surface grinder: views (*a*) and (*c*) are from the front, view (*b*) from the right-hand side. The two parts of (*a*) are in correct relative

position, but between them is inserted (*c*), from behind the lower part and showing the element that is particularly concerned with insuring reversal. Of the main machine members that are no more than suggested, carriage 4 has the cross-feed motion, right and left in view (*b*), then on this carriage table 5 has the longitudinal traverse for work under wheel, on the one flat slide shown and another V slide.

Beginning at the lower right-hand corner of the drawing, power is received by a belt pulley on shaft 2, then through pinion A and gear B or C shaft 3 is driven in either direction. Beyond this shaft there are two pairs of gears and an intermediate like

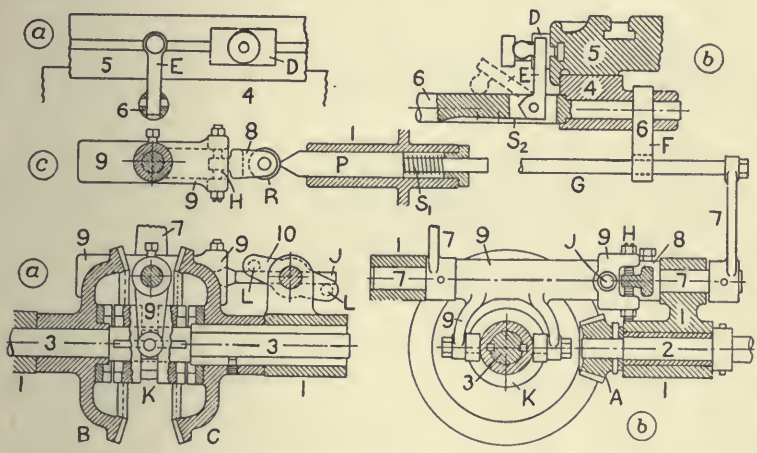

FIG. 199.—Automatic feed reverse for grinding machine.

the bull wheel of a planer, to the rack beneath table 5; and the hand-wheel drive comes into that part of the train. For the control mechanism begin at the top: the push of table dog D upon feed lever E is carried through shaft 6 and fork F to rod G of clutch lever 7-8-9. When the table is moving to the left lever E will of course not be in neutral position as here drawn, but swung toward D; and roller R, view (*c*), will be on the upper wedge-face of spring plunger P. As D pushes on E, swinging it to the vertical, plunger P is pushed back, storing energy in spring S_1. The clutch lets go just before the mid-position drawn, but there is plenty of momentum to carry the rig at least a little beyond neutral, whereupon plunger P acts upon the roller to insure completion of swing.

As a secondary device note first the quick throwout of automatic control, by swinging E to the dotted position in (*b*) and thus giving full freedom for hand control, by means of a lever on the outer end of shaft 6. The whole mechanism can be thrown out or held in neutral as actually drawn, by turning a knob on rod 10 until pins L, L lock radius rod J on clutch lever 9: when released double arm 10 stands vertical. Tooth clearance of the two clutches can be equalized by means of the double-screw adjustment between levers 8 and 9 at H.

The device in Fig. 200 goes a step farther and for its sure action does not require any motion of the table after the clutch lets go; it is therefore well suited to slow feed movements.

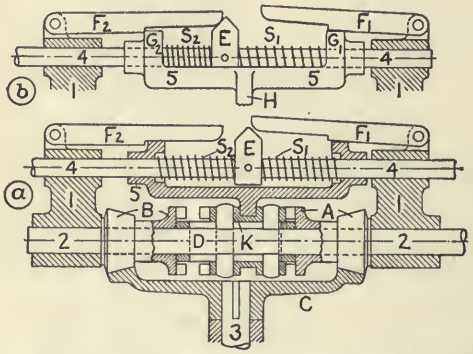

FIG. 200.—Trigger type of feed reverse.

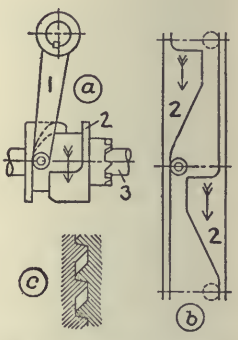

FIG. 201.—Actuating gear for clutches in Fig. 191.

Only the direct clutch-control mechanism is shown and that in diagram fashion, with all the parts swung into one plane. Here shaft 2 drives and 3 runs on to the table gears. The table dogs actuate (indirectly) rod 4, which slides within piece 5 but has a dog E fast to it. With clutch slide 5 to the right and held there by latch F_2, the table is moving so that at the end of its traverse rod 4 will be pushed to the left. In so moving it will compress spring S_2 and relax S_1, as seen in view (*b*), where latch F_2 is about to let go. When it does so the spring will snap the clutch to the left and latch F_1 will drop behind G_1 to hold it there until dog E comes back at the end of this new stroke.

These two grinding-machine mechanisms are alike in making the driven table supply energy to be stored for performing reversal. A different principle, used with the double clutch of Fig. 191, is to get movement for clutch shift from another

source than the driven member. In Fig. 201(*a*) clutch lever 1 is swung by cam 2 on continually running shaft 3: by the action of a timing cam 2 is clutched to 3 for just half a turn, then released. This gives a complete and very quick shift of the main clutch. The latter holds itself in place by friction, and in order to give freedom for movement of lever 1 by hand, as when trying out clutch adjustment, cam slot 2 is widened out as shown by development (*b*). View (*c*) gives the clutch-tooth profile for the kind of engagement required in the mechanisms just considered.

(*d*) **Stops and Trips.**—Under this title might be grouped a large number of devices, dissimilar in form and mechanism but related in their purpose and function. The following series of descriptive definitions outlines the whole field, and is so arranged as to lead to a fuller consideration of certain types associated with gear trains.

Safety Stops.—Besides its regular speed-control governor an engine is often equipped with an overspeed safety stop; if the main governor fails to control and speed rises say five per cent above normal, this device shuts off steam or gas and stops the engine. Sometimes it trips the shutoff valve by means of an electromagnet, in which case push buttons may be installed at several places in a factory, for quick stoppage by hand in event of accident or emergency.

Overload Stops.—In machines of digging, cutting or pressing function, which in positive motion are likely to encounter a sudden abnormal resistance, serious damage is obviated by providing a friction coupling that will slip, a light key or pin that will shear off, or even a relatively weak and easily replaceable member that will be sure to break instead of larger and more costly members.

Operation Stops.—Textile machines and the like, with function of spinning, winding or weaving, are regularly provided with devices by which the breakage of a thread, or perhaps no more than an excessive resistance, will sound an alarm or trip a clutch and stop the machine.

Travel Limitation.—In relation to machine-tool operation this title covers two phases. The first is automatic stoppage because the tool has passed completely over the surface upon which it acts; this may be simple and final or may be followed by automatic reversal, as in the last two articles. The second is precise

stoppage which will determine a dimension of the work piece, as the location of a shoulder on a turned piece or the depth of a hole; this may or may not be automatic.

Position Stops.—In turret lathes and multiple-spindle automatic lathes a highly important function is the accurate indexing of turret or spindle head. By indexing is meant the rotation of a machine member through a definite angle which is some aliquot part of a turn, and a full discussion of that subject will be found in Sect. 25. A rough type of position stop has already been considered, in the group of self-releasing clutches for presses, Figs. 167 to 171. Examples giving precise action are set forth in Arts. (*f*) and (*g*) following.

(*e*) **Stoppage of Feed Motion.**—This can be effected by throwing out a toothed clutch, by releasing a friction clutch, by dropping a worm out of mesh with its wheel or perhaps even by

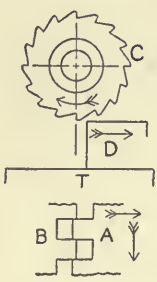

FIG. 202.

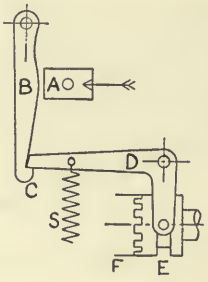

FIG. 203.
Tripping stops.

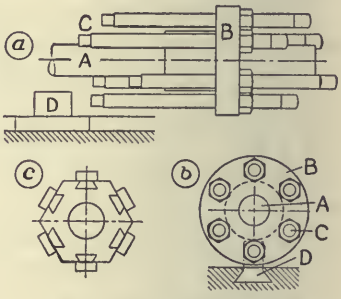

FIG. 204.—Positive stops for turret lathes.

opening a split nut on a screw. Having a unit like Fig. 195 right on the feed screw of a milling machine, it is a simple matter to transmit the action of a table dog directly to the clutch, slowly pushing it out of engagement at end of "stroke." With the common type of milling operation, as sketched in Fig. 202, this scheme is entirely satisfactory, for the work piece must be carried at least a little way past tangency to the cutter (or past a vertical through axis of cutter), so that cutting resistance will have ceased; further, there is no requirement of precision.

For accurate stoppage under cutting load, as on a lathe or drill, such a method of gradual disengagement is not good. As contact narrows unit pressure rises, to a high value just before release; this tends to wear the edges round, so that an originally

sharp action will become more or less blurred and indefinite in timing. The elements of a better scheme are shown in Fig. 203: lever B is so proportioned that the movement of dog A is magnified at catch C; release takes place under only a moderate pressure; and if member F is at the quicker beginning of a train (rather than its slower-moving end) the time value of release action will be diminished. Some such trigger device, with sharp and full action in release, must be used with a friction clutch or a drop worm.

On automatic lathes, or "screw machines," the reciprocating tool movement is commonly produced by cams, so that this question of release from a continually running drive does not arise. With the partly automatic turret lathe the simpler scheme is to have some automatic disengagement that will function just before the desired end of cut, leaving it to the operator to get accurate dimension by hand-feed finish. Of course all such machines are fitted with suitable dead stops, which by definite contact limit the movement of tool carriage. One example is shown by front view (*a*) and end view (*b*) in Fig. 204: bar A reaching back from the turret slide carries disc B in which are six adjustable stops (long set screws), one for each tool. Bar A is geared so as to index with the turret, and the lowest stop comes in contact with block D, clamped in a slot of the machine bed. View (*c*) suggests another form, a hexagon bar having a dovetail slot in each face, along which can be slid and set a little stop pin or block. A different scheme is to have a battery of fixed stop rods (clamped in the bed) and a series of latches on the turret slide, of which a particular one is dropped for action at each turret setting.

(*f*) **Turret Indexing.**—All the elements of a self-indexing lathe turret are well shown by the example in Fig. 205. Carriage 2 slides on bed 1 and carries turret 3 on stud 4. This stud has a square fit A for the turret and a close turning fit B with base 2. On its head C is cut a light spiral gear, to turn a stop bar of the type in Fig. 204, and beneath this head is fastened the star wheel D which appears more fully in views (*b*) and (*c*). Nut E holds the combination together with a running fit, while a hand nut above cap F is used to clamp the turret down hard during a cut.

In plan view (*b*), which is a section taken beneath the turret, an indexing movement is just completed, while partial view (*c*) shows conditions before it begins. After a cut the turret carriage

is moved to the right (by hand) until definitely stopped. Near the end of this movement lever 6 rides up on trip 7, pulling down lock pin 5. By the time a tooth of star wheel D touches pin G on lever 8, view (*c*), pin 5 is clear; and the final movement, to position at (*b*), gives the turret a 60-deg. rotation, limited and determined by stop pin H on the bed. A very small movement to the left trips 7 and lets spring S_1 snap the locking pin into its taper bushing in the bottom of the turret. A light spring S_2, not drawn, allows lever 8 to swing back as necessary but keeps it normally against pin H as at (*c*).

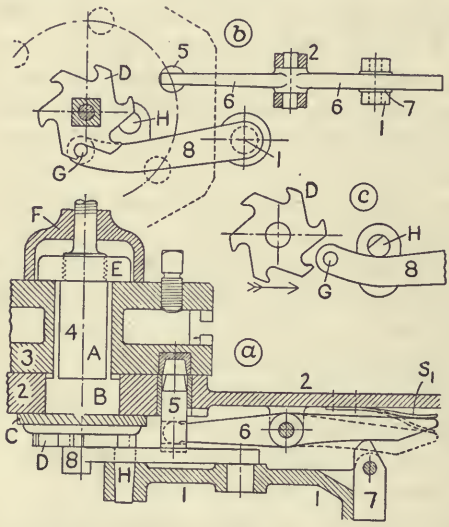

FIG. 205.—Indexing mechanism for lathe turret.

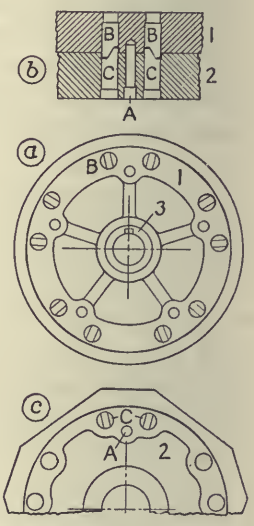

FIG. 206.—Turret lock, Bullard design.

The design whose essential features are given by Fig. 206 shows highly specialized development to meet requirements of precise and rigid positioning against unusually severe force actions. Piece 1 is the carriage, piece 2 the base of the turret, reversed in view (*c*). Intended for a machine comparatively large and with rather long cutting periods, this turret is wholly hand-operated. A swinging handle works clamp and release, and when swung to free position it draws turret 2 forward until index pin A clears the face of base 1—acting positively by a sort of cam. This forward movement also frees the turret for rotation on stud 3, because this stud is slightly tapered and the turret when clamped in place fits too tightly for turning.

On the front extension of stud 3 is keyed a gear, and indexing movement is effected by a pinion carried in the turret; one exact turn of its crank rotates the pentagonal turret through 72 deg., bringing index pin A just in line with one of the holes in base 1. Exact and rigid location is secure by the wedging fit of locking pins C, C with one of the five pairs B, B in the base.

(*g*) **Index Drive for Gear Cutter.**—On an automatic gear cutting machine (using formed milling cutters) an important function is the indexing of the gear blank between cuts. This is preformed by change gears in the train to the master worm wheel —see Fig. 339. To drive this train there is a device which, from a continually running source, can produce a driven movement of exactly one or two or four or perhaps eight turns, then hold position until the proper time in the next cycle of operations. A good example is fully shown by Fig. 207 and is worth close study.

In Fig. 207 side views (*a*) and (*c*) show mechanism to the left of plane YY, view (*b*), views (*d*) and (*g*) show mechanism to right of that plane: these are all left-side views, while (*f*) shows a detail viewed from the right. Similarly, front view (*b*) shows mechanism to the rear of plane XX, view (*a*), while the parts in view (*e*) are mostly in front of that plane.

Drive shaft 1, continually running, carries clutch A which mates with clutch B on short shaft 2. This clutch member B is splined on sleeve C, fast to 2; it is thrown in by spring S_1 and upon it is formed cam D for automatic release by action with pin E on lever 6. The main drive continues through pinion F and gear G to shaft 3, with the special start-and-stop cam pair shown in view (*f*). Shaft 2 can be started abruptly, but 3 carries much more mass and needs more gradual start and stop.

Operation is controlled by rock shaft 5, to which are keyed levers 6 and 8. As the tool carriage completes its return stroke, toward the left, a dog depresses trip lever 10 and through spring plunger 11 gives to shaft 5 a small rotation—but the dog then passes over 10, so as to leave it free to rise, and in the cutting stroke can come back without depressing 10. Provision is also made for hand tripping from two convenient places on the machine, one through arm 9, the other by a handle on the far end of 5, as it extends to the left from view (*b*).

The small rotation of 5, anticlockwise in views (*a*), (*c*) and (*d*), lifts the dogs on lever 8 so as to permit turning of the train,

swings lever 6 to the clear position in view (*c*), and by pressure on fixed pin H throws pawl 7 out of contact with its one-tooth ratchet on B, so that it will not click as B turns. Clutch B now engages and motion continues until the timing device lets lever 8 drop and pin E opens the clutch. The purpose of detent 7 is to prevent the stopped cam from sliding backward on pin E and thus permitting the clutch teeth to catch; and there is enough flywheel action in the mass of body B-C to carry it a little beyond the position of clutch release and let 7 engage.

The timing or counting mechanism consists of gear pair L-M and notched discs N on shaft 4, together with drum J formed on the side of large gear G. This drum has one gap under dog K, so located that toward the end of one revolution of shaft 3 lever 8 may be allowed to drop just in time to release the clutch and stop movement. Counting discs N under adjustable dog P serve to prevent stoppage until shaft 3 has made two or four or eight turns. The position of stoppage, determined by cam pair D-E and detent 7, is such that rollers R, R stand on a vertical line and thus hold cam T and gear G in very definite location.

The diagram in Fig. 208 will be a help in understanding the action of this counting device. The lower part of main view (*a*) is a development of the cylindrical face of disc-group N, treated as a solid cylinder with notches of varying (axial) width. Above it is a series of relative positions of the controlling edge of

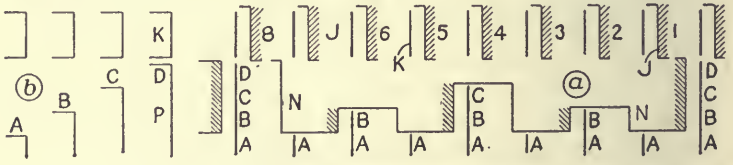

FIG. 208.—Action of counting discs.

the gap in ring J, covering a cycle of eight revolutions of driven shaft 3. At (*b*) are shown the different settings of dog P relative to K and to N. In the instantaneous positions of edges this diagram corresponds with view (*g*) of Fig. 207, which shows condition at the instant when lever 8 has just dropped: J and N have yet to turn to the final standing positions in view (*d*). The heavy cross lines (vertical) in Fig. 208 (*a*) represent dogs K and P.

An essential feature is the pair ratio of gears L and M, here 7 to 8 and due to 28 and 32 teeth. Now for one turn of J shaft 4

with discs N makes seven-eighths of a turn; and if at one-turn intervals for J we took a series of instantaneous views like Fig. 207 (*g*) as regards J and N, the effect would be a slow turning of discs N backward or clockwise, against their actual direction of

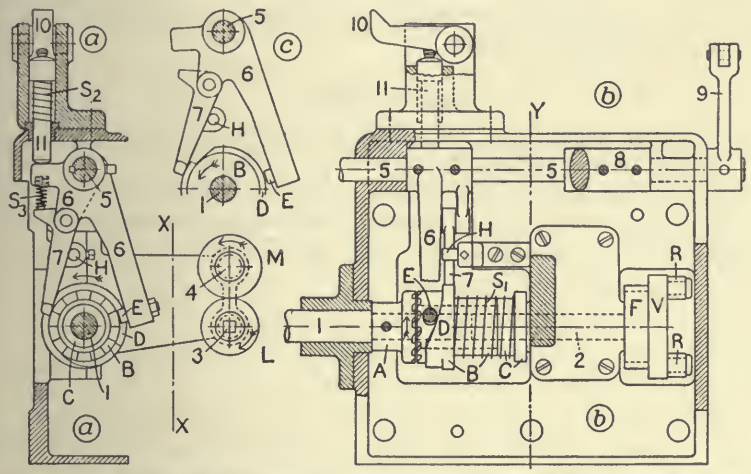

FIG. 207*a*.—Counting mechanism, index drive for Brown & Sharpe gear cutter.

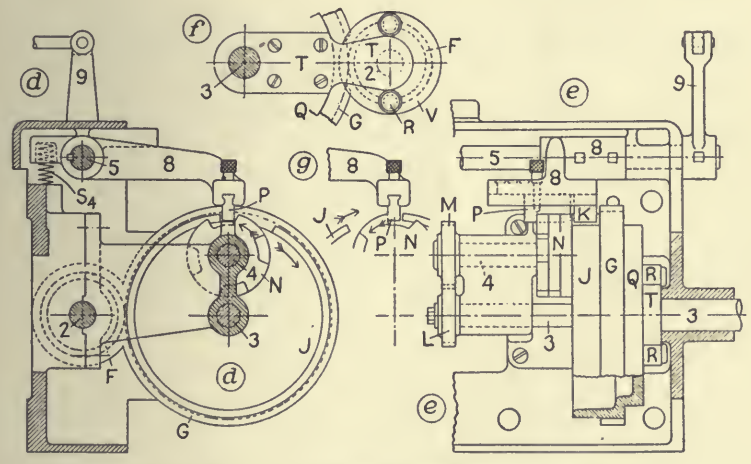

FIG. 207*b*.—Counting mechanism, continued.

rotation. In Fig. 208 (*a*) it is more convenient, however, to hold N fixed and show the relative movement by stepping J one-eighth turn to the left for each revolution. Understanding

this scheme, we see then that with setting A, view (*b*), dog P is clear of N, J controls, and there is stoppage at the end of each single revolution. Setting B requires two turns to bring N and J into proper coincidence, setting C four turns and setting D eight turns. For the smaller rotation series 1, 2, 4, pair L-M has a 3 to 4 ratio and for single turns dog P is set over the first disc of group N instead of outside.

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JONES, "Mechanisms and Mechanical Movements," chap. I, "Reversing Mechanisms," chap. IX, "Clutches and Tripping Mechanisms."

Brown & Sharpe pamphlets "Separate Parts," for machines named.

CHAPTER IV

GEAR TRAINS FOR SPEED VARIATION

SECTION 15. METHODS AND RELATIONS

(*a*) **Speeds in Series.**—In most machine tools, and to a less degree in some machines of other classes, it is desirable that a number of working speeds be obtainable from a single driving speed. The greatest range of variability is needed in machines having rotary final motion, such as lathes, drilling and boring machines and millers. The two typical requirements in the main drives of these machines are, change of linear velocity or cutting speed with a given diameter of work or tool, and maintenance of about the same speed with different diameters.

Both in the main train and in feed gearing (except for screw cutting) a certain number of speeds will cover their ground most effectively and uniformly if they fall into a geometrical series, with a constant step ratio or a uniform proportion of change. If, for instance, this ratio is 1.25, then any higher speed is fixed by adding 25 per cent to the next below, any lower speed by deducting 20 per cent from the one above.

From the viewpoint of calculation, whether in analysis or design, a most useful property of this series is found in the fact that the logarithms of its terms fall into an arithmetical series. Suppose, for example, that we want a well-spaced series of ten speeds from 40 to 400 r.p.m. There will be nine intervals or steps in the rise to ten times the first speed, or in the increase of log N by 1.0000; then if s is the uniform step ratio its value will be got by

$$\log s = 1.0000 \div 9 = 0.1111, \; s = 1.291;$$

and the speed series will be as given in Table 1.

Prob. 83. For each of the turning speeds in Table 1 find the diameter (as of a work piece in a lathe) that will give a circumferential velocity (cutting speed) of 60 ft. per min. What sort of a series do these diameters form?

Get a similar set of diameters for an arithmetical series of ten speeds from 40 to 400 r.p.m., compare it with that just found, and discuss usefulness.

TABLE 1

No.	Log N	N	No.	Log N	N
1	1.6021	40.0	6	2.1576	143.7
2	1.7132	51.7	7	2.2687	185.7
3	1.8243	66.7	8	2.3798	239.8
4	1.9354	86.2	9	2.4909	309.7
5	2.0465	111.3	10	2.6021	400.0

(*b*) **Stepped Cones.**—The commonest device for producing moderate changes of speed is the pair of stepped cones, made up either of belt pulleys or of spur gears. Generally the diameters of the steps are separated by equal differences, or they are in arithmetical progression. How this arrangement is related

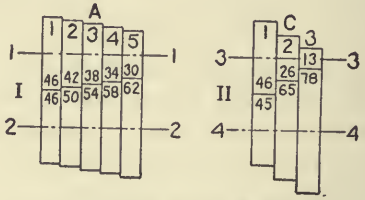

FIG. 209.—Short-step and long-step gear cones.

to the geometrical series of driven speeds will now be considered, using as an example the short-step cone pair I or A-B in Fig. 209; and at the same time the long-step pair II or C-D will also be analysed. It is to be understood, of course, that one cone of each pair (A or C) is solid but that in the other cone (B or D) any one gear at a time can be made fast to the shaft, the others running idle—see Figs. 187 and 138.

FORM 1

Step	Pair I A/B =	m	s	Step	Pair II C/D =	m	s
1	46/46 = 1.0000	1.190		1	46/45 = 1.0222	2.556	
2	42/50 = 0.8400	1.194		2	26/65 = 0.4000	2.400	
3	38/54 = 0.7037	1.200		3	13/78 = 0.1667		
4	34/58 = 0.5862	1.212					
5	30/62 = 0.4839						

Regarding the calculations for these cone pairs, as given in Form 1, the first thing to be noted is that the gear-pair ratio m,

the quotient of driver by follower, is the turning ratio of driven shaft to driver: it is therefore in the direction of what we shall call *value of the train* and is the reciprocal of r_n as used in Sect. 8 (*a*). This change, which will be general, is made because the viewpoint is here so predominantly kinematical. Step ratio s will regularly be taken as the quotient of a higher by a lower value of m or of N.

As to the numerical results in Form 1, it is seen that with the small range of m in pair I the value of s is substantially uniform for equal steps; but with the much greater range in pair II this result is got only by a decided departure from equality in the step differences, the tooth-number change being 20 between steps 1 and 2, only 13 between 2 and 3.

In order to get a more general view of the matter, consider the example worked in Table 2 and plotted in Fig. 210, for a ten-

TABLE 2

Step	d_1	d_2	m	s	s	m	d_1	d_2
1	50	50	1.0000			1.0000	50.00	50.00
				1.222	1.25			
2	45	55	0.8182			0.8000	44.44	55.56
				1.227	1.25			
3	40	60	0.6667			0.6400	39.03	60.97
				1.238	1.25			
4	35	65	0.5385			0.5120	33.86	66.14
				1.257	1.25			
5	30	70	0.4286			0.4096	29.06	70.94
				1.286	1.25			
6	25	75	0.3333			0.3277	24.68	75.32
				1.333	1.25			
7	20	80	0.2500			0.2621	20.77	79.23
				1.417	1.25			
8	15	85	0.1765			0.2097	17.34	82.66
				1.588	1.25			
9	10	90	0.1111			0.1678	14.37	85.63
				2.111	1.25			
10	5	95	0.0526			0.1342	11.83	88.17
	1	2	3	4	5	6	7	8

step pair with diameters changing by 5 at a time from 50 and 50 to 5 and 95. The first section of the table, columns 1 to 4, is concerned with this constant-difference pair. It appears that so long as m does not fall below one-third, or down to step No. 6, ratio s does not depart greatly from a mean value for that range; but in the extreme range of m the departure from a geometrical series is very marked.

For comparison with these "straight" cones the second section of Table 2 represents a stepped pair built up on the constant term ratio $s = 1.25$ for the speed series. The resulting geometrical

series for m (column 6) leads to the diameters in columns 7 and 8. To get these d's use the proportions,

$$d_2 : d_1 :: 1 : m \text{ and } d_2 : (d_1 + d_2) :: 1 : (1 + m),$$
$$\text{whence, since } d_1 + d_2 = 100,$$
$$d_2 = 100 \div (1 + m). \tag{49}$$

In Fig. 210 the two sets of diameters from Table 2 are plotted on a base which, assuming equal gear widths, represents distances along the axes of rotation. To be exact, it is the short diameter d_1 that is fully shown, by the distance from the base to line AB or curve ACD; diameter d_2 would reach the rest of the

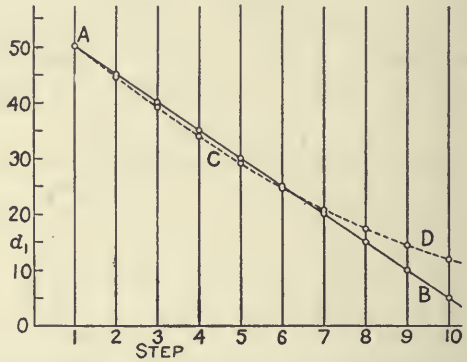

FIG. 210.—Comparison of cone-step diameters.

way up to a line parallel with the base at a distance of 100 units from it. Over to step 6 or 7 the straight line AB on the equal-step cone is in very fair agreement with the curve ACD for a geometrical series of speeds.

In building up a cone of gears there is no such flexibility as would be necessary for the close realization of a curve like ACD in Fig. 210. Because each difference in diameter must correspond to a certain small number of teeth, equality of step differences is the prevailing condition, except when there is so big a range of speed variation as appears in pair II of Fig. 209. With cone pulleys it would be easy enough to make the diameters anything desired; but these seldom run to values of diameter ratio m so far from unity that departure from simple equality of step differences is worth while. The cone-pulley problem is complicated by the variation in sum of diameters that has been considered in Sect. 9 (h) and (i); but with ordinary countershaft drives

this modification is so small that it can be disregarded in speed discussions such as those of Art. (*f*) following.

(*c*) **A Gear-cone Train.**—The complete train from which are taken the cone pairs in Fig. 209 is laid out in Fig. 211, with all the axes of rotation swung into one plane. The driving pulley on shaft 1 receives motion from a countershaft, but need not be stopped in order to stop the machine: that function is taken by the friction clutches within cone B, which have been shown by Fig. 187 and are operated by slip rod K. Cone pair A-B gives five closely spaced speeds with each of the widely different speeds

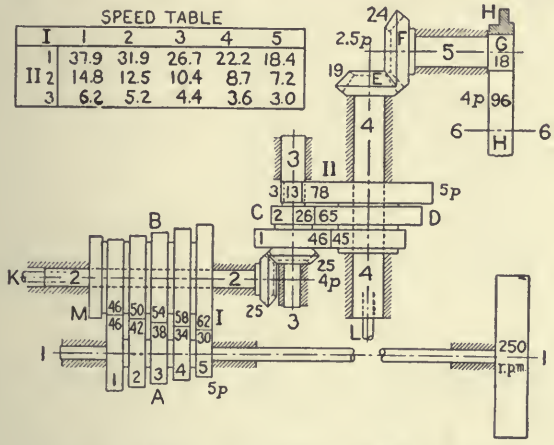

FIG. 211.—Main drive, Bullard boring mill of 1907.

of pair C-D, so that there are fifteen speeds in all. Pinion G drives annular gear H; this is fastened beneath the table of the machine, which turns on axis 6. The sliding key for cone D has been shown in Fig. 138. At M, on shaft 2, is a brake for quickly stopping the machine.

Omitting the miter gears which connect axes 2 and 3, the value of this train is

$$T = \frac{N_6}{N_1} = \frac{a}{b}\frac{c}{d}\frac{e}{f}\frac{g}{h} = m_a m_c t, \qquad (50)$$

the convention being to use T for the whole train and t for that part of it outside of the speed-changing units; also, the letter of the driver is considered a sufficient subscript to designate m. Substituting 250 for N_1, evaluating t and dropping the subscript from N_6, we have

$$N = 250 \cdot \frac{19}{24}\frac{18}{96} m_a m_c = 250 \cdot \frac{19}{128} m_a m_c = 37.11 m_a m_c. \qquad (51)$$

With the m values already recorded in Form 1, the whole series of speeds may easily be calculated by the method set forth in Form 2, where symbol k is used for the factor $N_o t m_c$, appearing as 37.93, 14.84 and 6.185 for the three values of m_c. Instead of relating N_1 and N_6 to axes 1 and 6, we now change to N_o for input speed and N for output speed, reserving the number subscripts to designate individual values of this final speed N.

FORM 2

Speed	A-B	C-D	$m_a \times k = N$	Speed	A-B	C-D	$m_a \times k = N$
1	1	1	$1.000 \times 37.93 = 37.93$	8	3	2	$0.704 \times 14.84 = 10.45$
2	2	1	$0.840 \times 37.93 = 31.86$	9	4	2	$0.586 \times 14.84 = 8.70$
3	3	1	$0.704 \times 37.93 = 26.69$	10	5	2	$0.484 \times 14.84 = 7.18$
4	4	1	$0.586 \times 37.93 = 22.24$	11	1	3	$1.000 \times 6.185 = 6.19$
5	5	1	$0.484 \times 37.93 = 18.36$	12	2	3	$0.850 \times 6.185 = 5.20$
6	1	2	$1.000 \times 14.84 = 14.84$	13	3	3	$0.704 \times 6.185 = 4.35$
7	2	2	$0.840 \times 14.84 = 12.47$	14	4	3	$0.586 \times 6.185 = 3.63$
8	3	2	$0.704 \times 14.84 = 10.45$	15	5	3	$0.484 \times 6.185 = 2.99$

(*d*) **General Methods.**—In Form 2 each group of five speeds, determined by the active step of cone pair C-D, is calculated independently. A slightly more convenient scheme, especially if we wish to make an accurate calculations by means of logarithms, is to work out the first five speeds, then modify them by the successive step ratios s_c of pair C-D. Form 3 outlines the operation and Form 4 shows a complete, condensed computation by means of logarithms.

FORM 3

A-B step	C-D step 1 $m_a \times k_1 = N_{1-5}$	2 $\div s_{12} = N_{6-10}$	3 $\div s_{23} = N_{11-15}$
1	$1.000 \times 37.93 = 37.93$	$\div 2.556 = 14.84$	$\div 2.400 = 6.19$
2	$0.840 \times 37.93 = 31.86$	$\div 2.556 = 12.47$	$\div 2.400 = 5.20$
3	$0.704 \times 37.93 = 26.69$	$\div 2.556 = 10.45$	$\div 2.400 = 4.35$
4	$0.586 \times 37.93 = 22.24$	$\div 2.556 = 8.70$	$\div 2.400 = 3.63$
5	$0.484 \times 37.93 = 18.36$	$\div 2.556 = 7.18$	$\div 2.400 = 2.99$

Lines 12 to 14 and 1 to 5 in Form 4 determine all the numbers in Form 1 except the s values for pair A-D; lines 6 to 11 combine all the factors outside of m_a, in Eq. (51), that are needed for getting the first group of speeds in Form 3; and the rest of the calculation runs right parallel with Form 3. This model is to be followed when the exact solution of problems is called for; with slide-rule work a combination of Forms 1 and 3 will be used.

FORM 4

	C-D step	1	2	3		$N_0 t m_{c1} = k_1$
1	log c	46 1.6628	26 1.4150	13 1.1139	6	log 250 2.3979
2	log d	45 1.6532	65 1.8129	78 1.8921	7	log 19 1.2788
3	log m_c	0.0096	9.6021	9.2218	8	colog 128 7.8928
4		9.6021	9.2218		9	log $N_0 t$ 1.5695
5	log s_c	0.4075	0.3803		10	log m_{c1} 0.0096
					11	log k_1 1.5791

	A-B step	1	2	3	4	5
12	log a	46 1.6628	42 1.6233	38 1.5798	34 1.5315	30 1.4771
13	log b	46 1.6628	50 1.6990	54 1.7324	58 1.7634	62 1.7924
14	log m_a	0.0000	9.9243	9.8474	9.7681	9.6847
15	C-D 1, k_1	1.5791	1.5791	1.5791	1.5791	1.5791
16	N_1 to N_5	**1.5791**	**1.5034**	**1.4265**	**1.3472**	**1.2638**
17	C-D 2, s_{12}	0.4075	0.4075	0.4075	0.4075	0.4075
18	N_6 to N_{10}	**1.1716**	**1.0959**	**1.0190**	**0.9397**	**0.8563**
19	C-D 3, s_{23}	0.3803	0.3803	0.3803	0.3803	0.3803
20	N_{11} to N_{15}	**0.7913**	**0.7156**	**0.6387**	**0.5594**	**0.4760**

Finally the whole series of speeds is collected into Table 3, where its continuity is tested, especially at the critical points of change

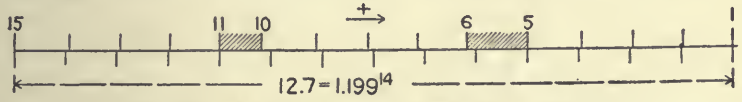

FIG. 212.—Logarithmic scale of speeds for Fig. 211.

in cone pair C-D; and in Fig. 212 this test is made graphical by laying out the series of values of log N, for comparison with the uniform graduation below the base line. We begin at the

left with the lowest speed N_{15} and run up to N_1, each interval expressing a value of log s. Let S be the overall speed ratio N_1/N_{15}, then log S = 1.5791 − 0.4760 = 1.1031 and S = 12.68. For uniform steps, log s' = 1.1031 ÷ 14 = 0.0788, s' = 1.199.

TABLE 3

Speed	Log N	log s	N	s	Speed	Log N	log s	N	s
1	1.5791		37.94		8	1.0190		10.45	
		0.0757		1.190			0.0793		1.200
2	1.5034		31.87		9	0.9397		8.70	
		0.0769		1.194			0.0834		1.212
3	1.4265		26.70		10	0.8563		7.18	
		0.0793		1.200					
4	1.3472		22.25				**0.0650**		**1.161**
		0.0834		1.212					
5	1.2638		18.36		11	0.7913		6.19	
							0.0757		1.190
	0.0922		**1.237**		12	0.7156		5.20	
							0.0769		1.194
6	1.1716		14.85		13	0.6387		4.35	
		0.0757		1.190			0.0793		1.200
7	1.0959		12.47		14	0.5594		3.63	
		0.0769		1.194			0.0834		1.212
8	1.0190		10.45		15	0.4760		2.99	
	1	2	3	4		1	2	3	4

Running from right to left in Fig. 212 (or down Table 3), we note that the four-step intervals 1–5, 6–10 and 11–15 are determined and subdivided wholly by cone pair A-B, while lengths 1–6 and 6–11 (or 5–10 and 10–15) are fixed by pair C-D. Evidently, the steps 5–6 and 10–11 which depend upon both gear units, are the critical points in the series. Relations covering this matter will now be developed.

(*e*) **General Relations.**—Symbol S has just been used for the ratio of highest to lowest speed in the whole train. Apply it now, with designating subscript, to the separate units A-B and C-D of our example, thus

$$\text{I.}\quad S_a = \frac{m_{a1}}{m_{a5}} = \frac{a_1}{b_1}\frac{b_5}{a_5} = \frac{46}{46}\frac{62}{30} = 2.067, \qquad (52a)$$

$$\text{II.}\quad S_c = \frac{m_{c1}}{m_{c3}} = \frac{c_1}{d_1}\frac{d_3}{c_3} = \frac{46}{45}\frac{78}{13} = 6.133. \qquad (52b)$$

Ratio S_a covers four steps, of which the average value will be $s_a = \sqrt[4]{S_a} = \sqrt[4]{2.067} = 1.199$; and the realized values keep very close to this average.

What we have done here is to take the two step pairs involved, the first and last of each cone pair, and go around them in back-

gear fashion. The result may be stated in this useful, general form:

Between the speeds produced by any side-by-side or "parallel" pairs of gears there is a step ratio which is equal to the train ratio that would be given by these pairs if functioning as a back gearing.

This principle applies to any pair of cone steps, separated or adjacent. The order of sequence should always be such that the ratio will be greater than one, not a fraction. A strong reason for thus taking S or s always from greater to less is to avoid negative characteristics for the logarithms that will be handled in close calculations, especially in the method now to be presented.

(*f*) **Extended Overall Ratio.**—Ratios S_a and S_c as got in Eq. (52) may be called the overall speed ratios of the respective cone units. As between a lower unit A-B and a higher unit C-D relations are further expressed as follows:

From A-B step 1 with C-D step 1 to A-B step 1 with C-D step 2 there are five speed intervals: therefore in order to get a smooth series the step ratio in C-D should be

$$s_c = s_a^5 = S_{ae} = S_a^{5/4} = 2.478, \tag{53}$$

as compared with actual 2.556 and 2.400, Forms 1 and 3. The idea is to carry forward or extend S_a over one interval beyond the range of its unit A-B, so that S_{ae} is the *extended overall ratio* of this first unit. The test question for the train is, *How closely does each step ratio of a higher unit agree with the extended overall ratio of the unit below it?* With S_a and each s_c fixed, as in Figs. 209 and 211, the measure of the critical intervals 5–6 and 10–11 is

$$s = s_c \div S_a; \tag{54}$$

for which we have in this example,

$s_{5-6} = 2.556 \div 2.067 = 1.237$, $s_{10-11} = 2.400 \div 2.067 = 1.161$. These special intervals, not quite so regular as the others, are emphasized by shading in Fig. 212.

If there are n steps or gear pairs in a cone pair or similar unit, the extended ratio is evidently

$$S_e = S^{\frac{n}{n-1}}. \tag{55}$$

Going back to Eq. (50) we may write the limiting or extreme values of this train as $T_1 = tm_{a1}m_{c1}$ and $T_{15} = tm_{a5}m_{c3}$; whence the overall speed ratio for the whole train is

$$S = \frac{T_1}{T_{15}} = S_a S_c. \tag{56}$$

This expresses the important fact that the speed-changing effect of the whole train is the product of the effects of its units.

As an illustrative application of these methods, analyze the lathe drive in Fig. 213, which has three two-step gear cones in sequence: the fact that the gear pairs are so separated as to make room for double friction clutches does not destroy the descriptive value of the word "cone." The train is laid out flat at (*a*), with an axis diagram (*b*) to show how it is folded into compact form in the machine. The gears are lettered in order of drive, but the cone pairs and clutches are numbered in order of influence upon

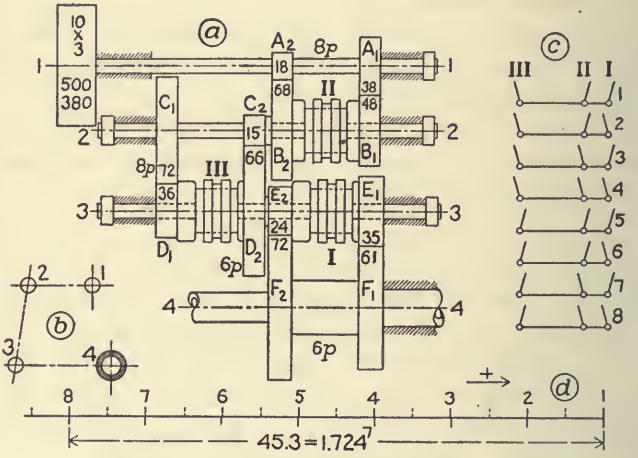

FIG. 213.—Gear drive for 16-inch lathe, Pratt & Whitney.

speed, I giving the smallest interval and III the largest. Clutch-lever diagram (*c*) shows all the settings from highest speed 1 to lowest speed 8: cone I changes continually, cone II for each two speeds, and cone III for the main division of four speeds.

Example 8. Find the determining speed ratios for Fig. 213.

Apply Eqs. (52) and (53) as follows:

$$\text{Pair I or E-F,} \quad S_e = s_e = \frac{35}{61}\frac{72}{24} = \frac{105}{61} = 1.721$$

$$S_{ee} = S_e^2 = 2.963$$

105	2.02119
61	1.78533
	0.23586
	2
	0.47172

Pair II or A-B, $S_a = s_a = \frac{38\,68}{48\,18} = \frac{323}{108} = 2.991$

$S_{ae} = S_a^2 = 8.945$

Pair III or C-D, $S_c = S^c = \frac{72\,66}{36\,15} = \frac{132}{15} = 8.800$

323	2.50920
108	2.03342
	0.47578
	2
	0.95156
132	2.12057
15	1.17609
	0.94448

The overall speed ratio of the train, the product of the partial unit ratios, is

$$S = N_1/N_8 = 1.721 \times 2.991 \times 8.8 = 45.30$$

	0.23586
	0.47578
	0.94448
7)	1.65612
2)	0.23659
	0.11830

And the average term ratio is

$$s = S^{1/7} = 1.724.$$

Very close approximation to an exact geometrical series is shown by the practical agreement of 2.963 with 2.991 and of 8.945 with 8.800. Understand that ratio S is a mechanical fact in each case; while that any S_e should equal the s of the next unit is a desideratum which may or may not be attained. With this machine a two-speed countershaft splits the rather large speed ratio $s = 1.724$. The square root of that number is 1.313, so that if 500 r.p.m. is the higher speed of the driving pulley the lower speed will be $500 \div 1.313 = 380$ r.p.m.

Prob. 84. For Fig. 213 find the highest and lowest spindle speeds with 500 r.p.m. of the belt pulley. Then using the factors just calculated get the full series of speeds, arranging them as in column 3 of Table 3 and adding the column of s values. Slide-rule work will be accurate enough.

$N_1 = 454.2$, $N_{16} = 7.63$.

SECTION 16. VARIOUS CONE-PAIR TRAINS

(*a*) **Belt Cones and Back Gears.**—Instead of a second cone with big steps, as in Fig. 211, a back gearing may be used to repeat or multiply the short series given by a cone pair. The three typical examples in Figs. 214 to 216 show different ways of carrying out this idea with cone pulleys. In every case the cone on the countershaft is exactly like the one of the machine, or is an equal cone, and that shaft has two speeds, so as to double the number from the train in the diagram. These examples show interesting variations in method of handling the speed series, and will serve as good practice problems.

As it happens, the belt-cone drive of Fig. 214 is an (older) alternate to the geared drive of Fig. 213: it gives practically the same series, using the second speed of countershaft to divide its

long interval. In the milling-machine drive of Fig. 215 the cone steps are close together, ratio s being only about 1.24, and the short cone series is repeated by changing countershaft speed before throwing the back gear in or out. This means that the countershaft speed ratio must equal the extended overall ratio of the belt cones, the back-gear ratio being then twice as great in effect, or equal to the square of the extended cone ratio. The series of sixteen speeds is in four sections, namely, high counter direct, low counter direct, high counter back-geared, low counter back-geared.

The lathe drive in Fig. 216 combines the scheme of Fig. 215 with the use of a two-speed or double back gear, got by means of

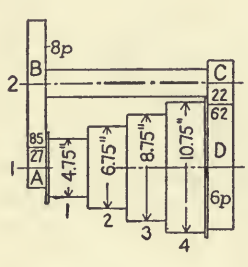

FIG. 214.—Plain cone and back gear.

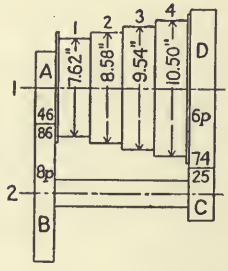

FIG. 215.—Back gear with cone of half range.

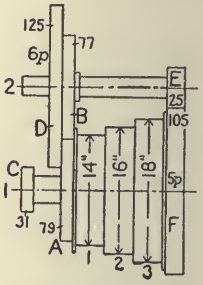

FIG. 216.—Double back gear and half-range cone.

the doubled pair A-B and C-D, in effect two-step gear cones. There are here eighteen speeds in six groups of three or in three main groups of six, characterized by direct, low back-gear and high back-gear drive.

Prob. 85. Calculate the determining ratios for Fig. 214, namely, S_p and average s for the cone pulleys, extended S_{pe} and back-gear ratio S_g. Then get all the spindle speeds, with 200 and 152 r.p.m. for the countershaft. Write the series of speeds and show realized values of s.

Prob. 86. In similar fashion get ratios for Fig. 215, then the full speed series with 300 as the higher r.p.m. of countershaft; what will its lower speed be for a smooth series? Again show all actual values of s.

Prob. 87. Find principal ratios for Fig. 216, checking the two back gears against each other. With 230 and 114 r.p.m. for countershaft find the eighteen speeds and test for smoothness of series. What change in pulley diameters 1 and 3 is desirable?

(*b*) **Gear Cones with Back Gearing.**—Figure 217 is a good representative example of the combination of gear cones with

back gearing and also serves as a good introduction to a study of the different forms of gear cones. Short-step pair A-B gives the speed intervals, long-step pair C-D doubles the first range, and back gear E-F-G-H repeats the whole preceding train. Cone A has three of its gears in sliding member M, with a jaw clutch for A_1. One gear is common to both cone pairs, serving as B_4 and as C_1. Gears D_1 and D_2 are loose on sleeve S, which itself turns on spindle 3, either one being engaged by clutch N. Sleeve T, splined to S, carries gear E and a clutch for big gear H which is keyed to the spindle. Pinion G is splined to the long sleeve hub of F and the shifting rig is so designed that G is slid out of mesh with H before clutch P can engage, and vice versa.

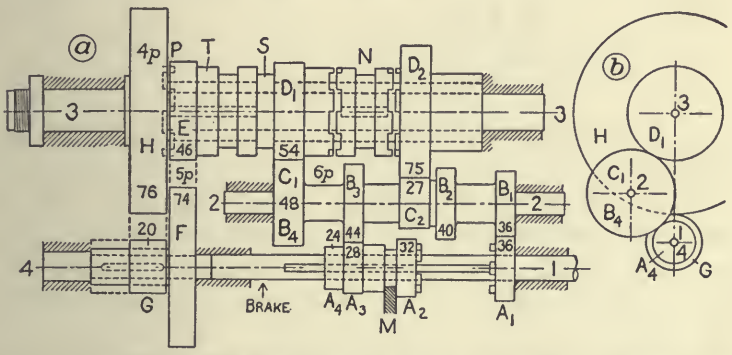

FIG. 217.—Le Blond milling-machine drive with four-step cone, two-step cone and back gear, giving 16 speeds with 15 gears.

In the three trains thus far shown we have examples of close cones with internal engaging devices, Fig. 211, of clutch-type cones opened out so as to give plenty of room for the engaging rig, Fig. 213 and pair C-D in Fig. 217, of the sliding cone in steps 2, 3 and 4 of pair A-B in Fig. 217, and of mixed-type cones in the whole of this pair A-B, since step 1 is of the clutch type. Note the difference that with key or clutch engagement all the gears remain in mesh, while with sliding cones only the active pair has tooth contact.

(*c*) **Sliding Cones.**—A very common application of the principle of gear cones is found in the sliding-change type introduced by Fig. 217, diagrammed in Figs. 218 to 222, and variously shown in the examples which follow. Frequently the word "cone" loses all semblance of its proper geometric meaning, but is conveniently retained in the sense of a series of gears on an axis, with

diameters changing by equal or nearly equal amount, whether or not diameters successive in size are in sequence along the axis. In every case the gears of one cone are secured to their shaft, while those of the second are fast on a sleeve which is splined to the other shaft. Questions arise as to sequence of pairs and of engagements, space occupied (measured along the

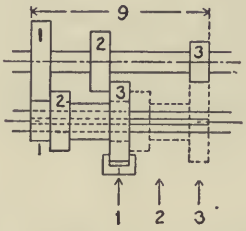

FIG. 218.

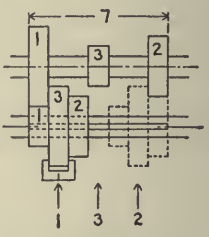

FIG. 219.

Free-shift or open sliding cones.

axis), and whether there is freedom or locking while changing engagement.

On considerations of sequence these diagrams would be differently grouped, Figs. 218, 220 and 221 having direct order, Figs. 219 and 222 indirect or broken order; but note that in Fig. 221 the order of engagement is not the same as the direct order of

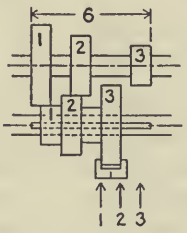

FIG. 220.

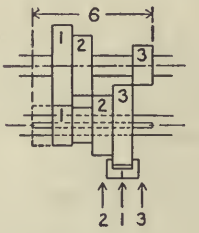

FIG. 221.

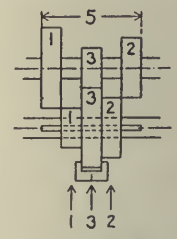

FIG. 222.

Locked-shift or close sliding cones.

the steps. In general, the indirect order gives the more compact arrangement, as shown by the dimensions on the figures, which have the gear breadth as their unit. In a working design this unit is a little bigger than the width of a gear, to insure a slight clearance.

Prob. 88. Change Fig. 218 by putting steps 2 and 3 of the sliding cone together, as in Fig. 221, and see what will be the effect upon the other cone and upon the sequence of engagements.

Prob. 89. For a four-step direct-order open sliding cone pair (Fig. 218 extended) find what spacing or grouping will give the best and most compact arrangement and what is the shortest overall length (in gear-breadth units) that can be obtained.

Prob. 90. For the four-step pair in Fig. 226 try the effect of separating steps 4 and 3 of cone B, changing the grouping to 24–31: get the spacing in cone A, the order of engagements and the overall length.

(*d*) **Locked and Free Shift.**—In Fig. 220 the sliding cone is drawn at half shift, and it is evident that the train is locked when two pairs of wheels are thus in mesh at the same time. This more compact type of arrangement is sometimes used on machine tools, but would be entirely unsuitable for the gear

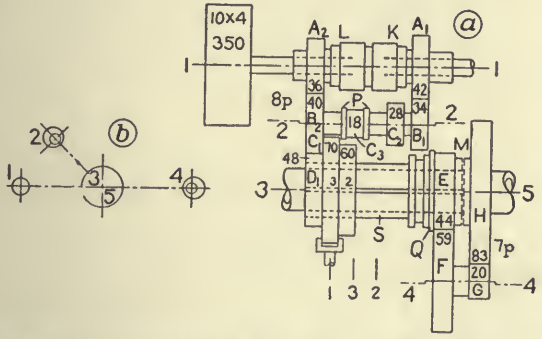

FIG. 223.—Gear train of a 14-inch lathe with two-step clutch cone, three-step close sliding cone and back gear, giving 12 speeds with 13 gears.

box of an automobile. To make a change the machine must be stopped and then slowly turned by hand, "feeling" for a position where tooth will enter space on the pair to be engaged.

Locking of the train, with a close sliding cone, can be avoided by separating the axes so that all gears will be out of mesh while the change is being made. The point of special interest in Fig. 223 is the application of this idea, together with a system of safety collars to prevent double engagement. As more clearly shown at (*b*), intermediate shaft 2 has a regular back-gear eccentric throwout, which draws it away from axis 3 or 5. At each side of pinion C_3 is a collar P flush with the tips of the teeth. For anything else than full shift one of these collars will strike gear D_3 and shaft 2 cannot therefore be thrown back into place. A similar collar Q beside gear E safeguards the back gearing. This is another example of the cone-and-back-gear train (compare Fig. 217), with a logical use of friction clutches K and L

on the first shaft and a jaw clutch M at spindle gear H. The axis of sleeve S, which turns on the spindle is first counted as No. 3 in the series of five, then returning through the back gearing this same axis is taken to be No. 5.

(*e*) **Trains with Sliding Cones.**—Figure 224 shows a combination of two direct-order three-step cone pairs like Fig. 218,

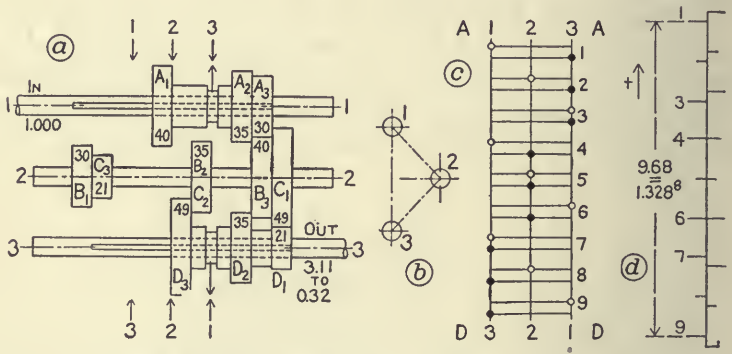

FIG. 224.—A symmetrical nine-speed feed train.

and is used to illustrate the complete series of diagrams for such trains as are being discussed; all have already been given in one figure or another, but not as yet thus grouped together. First comes the flat layout (*a*) of the train, then the actual shaft or axis diagram (*b*). The shifting of the slides for the sequence

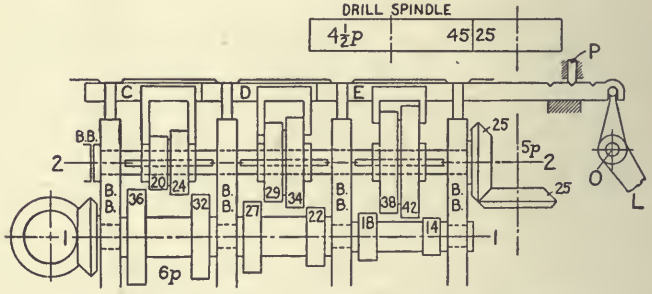

FIG. 225.—A six-step pair, with three sliding sections.

of speeds is shown at (*c*), where the open circle is for short-step cone A, the blacked circle for long-step cone D: by using different types of levers for the two slides it is easy to secure that a swing to the right shall give higher speed, to the left lower speed, for both cones. In the logarithmic speed scale (*d*) the graduations

on the left side of the line are from the train as given at (*a*), while those on the right side show the effect of using the tooth numbers 20 and 50 instead of 21 and 49 in steps 1 and 3 of pair C-D. Even so small a change has a large influence upon speed intervals 3-4 and 6-7.

The last two examples, Figs. 225 and 226, are extreme cases, the first showing an unusual number of steps, but with simple

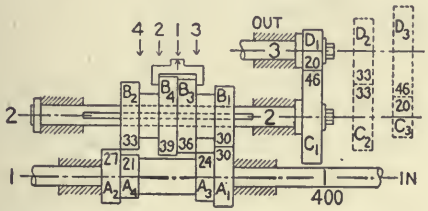

FIG. 226.—A four-step sliding cone.

two-gear slides, the second a single four-gear slide. They are given here in order to round out the exhibit of sliding cones and as added material for the group of problems which follows; and they also illustrate strikingly the contrast between serial and selective change, which is the next matter to be taken up. In Fig. 225 there are three shift rods C, D, E, one for each gear slide; and lever L can engage any one of these at a time, being

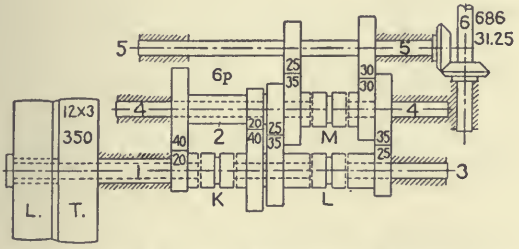

FIG. 227.—Cone and back-gear train for a radial drill.

shifted along its fulcrum axis O for that purpose. At P is a spring plunger or locking pin for each rod, and shafts 1 and 2 have ball bearings at the places marked B, B. In Fig. 226 certain shifts can be made only by passing through intermediate engagements. Note the use of change wheels C and D instead of a second cone pair.

Prob. 91. Trace out the working of Fig. 227 and draw a diagram similar to Fig. 224 (*c*) for the three double clutches K, L and M. Sketch the train and letter the gears after the manner of Fig. 217.

Prob. 92. Trace the working of Fig. 228 and state how the different combinations are made, describing the clutch changes and gear shifts.

Prob. 93. Similarly analyze and describe the working of Fig. 229.

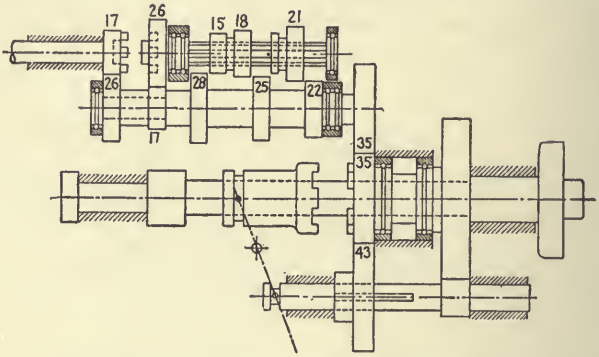

FIG. 228.—Eight-speed head for a 24-inch lathe.

Prob. 94. (*a*) Fig. 224 or (*b*) Fig. 226: get the s values for the first (short-step) unit, the S for each of the others, and try out s for the critical intervals dependent upon two units—method of Sect. 15 (*e*) and (*f*).

Prob. 95. Adapting Form 4 to the particular train, make a complete calculation by logs for (*a*) Fig. 217, (*b*) Fig. 223 or (*c*) Fig. 227, arrange results as in Table 3, and lay out the log diagram of speeds.

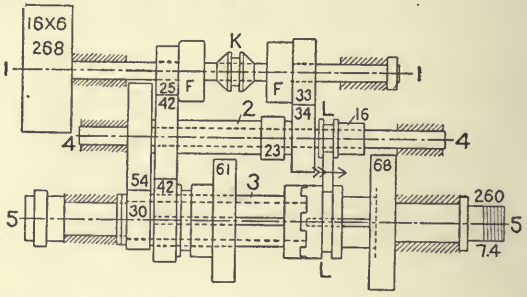

FIG. 229.—A twelve-speed lathe head.

Prob. 96. Analyze (*a*) Fig. 228 or (*b*) Fig. 229 as directed in Prob. 94 and add the log scale of speeds, getting it from this less complete calculation. With Fig. 228 determine suitable tooth numbers for the last back-gear pair, assuming same total number 78 as in preceding pair.

(*f*) **Serial and Selective Change.**—A question which has to do with the ease and quickness of manipulation of a change train may be stated thus: If the control levers or handles are

set for a particular speed, can they be thrown directly into the positions for any other speed or is it necessary that the gears pass through certain intermediate settings in order to reach the one desired? Contrasting examples are, on the one hand (serial), the close cones of Fig. 211, the four-step pairs in Figs. 217 and 226, and the three-step sliding cones in Figs. 224 and 132; on the other hand (selective), the two-step cones of Figs. 213 and 227 and the subdivided six-step pair in Fig. 225. In general, a speed-changing unit of the cone-pair type gives selective change when each shifting member controls but two speeds or has but two active settings. In their very nature the cone-and-tumbler unit and the back gear are selective.

The hindrance to quick change offered by a serial arrangement depends very much upon the type of engagement. A series of friction clutches, as in pair A-B of Fig. 211, may be no obstacle at all but rather a help, because the successive brief grippings of the clutches give several small accelerations to the driven member, better than a single big one. With sliding keys such as are shown in Fig. 138 there is practically the same action. But when it is necessary definitely to enter and slip through each engagement, as with the old key device of Fig. 136 or with any sliding cone of more than two steps, the time required is proportional to the number of engagements.

Among machine tools the demand for quick and convenient speed change is strongest in lathes and boring mills of the turret type, used for repetitive work and performing a series of operations on each piece; during this cycle of operations the speed may be changed several times. In a milling machine, on the other hand, where the same operation is likely to be repeated on a number of work pieces, changes are comparatively infrequent and to have one consume a little longer time does not involve appreciable loss.

The element of convenience is a good deal affected by the number of handles to be manipulated. A glance over the trains preceding and following will show three as the most usual number, with range from two to four. Occasionally there is an interlocking rig which controls several train units by means of a single handle, but commonly the simple scheme of independent handles is used.

(*g*) **A Special Cone Type.**—The lathe train in Fig. 230 has a number of distinctive features, of which two have special interest

in connection with the discussion of gear-cone arrangements. The first is the operation of a three-step cone pair by means of a double friction clutch. The cone clutches inside of wheels B_1, B_2, D_1 and D_2 are indicated on the drawing; these belong to the higher-speed steps. In wheels B_3 and D_3 of the lowest-speed steps there are ratchets, driving from the wheels to drums on shafts 2 and 3. When the clutches are free or at neutral these ratchets engage, so that shafts 2 and 3 never stop while shaft 1 is running. If step 1 or 2 is thrown in the shaft simply

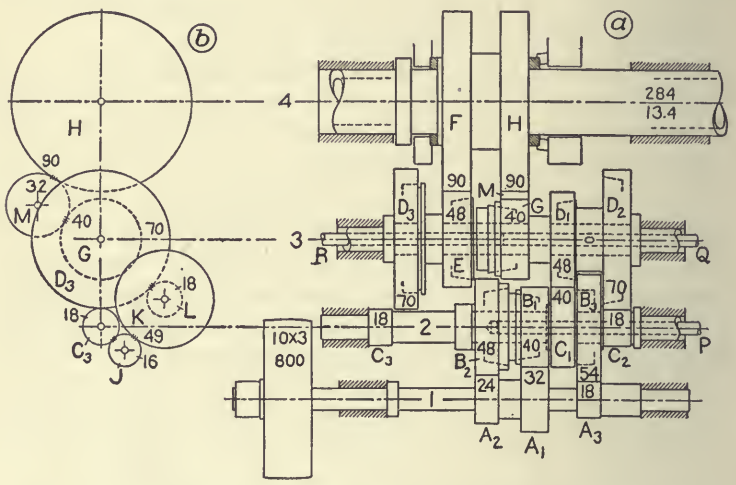

FIG. 230.—Hartness lathe drive, with a compounded cone-step pair.

runs ahead of wheel B_3 or D_3. A special silent pawl, held clear by friction, obviates unpleasant clicking.

The other point of special interest in Fig. 230 is the introduction of compound gearing into step 3 of cones C-D in order to produce a big speed change. Step 2 has, in 18 teeth for C_2, about as small a pinion as may be used. To get a lower speed by step 3 the gears J-K-L shown at (*b*) are interposed between C_3 and D_3. Idler J has no effect on speed, but is needed to keep the direction of turning right. Movement of the spindle is controlled by rod R and the clutches in E and G, which give "forward" through E-F, "stop" at neutral and "reverse" through G-M-H.

Prob. 97. Get the full speed series of Fig. 230, arranging results as in Table 3 and laying out the log scale of speeds.

Prob. 98. Readjust the tooth numbers of the train in Fig. 230 so as to get as nearly as possible an exact series of nine speeds on $s = 1.5$.

Suggestion: Keep A_3B_3 and C_2D_2 unchanged; in the C_3D_3 train modify gear K.

(*h*) **Some Tendencies of Design.**—Speed-changing gear trains are used mostly in machine tools and motor cars. The two lines of design have had coincident development, most active during the period from 1900 to 1912. In the progress based on fuller experience there is a definite (and normal) tendency toward simplicity, compactness, ruggedness and the elimination of less suitable forms. Thus many early gear boxes for automobiles had all gears in mesh, with individual jaw or even friction clutches, or sliding gears with serial engagement. Now the type repre-

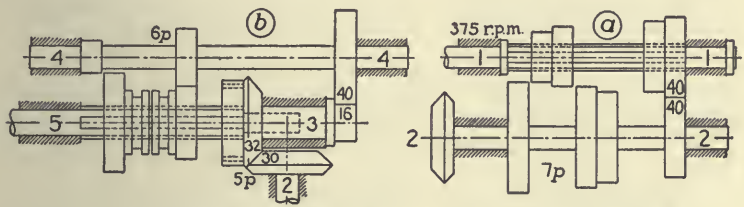

FIG. 231.—Later substitute for train in Fig. 211: see Prob. 124.

sented by Fig. 133 is universally prevalent—fully selective, having sliding engagement, and operated by a single control lever which can connect with any one of the shifting rods.

Machine-tool drives are far more various in function, form and type of construction. But those that have to meet the requirement of frequent change, likely to be made under load, tend to conform to the motor-car type of arrangement and detail, including coarse-toothed, face-hardened gears of high-grade alloy steel. A striking example is given in Fig. 231, which shows the train that has superseded the one in Fig. 211. The primary gear box (*a*) is a four-step cone pair with selective engagement of the type in Fig. 225, the secondary box (*b*) is a double back gearing. A single handle controls each unit, a third handle the clutch at the driving pulley on shaft 1; and these are interlocked so that no gear changes can be made unless the clutch is released.

Most of the main-drive trains that have been shown are from machines of small or moderate size, which even in a shop running on electric power are at least as likely to be driven in groups through belting as to have individual motors. With larger

machines, for which the separate motor is more strongly indicated the scheme suggested by Figs. 240 and 241 is very effective: the gear train provides a few big speed intervals, which are subdivided by a variable-speed motor into smaller steps than are practicable with a gear train.

Of course, the fully geared machines designed for constant-speed drive, and here shown with a belt pulley, can equally well be arranged for motor connection, using either gear or chain transmission as convenient.

(*i*) **Cone-and-tumbler Trains.**—As shown by Fig. 140, the single cone used with a tumbler must carry the geometrical series

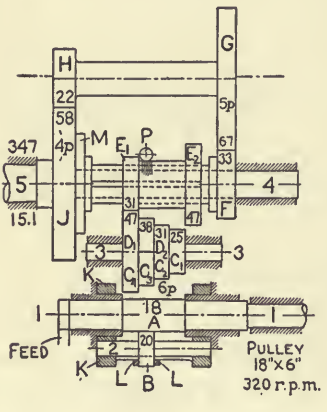

FIG. 232.—Brown & Sharpe milling machine.

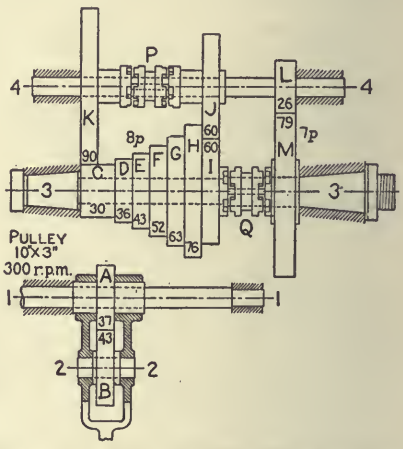

FIG. 233.—Hendey milling machine.

in the diameters of its gears. Cones for screw cutting are stepped somewhat differently, as can be seen in Sect. 20, Figs. 259, 260, etc. In trains the cone-and-tumbler unit has the same function as a short-step cone pair—but may be given a much longer range, as in Fig. 236—and is similarly combined with long-step cones or with back gearing.

Two main trains for milling machines are given in Figs. 232 and 233, with a four-step and a six-step tumbler cone. A special feature of Fig. 232 is the use of two steps of tumbler cone C as elements of cone pair D-E; other notable features are, the long driving pinion A, the stout tumbler frame K, the shifting of tumbler wheel B by a separate finger piece L, the use of little pinion P to shift gear cone E by means of a cylindrical rack cut

on its sleeve, and the fact that clutch M is operated by the back-gear lever in such a way as to make locking of the train impossible. The tumbler rig of Fig. 233 is like that shown in Fig. 141 as to general form, but is made stiffer and stronger because it must withstand heavier forces, Fig. 141 being from a lathe feed train. With direct drive (cone clutched to spindle) and two back-gear ratios this train gets eighteen speeds from the six-step cone.

(*j*) **Compound Tumblers.**—Examples of the compound tumbler are given in Figs. 234 and 235. The second gear C in Fig. 234 is used only on cone step F, the four tumbler combinations being

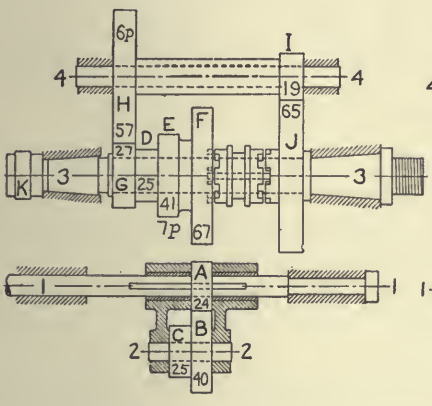

FIG. 234.—Lathe drive with two-step tumbler.

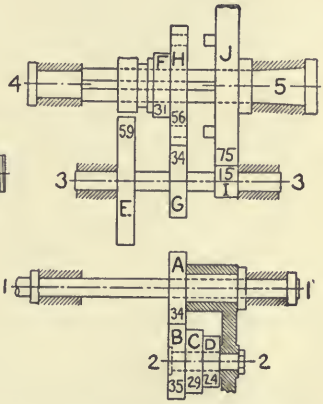

FIG. 235.—Miller drive with three-step tumbler.

ABD, ABE, ABF, and ABCF; the train gives only eight speeds, but its interval is split by a two-speed countershaft. Of Fig. 235 it may be said that the cone now really becomes the tumbler, since its small wheels are engaged first with large wheel G, then with E. Shaft 3 and spindle 4 are connected by a long-step three-gear cone of mixed sliding and clutch type.

An interesting amplification of the principle of the compound tumbler is shown in Fig. 236, where a four-wheel swinging cone is used with a five-wheel fixed cone to get eight speeds. Diagram (*c*), related to both views (*a*) and (*b*) but widened out of proportion with (*a*) so as to make room for the lettering, shows the drilling for a latch pin like the one in Fig. 141 and also gives the series of engagements, in order from high to low speeds. The design of cone pair BC-GD, whose proportions are interlocked

with those of the tumbler cones, is described at the end of Sect. 19 (*b*).

The final example in Fig. 237 has a simple tumbler BC and a compound tumbler DEF on the same swinging frame. Cone A is drawn clear of the other gearing, but its true position is back

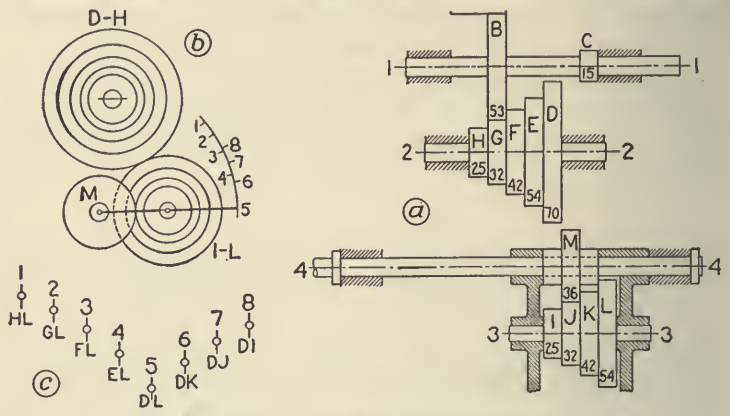

FIG. 236.—Miller drive with cone and tumbler cone.

of the tumbler rig, so that either B or D can engage its steps. Note that here the cone shifts instead of the tumbler, also that the cone is driver. The higher speeds are given through B, the lower through D. For one revolution of B shaft 4 makes 28/20 = 1.4 turns, while for one of D it makes 21/30 = 0.7 turn, so that the gear ratio from B to D is 2. The extended overall ratio of

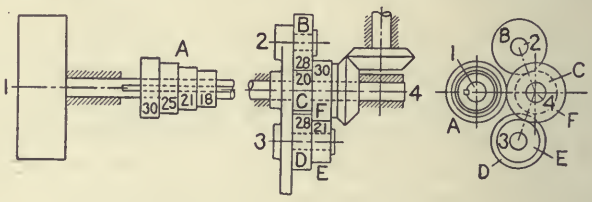

FIG. 237.—A double-tumbler feed drive.

cone A is likewise evidently 2, since the next step would be either 36 on the big end or 15 on the small end.

Prob. 99. For tumbler cone C in Fig. 232 find the three individual values of s and the exact mean value, using only steps 1 and 4 in the last determination. Prove, by numerical trial and by algebraic deduction, that with two steps of cone C used as here for cone D of pair D-E, the step ratio of the latter is equal to the extended ratio of the cone-and-tumbler unit.

Prob. 100. For the tumbler cone C-H of Fig. 233 find the five individual values of s, then get a mean value from steps C and H and another mean value from cone pair IC-JK.

Prob. 101. Determine the complete speed series of Fig. 232 or of Fig. 233, following the directions given in Probs. 94 and 96.

Prob. 102. In Fig. 234 get the three values of s for the four "speeds" of the cone-and-tumbler unit, incidentally formulating a statement relating cone CB to cone DEF as regards step ratio. Get S for the back gearing and find the fourth value of s.

Prob. 103. Determine the step and unit ratios (the latter the S's) in Fig. 235, applying the methods of Sect. 15 (*e*) and (*f*) and covering the whole train. Formulate the conditions under which gears E and G with the tumbler will give a proper speed series.

Prob. 104. Make a full analysis of the speed series in Fig. 236, getting the series of s's for the whole tumbler unit and trying its extended ratio S_e against the s of cone pair BC-GD.

SECTION 17. COMPLEX UNITS AND TRAINS

This title is used as a short name for a group of mechanisms in which change of speed is accompanied by reversal in direction of drive through some part of the train.

(*a*) **Compounded Cones.**—A combination which gives four speeds from three pairs of gears apparently arranged in cone

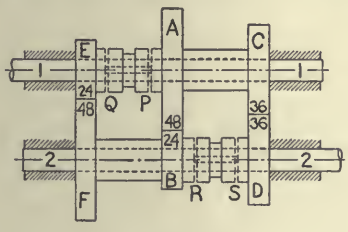

FIG. 238.

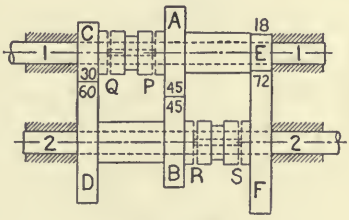

FIG. 239.

Back-geared or compounded cones.

fashion is shown in Figs. 238 and 239. Each shaft carries a single and a double loose wheel and a double jaw clutch. There are three direct transmissions, each by a single pair of gears, and a fourth transmission involving all three pairs in what may be called a doubled back-gear action. The several combinations and the resulting speeds of shaft 2 for unit speed of shaft 1 can

Clutches.....	P R	P S	Q R	Q S	P R	Q R	P S	Q S
Gears.....	A-B	C-D	E-F	EFBACD	A-B	C-D	E-F	CDBAEF
Speed N_2....	2	1	½	¼	1	½	¼	⅛

best be set forth in tabular form. These two trains are designed to give a binary series of speeds, or one having the term ratio $s = 2$. The use of this series in lathe feed gears is described in Sect. 20 (*d*).

Two applications of this cone pair are shown in Figs. 240 and 241, forming the complete gear trains of lathes that are to be driven by variable-speed electric motors. The motor gives small changes over a moderate range, the gearing makes the big changes, repeating the motor range. Both trains are of the type of Fig. 239 (not 238), the first exactly, the second modified in detail because the drive comes on gear C instead of shaft 1. Figure 240 produces an exact geometrical series with the term ratio 2.5, Fig. 241 approximates 3.33.

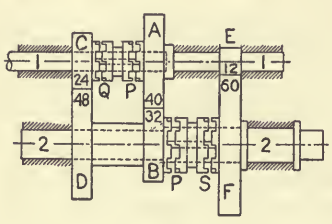

FIG. 240.

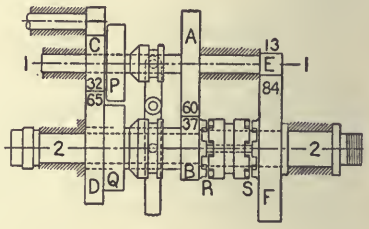

FIG. 241.

Four-speed lathe drives, compounded cones.

The lathe drive in Fig. 242 has two of these compounded cone units in sequence, one for small changes, the other for big ones. The scheme is economical of gear wheels (16 speeds with 14 wheels) but extravagant in clutches, and there are four levers to manipulate. The excessive compounding at certain speeds means low machine efficiency; thus for the lowest speed of spindle all seven pairs of gears are active and the friction loss must be high.

Figure 243 shows a combination of compounded cones with a compound tumbler, intended for feed-gear use. From the purely kinematic viewpoint this has the merits of flexibility and compactness, but some of its combinations put an excessive number of gears into action. All of the wheels from D to K are loose on their shafts 3 and 4. Either of the single wheels D or K can be clutched to shaft 3, but the sleeve pairs E-F, G-H, I-J are always loose. The tumblers B and C can engage gears D, G, H and K on shaft 3. The positions of arm M are numbered in the order from low to high speed. The low half of the series is got with clutch P, since from

D toward K the train speeds down; with Q in and drive from K toward D it speeds up. The active trains and the turning ratios for extreme speeds are, low, ABCDEFGHIJK, 0.127, high, ABKJIHGFED, 3.97.

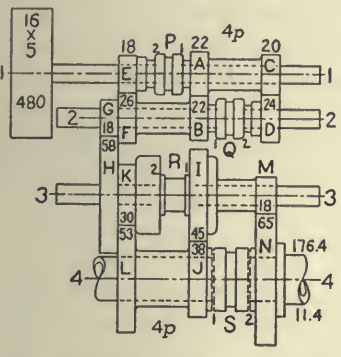

FIG. 242.—Train for a 20-inch lathe.

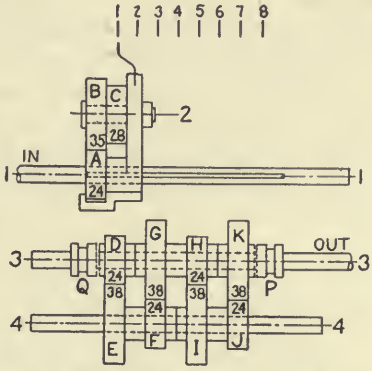

FIG. 243.—Feed gear with multiple compounded cones.

(*b*) **Complex Trains.**—The two examples in Figs. 244 and 245 show fuller development of the idea of making a part of the train drive in either direction. The first is special and comparatively simple, but the second introduces a type of train that is far more

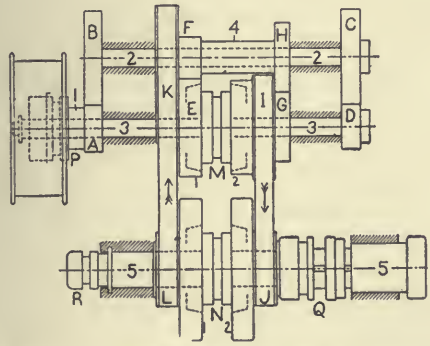

FIG. 244.—A quick-change two-speed reversing train.

complex than the usual straightforward combinations of cone pairs, tumbler cones and back gearing.

The simple purpose of Fig. 244 is to provide a quick change (effected automatically) of spindle 5 from high to low speed and from forward to backward running. The pulley drives sleeve

1 and gear A through a friction clutch. Train A-B-C-D, always in action, carries motion to shaft 3 which is driver for the quick-change train, C and D being change wheels. Through equal gears G and H chain wheels I and K are made to turn at the same speed but in opposite directions. Gear E is twice the size of F; then with clutch M_1 engaged wheels I and K turn twice as fast as shaft 3, with M_2 engaged they have the same speed as 3. Clutch N_1 gives forward motion, N_2 reversal. These double clutches have been illustrated in Fig. 191 and their operating device in Fig. 201. Collars Q and R are concerned with spindle devices for holding and feeding the work, or with chuck and bar feed.

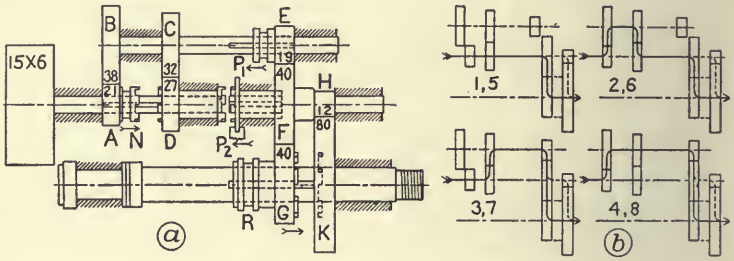

FIG. 245.—A lathe train of complex type.

The eight-speed lathe drive of Fig. 245 has four shifting members, but since P_1 and P_2 must be worked by the same lever in order to avoid danger of locking the train, there are but three handles to manipulate. The little action diagrams at (*b*) illustrate the following condensed description:

Wheels A, B, C, D form a low-ratio back gearing, covering the short speed intervals 1–2 and 5–6, and serve also as a two-step cone pair for the drive through E, here producing the equivalent speed intervals 3–4 and 7–8. Change from direct drive through F to back-gear drive through E covers the double interval 1–3, 2–4, etc. Finally, gears F, G, H, K form a two-step cone pair with its S equal to s^4.

Prob. 105. Get the s series in Figs. 240 and 241 and prove that if the three direct speeds are in geometrical series the fourth, compounded speed must be correct for the same series.

Prob. 106. Determine the sixteen speeds of spindle in Fig. 242, writing out for each of them the active gear train, as ABGHIJ for the highest speed.

Prob. 107. Test the speed ratios of Fig. 243, to see whether the series is essentially geometrical. With the symmetrical arrangement on axes 3 and

4 why are not the extreme turning ratios 3.97 and 0.127 equally distant from unity? That is, why are they not reciprocals?

Prob. 108. Trace out in detail the working of Fig. 244, sketching a group of diagrams like Fig. 245 (*b*) to show the effect of the four different clutch combinations.

Prob. 109. With 200 r.p.m. of driving pulley find all the spindle speeds for Fig. 245, then test the series by getting the successive values of s.

SECTION 18. DESIGN OF SPEED TRAINS

(*a*) **Gear-cone Relations.**—In Table 2 and Fig. 210 we found by simple trial the effect of a constant step difference with a (necessarily) constant sum of diameters. For a more general treatment of the subject proceed as follows:

Start with a gear pair of given speed ratio m but of indeterminate diameters or tooth numbers, so that the two wheel sizes will be driver mx and follower x, having the sum $q = (m + 1)x$. Let y be the step difference and s the step ratio, to the lower speed ratio $m \div s$. The four wheels, taken in the back-gear order of Eq. (52) will now be mx, x, $x + y$ and $mx - y$, as seen in Fig. 246; so that for s we have

$$s = \frac{mx}{x} \frac{x + y}{mx - y} = \frac{m(x + y)}{mx - y}, \tag{57}$$

whence

$$x = y \frac{s + m}{m(s - 1)} \text{ and } q = y \frac{(m + 1)}{m} \frac{s + m}{s - 1}. \tag{58}$$

The effect of this relation is shown by Example 9, where s and y are fixed and for different assumed m's the corresponding values of q are found.

Example 9. Let $s = 1.25$ and $y = 4$ and for the three cases $m = 2$, $m = 1$ and $m = 0.5$ find tooth numbers for the wheels and tooth sums for the pairs.

Case (1) $m = 2$	Case (2) $m = 1$	Case (3) $m = 0.5$
$x = 4 \times \frac{3.25}{0.5} = 26$	$x = 4 \times \frac{2.25}{0.25} = 36$	$x = 4 \times \frac{1.75}{0.125} = 56$
$mx = 52$	$mx = 36$	$mx = 28$
$q = 78$	$q = 72$	$q = 84$

These results are applied and checked in Fig. 246.

In Table 2 and Fig. 210 it has been shown that with a given y the value of ratio s increases as m gets away from 1. Here we see that the growing influence of y could be balanced by making q

larger. The impossibility of such variation is only another expression of the necessity of using an approximate solution, giving average conformity to the geometrical series. General expression (58) is, however, a great help in relating fundamental dimensions.

TABLE 4

s	q_0	s	q_0	s	q_0	s	q_0
1.05	820	1.23	194	1.34	138	1.6	86.7
1.10	420	1.24	187	1.35	134	1.65	81.6
1.12	353	1.25	180	1.36	131	1.7	77.1
1.14	306	1.26	174	1.38	125	1.8	70.0
1.16	270	1.27	168	1.40	120	1.9	64.4
1.17	255	1.28	163	1.42	115	2.0	60.0
1.18	242	1.29	158	1.44	111	2.2	53.5
1.19	230	1.30	153	1.46	107	2.4	48.5
1.20	220	1.31	149	1.48	103	2.6	45.0
1.21	210	1.32	145	1.50	100	2.8	42.2
1.22	202	1.33	141	1.55	93	3.0	40.0

It is evident from Example 9 that required sum q is least when m is near unity, or at the middle of a pair of equal cones. Since short-step cones are likely to be symmetrical about $m = 1$ (or nearly so) and do not range very far from this middle state, a useful result is got by evaluating (58) for $m = 1$, in the form

$$q_0 = 2y\frac{s+1}{s-1}. \tag{59}$$

Values of this q_0, computed with $y = 10$ and for s from 1.05 to 3, are given in Table 4 and can be used for quickly getting related values of q and y. With any other y (than 10), multiply the tabular number by $y/10$: thus at $s = 1.25$ and $y = 4$ the adjusted number will be $q_0 = 180 \times 0.4 = 72$, as in Fig. 246 (*b*). The extreme turning ratio of a primary cone will not often go much beyond $m = 2$ or $\frac{1}{2}$, so that Example 9 gives a fair idea of the probable extent to which it may be necessary to modify (increase) q_0 from the table: as between the 78 of case (1) and 72 of case (2) an adjusted value of 74 or 75 is indicated—compare Example 11, below.

(*b*) **Design of Short-step Cones.**—Procedure can best be shown by means of a couple of numerical examples, taking first

the case of equal cones, then starting with a particular value of m at one end of the pair.

Example 10. Design a five-step equal cone pair for the average ratio $s = 1.25$ with $y = 4$.

With $m_3 = 1.00$ for the middle step the turning ratios for an exact geometrical series on $s = 1.25$ would be,

$$m_1 = s^2 = \frac{25}{16} \qquad m_3 = 1 \qquad m_4 = \frac{1}{s} = \frac{4}{5}$$

$$m_2 = s = \frac{5}{4} \qquad\qquad m_5 = \frac{1}{s^2} = \frac{16}{25}$$

One half of this symmetrical cone, say steps 1 to 3, will be enough to lay out. Passing over step 2 use $y = 8$ with $s = 25/16 = 1.563$, since the particular s is the same as m_1. For $s = m$, Eq. (58) gives

$$x = y\frac{2}{m-1} = 8 \times \frac{2 \times 16}{9} = 28.44$$

and $mx = 44.44$. Against the sum $q = 72.9$ interpolation in Table 4 gives 73.1. This odd-number result, with fractional numbers for the wheels,

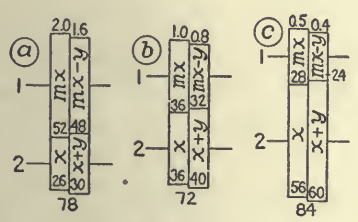

FIG. 246.—Finding tooth sums.

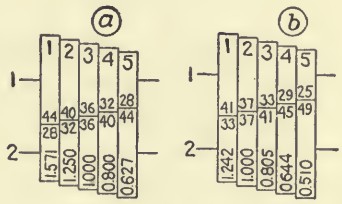

FIG. 247.—Designing gear cones.

illustrates the kind of difficulties encountered when attempting an exact solution of such problems. Necessarily using the $q = 72$ of case (2) in the last example, we get the results shown on Fig. 247 (*a*). From $m_1 = s^2 = 44 \div 28$ mean $s = 1.253$.

Example 11. Again for a five-step pair on $s = 1.25$ with $y = 4$, let m_1 have a given value, as 1.25.

The ratio from step 1 to step 5 will be $S = s^4 = 2.444$. In Eq. (58), with y now taken as 16 between extreme steps, this gives

$$x = 16 \times \frac{1.25 + 2.444}{1.25 \times 1.444} = 32.7,$$

and with $mx = 41$ the sum q is practically 74. The cones will be as shown in Fig. 247 (*b*), having $S = 2.435$ as against the 2.444 sought.

Prob. 110. Proportion a five-step equal cone pair on $s = \sqrt{1.5}$, (*a*) with $y = 4$; (*b*) for small-number wheels, but with not less than 16 teeth in the smallest pinion.

Prob. 111. Design a four-step cone pair on $s = 1.3$, with $y = 5$ and highest turning ratio $m_1 = 1.2$, then test out all the step ratios, namely, each s, total S and mean s.

Prob. 112. Design a four-step cone pair on $s = \sqrt[3]{2}$, with $m_4 = 0.25$ as the lowest turning ratio, the smallest pinion not to have less than 15 teeth.

Prob. 113. Proportion a five-step cone pair on $s = \sqrt[5]{4}$ with $y = 4$, having the turning ratio of the middle step $m_3 = 0.8$ about.

Prob. 114. Proportion a six-step pair on $s = 1.4$, with $m_4 = \frac{1}{3}$ and not less than 16 teeth in any wheel.

Prob. 115. Lay out a five-step pair on $s = \sqrt{2}$, with $m_1 = 1.2$ and not less than 20 teeth in the smallest wheel.

Remark. The value of s and the range of m are both so large in this problem that the best results can be got by varying y somewhat. One good solution has the step differences 8, 7, 7, 6.

(*c*) **Cones with Constant Step Ratio.**—That cones with constant q can be made to give an exact geometrical series has been shown by Figs. 238, 239 and 240. Consider the last of these, calling A-B step 1, C-D step 2 and E-F step 3. The successive m's, on step ratio $s = 2.5$, are

$$m_1 = 1.25 = \frac{5}{4},\ m_2 = 0.5 = \frac{1}{2},\ m_3 = 0.2 = \frac{1}{5}.$$

Now if a number is to be divisible into two parts which shall bear to each other the ratio 5:4 or 1:2 or 1:5, the number itself must be a multiple of 9 or of 3 or of 6. In this case then q must be a common multiple of 9, 3 and 6. Its least value is 18, but that has to be multiplied to 72 in order to get a sufficient number of teeth in the smallest pinion E. For Fig. 238 the similar least common multiple is 6, for Fig. 239 it is 30.

This scheme of exact relationship can be applied only in some cases, with simple values for m and s. Take, for instance, the m numbers $\frac{5}{4}$ and $\frac{25}{16}$ in Example 10: exact realization could be got only by making $q = 2 \times 9 \times 41 = 738$, which is out of the question. On the other hand, the binary-series pair in Fig. 239 can easily be expanded into a five-step arrangement giving the m series 4, 2, 1, $\frac{1}{2}$ and $\frac{1}{4}$; and by doubling q to 180, so that the smallest pinion will have 20 teeth rather than 10, ratios 8 and $\frac{1}{8}$ can be added to the series, making seven terms in all.

Prob. 116. Having given the middle turning ratio of a three-step cone pair, find q for exact value of s in each of the following cases: (*a*) $m_2 = \frac{1}{2}$, $s = \frac{3}{2}$; (*b*) $m_2 = \frac{1}{2}$, $s = \frac{5}{4}$; (*c*) $m_2 = \frac{1}{2}$, $s = \frac{4}{3}$; (*d*) $m_2 = \frac{2}{3}$, $s = \frac{4}{3}$; (*e*) $m_2 = \frac{1}{3}$, $s = \frac{4}{3}$.

(*d*) **Long-step Cones.**—The laying out of gear proportions for big speed changes is a somewhat more flexible and less definite

operation than that described in Arts. (*a*) and (*b*), with more of the cut-and-try element: and yet the principle that an arithmetical series of step diameters will produce a geometrical series of speed prevails to a large extent. In most cases it gives a good first approximation, subject to adjustment for closer fit.

Suppose that we have a first cone pair 1 with n_1 steps or wheel pairs and step difference y_1, a second pair 2 with n_2 and y_2, and perhaps a third pair 3 with n_3 and y_3. Then if these cones are approximately symmetrical, and if sum q does not change from pair to pair, it is nearly true that for a geometrical series,

$$y_2 = n_1 y_1, \; y_3 = n_2 y_2. \tag{60}$$

To see what this means, consider the example in Fig. 248, taken from Fig. 224. The three-step cone pair at (*a*) has q =

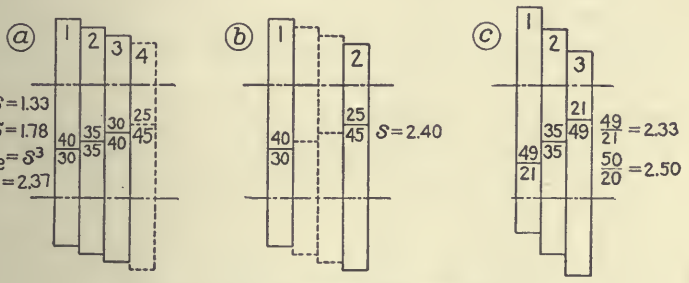

FIG. 248.—Evolving a second cone unit.

70 and $y = 5$; it uses this difference 2 or $(n - 1)$ times to produce its own, internal change effect S, Eq. (52); and one more step, making a total of ny or 15, will give the extended effect S_e to which s of the next unit ought to be equal. In sketch (*a*) of Fig. 248 a fourth step is added in dotted lines, then at (*b*) emphasis is placed upon the extreme wheel pairs 1 and 4, changed to 1 and 2 of a new unit (or partial unit) which properly covers the original unit. Change to the symmetrical three-step pair at (*c*) really extends the eccentricity and increases the influence of y; consequently the series is made smoother by dropping from 15 to 14: compare the log-scale showing of Fig. 224.

(*e*) **Relations in Gear-cone Trains.**—Reviewing other pure cone trains already given, we find further illustration of the principle just developed.

In Fig. 226 four-step pair 1 or A-B has $y_1 = 3$ in $q_1 = 60$; and the indicated y_2 of 12 in 60 is carried over almost exactly as 13

in 66 for the change gears. It is because pair 2 or C-D is symmetrical while A-B is not that there is no need for a small diminution of y_2 such as gave better results with Fig. 224. Noting the fact of change in q we see that the more general form of Eq. (60) is

$$\frac{y_2}{q_2} = \frac{n_1 y_1}{q_1}, \text{ etc.} \tag{61}$$

With Fig. 213 a first step is to bring the two pairs of unit 3 to a common pitch, changing the 6-pitch pair 15-66 to the 8-pitch equivalent 20-88. Then the several differences are, 11 in 96 or 11.5 per cent, 20 in 86 or 23.2 per cent and 52 in 108 or 48.2 per cent. The several ranges of m are, unit 1, 0.57 to 0.33, mean 0.43; unit 2, 0.79 to 0.26, mean 0.45; unit 3, 2.0 to 0.23, mean 0.68—these being geometric means, of the value $m_m = \sqrt{m_1 m_2}$. With practically the same mean m for 1 and 2 the percentage of difference is almost exactly doubled, while with m_m a good deal nearer to 1.00 in 3 than in 2 there is even more than full doubling, in spite of the much greater range of m.

In Fig. 211 the first speed change of unit 2 is (practically) from $m = 1$, and y_2 is just equal to $5y_1$ but with q_2 smaller by 1. In the second interval, however, the very great swing of m from unity drops y_2 from 20 to 13. In our full illustrative analysis of this train, Sect. 15, it was seen that unit 2 is not perfectly adjusted to the series, since its s values are 2.56 and 2.40, as against 2.48 for S_e from unit 1. The process of closer adjustment in such a case is shown by Form 5, where the scheme of

FORM 5

Case	(*a*)	(*b*)	(*c*)
y =	20 13	20 14	20 14
q =	91 46——26——13 45——65——78	93 47——27——13 46——66——80	94 47——27——13 47——67——81
s =	2.56 2.40	2.50 2.53	2.48 2.51

representation ought to be made clear by comparison with pair II of Fig. 209. The original proportions appear at (*a*). At (*b*) sum q is increased so as to diminish the influence of $y_{12} = 20$, and at the same time y_{23} is raised in order to make s_{23} bigger. The further slight adjustment of q at (*c*) simply lowers both s values together.

Prob. 117. Proportion a three-step cone pair to be used as second unit in connection with the pair of Prob. 111, or with $s = 1.3^4$; let $m_2 = 0.8$ and do not use less than 20 teeth in any gear.

Prob. 118. Find exact proportions for a three-step cone pair to be used in sequence with that of Prob. 113, or to have $s = 4$ exactly. Get tooth numbers for the three cases $m_2 = 1$, $m_2 = 0.5$, $m_2 = 0.8$, all of which are realizable requirements.

(*f*) **Back Gearing.**—Compare the back gear outlined in Fig. 249 (*a*) with the two-step cones (*b*) which contain the same wheel pairs. There is an important difference in the fact that the back gear delivers either the input speed unchanged or else a fully reduced speed, while with the cones one pair speeds up, the other down: but so far as speed-changing effect is concerned the two units are equivalent. This equivalence suggests the principle that step difference y will measure and determine the effect of a back gear just as it does that of a cone pair. The very simple example in Fig. 217 confirms and exactly embodies this idea.

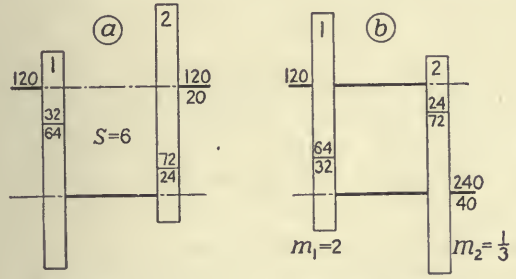

FIG. 249.—Back gear and two-step cone unit.

When analyzing a train such as Fig. 217 the procedure must be like that with Fig. 213 in the last article. If necessary the pairs are first to be brought to a common pitch, here changing G-H from 20-76 to 25-95. The step differences should be changed to percentages, or y in q converted to y' in 100: in this train $4/72 = 5.6/100$, $21/102 = 20.6/100$ and $49/120 = 40.8/100$. Discussion of these in relation to mean m is left to the reader: but one point to be understood is that when getting m_m for a back gear the two pairs must be treated as if side by side in a cone pair, not taken in order of driving sequence. Preferable going from outer axis 4 to spindle axis 3-5 in Fig. 217, the ratios are $m_1 = 74/46 = 1.61$ and $m_2 = 20/76 = 0.263$, whence $m_m = \sqrt{1.61 \times 0.263} = 0.65$.

(*g*) **Design of Back Gears.**—For a back gear not associated with gear cones, especially if it has a high change ratio S, the data for a design are likely to be this S and some indication as to wheel diameters. Thus for the example in Form 6 the desired back-gear ratio is 10, the diametral pitches are to be 7 for pair A-B and 5 for pair C-D, and the diameter of wheel D may be from 14 to 15 in. Tooth sums q_{ab} and q_{cd} must now be multiples of 7 and 5 respectively. Usually and reasonably the second pair has the larger ratio of diameters, or the larger share in producing S: and with 70 to 75 teeth in wheel D the sum q_{cd} can hardly be anything else than either 90 or 95.

FORM 6

Case	(*a*)		(*b*)		(*c*)		(*d*)	
2----B——C	97	20	101	23	93	20	90	18
1----A D	36	75	32	72	33	70	36	72
$p)q=$	7)133	5)95	133	95	7)126	5)90	126	90
$S = R =$	10.10		9.88		9.865		10.00	

Now for a back gear the change ratio S is identical with the train effect, as noted under Fig. 249; and since S has consistently been taken from higher to lower speed the train ratio should be, not T as in Sect. 15 (*c*), but its reciprocal R, from r_n in Sect. 8 (*c*). The scheme of solution for Form 6 is, then to assume tooth numbers c and d, divide r_{cd} into R to get r_{ab}, and divide q_{ab}. In (*a*) case $r_{cd} = 75/20 = 3.75$; $r_{ab} = 10/3.75 = 2.667$; $a = 133/3.667 = 36.3$, by Eq. (49); and with the nearest integer 36 for a the realized R is 10.10. Solution (*b*) is another trial, with the same sums 95 and 133; and the use of 32 for a (instead of exact 31.7) makes R come out a little less than 10. A disadvantage of both (*a*) and (*b*) is the high prime numbers for wheel B, 97 and 101. Trial (*c*) shows up better, having 31 as the largest prime factor; and solution (*d*), using obvious factors of R, is exact.

FORM 7

Case	(*a*)		(*b*)		(*c*)		(*d*)		(*e*)		(*f*)	
2	65	30	66	31	67	32	65	24	66	25	67	26
1	35	70	34	69	33	68	35	56	34	55	33	54
$q =$	100	100	100	100	100	100	100	80	100	80	100	80
$R =$	4.333		4.321		4.314		4.333		4.271		4.217	

Form 7 shows an obvious method for fine adjustment in a back-gear train, by means of nearly compensating changes in the two pairs. The scheme differs from Form 6 in that both q and y are kept constant. The contrast shown by Fig. 249 has further illustration here; for whereas the tooth changes directly modify the m's of the wheel pairs, they exert only a secondary effect upon the m ratio which is also R for the train. With equal q's for the two pairs, cases (a) to (c), the variation of R is so small as to be of little use; but when the two pairs have different pitches, so that the tooth changes are made upon different base numbers, the variation is of useful size.

Prob. 119. Complete the analysis of Fig. 217 by finding the m_m associated with each y', then discuss the influence of m range upon the approximation to Eq. (60) or (61).

Prob. 120. Make a complete analysis of (a) Fig. 223, (b) Fig. 228, (c) Fig. 229 or (d) Fig. 232 by the method described for Fig. 217, then discuss results.

Prob. 121. Design a suitable back gear for Fig. 225, making a twelve-speed train. Can it be placed upon the cone-pair axes, like the first small back gear in Fig. 228?

Prob. 122. Analyze the double back gear of Fig. 216, calculating the two gear ratios R_1 and R_2 and finding how well the speed ratio S of cone pair AC-BD agrees with R_1 and $\sqrt{R_2}$.

Prob. 123. Similarly compute the double- and triple-geared lathe of Fig. 129; get the two R's, see how well they work together, and test the relation of the belt cones to them.

(h) **Tumbler Cones.**—The simple problem of laying out tumbler cones for a geometrical series of speeds (and of diameters) is illustrated by two examples in Form 8. Start with the smallest wheel, here having 20 teeth in each case, using t for teeth in wheel. In line t' show the exact geometrical series, with fractional numbers. Opposite t write the line of integers that best approximate the exact numbers, then in the third line show the values of s realized with these actual wheels. While exact solution is gener-

FORM 8

Case (a)	$t' = 20$	25	31.25	39.06	48.83	61.23
$s = 1.25$	$t = 20$	25	31	39	49	61
	$s = 1.250$	1.240	1.258	1.256	1.245	
Case (b)	$t' = 20$	24	28.8	34.56	41.47	49.76
$s = 1.20$	$t = 20$	24	29	35	42	50
	$s = 1.200$	1.208	1.207	1.200	1.191	

ally not possible, satisfactory approximation can easily be attained. The biggest variation of t from t' here shown is about 1.3 per cent, for the 35- and 42-tooth wheels in the second example.

(*i*) **Problems in Train Design.**—Three typical kinds of problems are represented by Nos. 124, 125 and 126 which follow, differing in the amount of independent work required of the student. On these as models other problems can be selected or devised. In item 4 of Prob. 126 is introduced a condensed notation which calls for explanation. The scheme is to designate in sequence the speed-changing units of the train, using C for cone pair, B for back gear and T for cone and tumbler: the figure after the letter indicates the number of speeds produced by the unit, and the product of these factors gives the number of speeds for the train. Thus Fig. 213 is C2C2C2, eight speeds; Fig. 211 is C5C3, fifteen speeds; Fig. 232 is T4C2B2, sixteen speeds, and so on. A common back gear is B2, a double back gear, B3. The sequence may be either of drive or of speed-changing influence; these happen to be the same in the three examples just given (really, the last two) but differ in Prob. 126.

Prob. 124. Find suitable tooth numbers for the wheels not given in Fig. 231, to make a series on $s = 1.3$ as nearly as possible. Test exactness of series by the methods of Sect. 15 and calculate the highest and lowest speeds of shaft 5.

Prob. 125. Look up the description of the Cincinnati Milling Machine in American Machinist for Oct. 8, 1908, and from the information there found assign suitable tooth numbers to all the gears in the main drive.

Prob. 126. Lay out a speed-change train to meet the following specifications:

1. Train to be used in a.....*milling machine*
2. Driving pulley to have a constant speed of.....*300* r.p.m.
3. Spindle to have..*sixteen*..speeds in geometrical progression, ranging from..*350*..r.p.m. to..*14*..r.p.m.
4. Make the train, in order of..*drive*..*B2C4B2*.....following the general scheme of.....*the lathe train in Fig. 228.*
5. Use the same pitches as in..*Fig. 217*..and in the matter of wheel sizes or tooth numbers be guided by practice as shown by the examples that have been given.

SECTION 19. THREE-PART UNITS

(*a*) **Three Two-step Cones.**—The outstanding example of this type of speed-changing device is the combination of three gear cones in sequence, the first acting on the second, the second on the third, as in Figs. 250 to 253. To have at least two wheels

on the intermediate shaft serve as members of both cone pairs, with consequent interlocking of proportions, is an essential characteristic of these arrangements. Single wheels thus in common have been seen in Figs. 217, 223, 224, 229 and 233. Three-part units involving tumbler cones have appeared in Figs. 232 and 236, the first a very simple special case, the other to be discussed presently, at the end of Art. (*b*).

Diagram (*a*) of Fig. 250 shows the simplest possible combination of three cones, giving four speeds with six gears: the proportions are worked out for a geometrical series on the term ratio $s = 1.3$. In column under t_m are given the average tooth

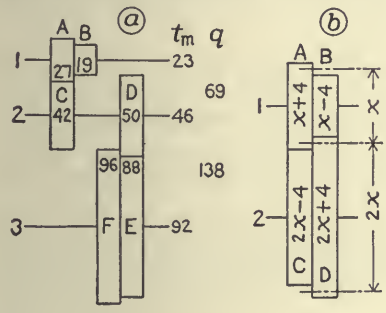

FIG. 250.

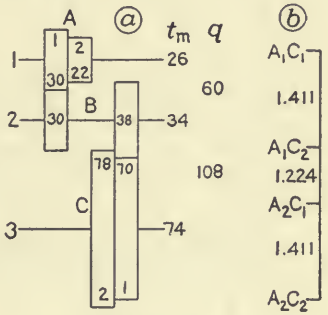

FIG. 251.

The 2-2-2 cone unit.

numbers for the wheels on each axis, while q is the tooth sum for each cone pair. To have the series of t_m values in the proportion 1 : 2 : 4 is the simplest way to make q twice as big for pair 2-3 as for pair 1-2; and because of this doubling of q the common step difference $y = 8$ has only half as much influence in the second pair as in the first. The results of calculation from the figure are, for pair 2-3, $s = 1.299$, for pair 1-2, $s = 1.692 = 1.301^2$.

The design of pair 1-2, which is the critical part of the train because it contains the smallest wheels, is partly illustrated by sketch (*b*). With $s = 1.3$, the doubled ratio for pair 1-2 will be $1.3^2 = 1.69$. Table 4 is used as an aid in deciding upon y: for $s = 1.69$, $q_0 = 78$; one third of this (since t_{m1} is half of t_{m2} or one-third of q_{12}) gives 26 for t_m on axis 1, and deducting half of tabular y (or 5) we get 21 teeth for wheel B. Since, however, m will be around 0.5, s will be somewhat greater than s_0; and a desire to keep down the size of the train leads to the use of 8 instead of 10 for y.

Now let x represent t_m for cone 1, when the tooth numbers for the four gears may be written as on Fig. 250 (*b*). Then

$$s = \frac{x+4}{2x-4}\,\frac{2x+4}{x-4} = \frac{x^2+6x+8}{x^2-6x+8} = 1.69; \qquad (62)$$

and the solution of this quadratic equation gives $x = 23.05$, from which the train at (*a*) is built up.

Unless this gearing is so proportioned as to double q in the second pair, it will not give a smooth series. Thus in Fig. 251, which is based on an actual lathe, the differences are 8 in 108 and 8 in 60, equivalent to 7.4 and 13.3 per cent. The result of this failure to get the 2:1 ratio (modified by difference in range of m for the two pairs) is the irregularity of intervals shown by the log plot at (*b*).

Prob. 127. Find a new value of t_m for cone *C*, Fig. 251, that will give equal divisions of the larger intervals of pair A-B.

Prob. 128. In train like Fig. 250 let t_{m1} be first 0.6 of t_{m2}, then 0.4 of t_{m2}, and for each condition find the value of t_{m3} that will make $q_{23} = 2q_{12}$.

Prob. 129. Design a three-cone unit like Fig. 250 for the average step ratio $s = 1.26$.

(*b*) **Combined Three-step Cones.**—The three-cone unit in Figs. 252 and 253 is the one that has been most used by designers,

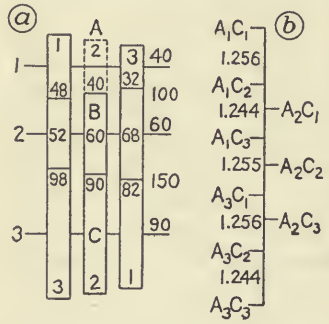

FIG. 252.

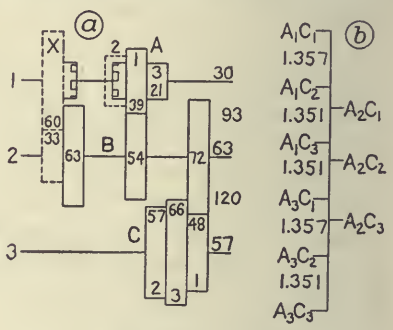

FIG. 253.

The 2-3-3 or 3-3-3 cone unit.

oftener with three steps than with two in the first cone, although two give the better arrangement. Pair B-C determines the small changes; and its extended ratio, covering three intervals, should be equalled by the two-step pair A-B as distinguished by full lines in cone A. The first layout, Fig. 252, follows the idea of Fig. 250 in putting the three t_m's into a geometrical series; this gives the same range of m in both pairs and, by means of the term

ratio 1.5 secures that $y = 16$ in $q = 100$ shall have three times the effect of $y = 8$ in $q = 150$. The six speeds got with steps 1 and 3 of cone A fall into the excellent series laid out on the left side of the log line at (*b*). Adding the dotted wheel A_2, the step ratios in pair A-B are now each about 1.5 times the size of s in pair C-D, which throws the three speeds from A_2 at about the middle of three of the main intervals.

This unit is not under such close restriction to certain relative proportions as is that in Fig. 250. On the contrary, the t_m's of cones B and C may have a pretty wide range of ratio and suitable dimensions yet be found for A_1 and A_3. Thus in Fig. 253 the mean step ratio of pair B-C is

$$s = \sqrt{\frac{72}{48}\frac{66}{54}} = 1.354,$$

and its extended ratio is $S = s^3 = 2.482$. Let x and $x + 18$ be the tooth numbers for A_1 and A_3, then the condition to be met is,

$$S = \frac{72}{x}\frac{18 + x}{54} = 2.482, \qquad (63)$$

from which $x = 20.89$. This is very closely represented by 21, and the integral numbers on the figure make $S = 2.476$. Again the main series of six speeds is very good, but now the extra three from A_2 are less regularly placed than in Fig. 252. A far better way to get nine speeds is to extend pair A-B by adding the independent step X, in true series with steps 1 and 3.

The operation of calculating wheels B and C for Fig. 236 is just like that in Eq. (63). The gear ratio of the tumbler rig from H to L, covering seven intervals, is

$$S = s^7 = \frac{70}{25}\frac{54}{25} = 6.048,$$

so that $s = 1.293$ and $S_e = s^8 = 7.821$. Using the established D and G, let $c = x$ and $b = x + 38$; then for the cone pair

$$S = \frac{70}{x}\frac{x + 38}{32} = 7.821.$$

From this $x = 14.75$; then the use of $x = 15$ makes S come out 7.729, and for the critical interval $s = 7.729 \div 6.048 = 1.278$, as against the prevailing 1.293.

Prob. 130. Determine a six-speed three-cone unit like Fig. 252 and Fig. 253, for mean $s = \sqrt[3]{2.5}$, letting $t_{m1} = 0.75\, t_{m2}$ and using $y = 8$.

Prob. 131. Investigate the feasibility of an exact algebraic solution of the last problem, when the data are s, y and one turning ratio. Take

s and y the same as there (numerically) and let the highest turning ratio be 1.00: the unknown quantities will be t_{m1}, t_{m2} and t_{m3}.

Prob. 132. Trace out the working of Fig. 254, lettering the gears in proper sequence and making out a tabular statement of all the combinations, in a series from high to low speed.

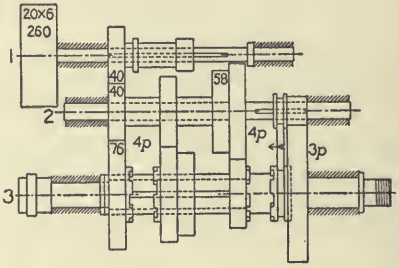

FIG. 254.—Twelve-speed train for 36-inch lathe.

Prob. 133. Compute and tabulate the complete speed series for Fig. 254, first supplying the missing tooth numbers.

(*c*) **Combined Pairs with Unequal Intervals.**—A special type of three-part combination, more complex in its scheme of speed serialization than any train that has so far been discussed, is

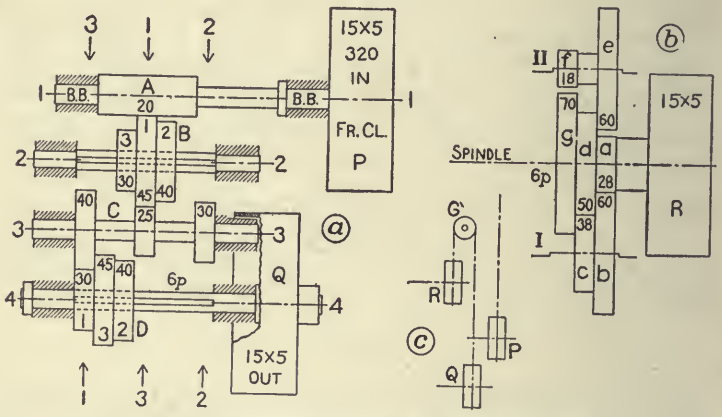

FIG. 255.—Main drive of Becker milling machine.

shown in Fig. 255 and in Fig. 256. In the first example sliding "cones" B and D are exactly alike, so that the two step differences of 5 and 10 have the same influence in both of pairs B-C and C-D. The smaller step changes speed by the proportion s,

the larger by s^2, therefore the overall ratio for either pair is $S = s^3$. With nothing like an extended ratio from the first pair to the second, but simply using the two internal S ratios, the combination covers six intervals and gives seven well-spaced speeds. Of course the whole unit will have an $S_e = s^7$, to be carried over to the double back gearing at (*b*). Counting direct drive by pulley R, the spindle has 21 speeds. Diagram (*c*) shows the arrangement of belting: the belt to pulley P comes from overhead shafting, then within the machine there is belt drive from Q to R, over the guide pulleys at G—see Fig. 113.

FORM 9

Interval.....	1-2	2-3	3-4			4-5	5-6	6-7
From setting...	B1-D1	B2-D1	B1-D2	B1-D2	B1-D2	B2-D2	B2-D3	B3-D2
to setting....	B2-D1	B1-D2	B1-D3	B2-D2	B3-D1	B2-D3	B3-D2	B3-D3
Pair change....	b_{12}	$-b_{12} + d_{12}$	d_{23}	b_{12}	$b_{13} - d_{12}$	d_{23}	$b_{23} - d_{23}$	d_{23}
Net effect.....	s	s^{-1+2}	s	s	s^{3-2}	s	s^{2-1}	s

To get a simple notation for the analysis of Fig. 255 in Form 9, let b_{12} represent the speed change (ratio) effected by shift 1-2 of cone B, and so on: then the values or effects of the four step changes are,

$$b_{12} = s, \qquad b_{23} = s^2, \qquad d_{12} = s^2, \qquad d_{23} = s.$$

In Form 9 each little column describes the operation of shifting from one speed to the next. The numbers in the top line (and in the first column of Form 10) represent the successive speeds, while those used with B and D refer to positions of the gear cones. The combinations symbolized in Form 9, by B1-D1, etc., are in Form 10 expressed numerically, with calculation of train value t and step ratio s.

Now out of three groups of three gears nine combinations may be selected; but this train gives only seven speeds, because three of the combinations are identical in effect, having the value $t = 1$. These are B1-D3, B2-D2 and B3-D1, alike in using one wheel of cone C as a simple idler. Practically the control rig provides for but one of these combinations, namely B2-D2. There are three possible ways of covering speed interval 3-4 (all shown in Form 9) and three for interval 4-5 (only the used one shown).

The general idea of this train is to use the short interval of each pair directly and also to make it split the long interval of the

FORM 10

Fig. 255						Fig. 256				
Speed	B	D	Train	t	s	Q	B	D	Train	t	s
1	1	1	$\frac{45}{25}\frac{40}{80} = 2.400$			$\frac{85}{35}$	1	1	$\frac{20}{30}\frac{75}{40} = 1.250$		
				1.350							1.364
2	2	1	$\frac{40}{30}\frac{40}{30} = 1.778$			$\frac{80}{60}$	1	2	$\frac{20}{30}\frac{55}{40} = 0.917$		
				1.317							1.344
3	1	2	$\frac{45}{25}\frac{30}{45} = 1.350$			$\frac{75}{65}$	2	1	$\frac{20}{55}\frac{75}{40} = 0.682$		
				1.350							1.364
	1	3					1	3			
4	2	2	$\frac{40}{30}\frac{30}{40} = 1.000$			$\frac{70}{70}$	2	2	$\frac{20}{55}\frac{55}{40} = 0.500$		
	3	1					3	1			
				1.350							1.364
5	2	3	$\frac{40}{30}\frac{25}{45} = 0.741$			$\frac{66}{75}$	3	2	$\frac{20}{75}\frac{55}{40} = 0.367$		
				1.317							1.344
6	3	2	$\frac{30}{40}\frac{30}{40} = 0.563$			$\frac{60}{80}$	2	3	$\frac{20}{55}\frac{30}{40} = 0.273$		
				1.350							1.364
7	3	3	$\frac{30}{40}\frac{25}{45} = 0.417$			$\frac{55}{85}$	3	3	$\frac{20}{75}\frac{30}{40} = 0.200$		

other pair. Four of the six changes are got by single shifts, but 2-3 and 5-6 require movements of both cones, as would also some of the superfluous changes at the middle of the series.

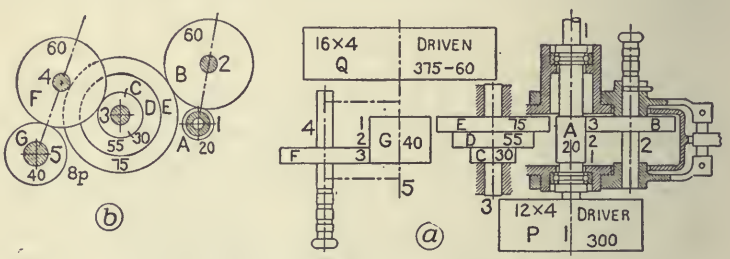

FIG. 256.—Older form equivalent to Fig. 255.

The net effect of a double shift can be shown in simple fashion by the addition of plus and minus exponents on s, in the last line of Form 9; each of these combined movements multiplies t by s^2 and divides by s.

In Fig. 256 there are two tumblers B and F, working with a common cone CDE. The latter has the step ratios $s^2 = 55/30 = 1.833 = 1.357^2$ for CD and $s = 75/55 = 1.364$ for DE. The speed series is developed very much as for Fig. 255, so that a description like Form 9 would differ only in a few details. The calculation for both trains in Form 10 is self-explanatory, except

as to the column headed Q. In Fig. 255 the sum of the tooth numbers of all the gears in action is constant (counting an idler twice); then the fraction or ratio under Q is got by adding together the teeth of drivers for numerator and the teeth of followers for denominator. It is now seen that there is a progressive change in these sums which is just like the equal step changes in a common cone pair.

Prob. 134. Design a unit like Fig. 255 (a), for $s = 1.26$ with $y = 5$ as the smallest step difference.

Prob. 135. Design a train like Fig. 256, for $s = 1.20$, with the highest turning ratio $t_1 = 1.2$.

SECTION 20. SCREW AND FEED TRAINS

(a) **Lathe Feed Gearing.**—The feed gearing of a simple lathe of the older type has been described in Sect. 10 (a), under Figs. 126 to 128. Any ordinary engine lathe has the two feed systems there shown, one for general work, the other for screw cutting. The now prevalent quick-change lathe has an elaborate gear train in place of the change gears in Fig. 127 (d); and this same train, or a part of it, serves as feed-rod drive.

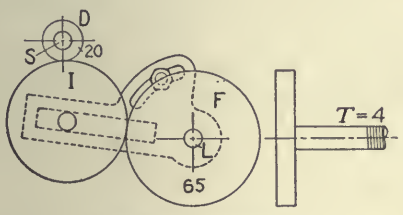

FIG. 257.—Simple.

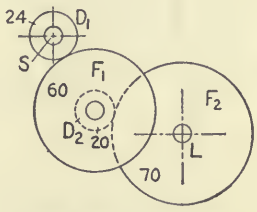

FIG. 258.—Compound.

Typical lead-screw trains.

The two typical forms of plain change gearing are shown in Figs. 257 and 258: they are distinguished by having a simple idler I or a compound intermediate F_1D_2. Stud S, which is on axis 2 in Fig. 127, turns at the same speed as the spindle, either directly or reversed. Let S = turns of stud, spindle or work piece and L = corresponding turns of lead screw. Use D and F to represent the tooth numbers of drivers and followers in the wheel train. Then for Figs. 257 and 258,

$$\frac{L}{S} = \frac{D}{F} \text{ and } \frac{L}{S} = \frac{D_1 D_2}{F_1 F_2}. \qquad (64)$$

Now let P = pitch of lead screw (measured on the inch scale) and T = threads per inch on the lead screw, also p = pitch and t = threads of the screw to be cut. For turns L and S equal measures of the distance traversed by the tool are,

$$PL = pS. \tag{65}$$

Whence

$$p = P\frac{L}{S} = P\frac{D}{F}; \tag{66}$$

and since t or T is the reciprocal of p or P,

$$t = T\frac{S}{L} = T\frac{F}{D}. \tag{67}$$

Here D/F is to be considered a general symbol for turning ratio, whether simple or compound.

In choosing change wheels for a lathe like Fig. 126 the scheme is to take the ratio t/T and select wheels that will give F/D the same value. Thus to cut a 13-thread screw with the 4-thread lead screw of that lathe, F/D must be 13/4 or D/F be 4/13. A factor 5, which converts the "terms" of the ratio to 20 and 65, gives suitable tooth numbers. If 35 threads are to be cut on that same lathe it will be necessary to use the compound intermediate. Take the ratio $DD/FF = 4/35$ and split it into the obvious factors 2/5 and 2/7, which can be realized with wheel of 24, 60, 20 and 70 teeth.

(*b*) **Tumbler Cones for Change Gearing.**—The primary unit of nearly all quick-change lead-screw trains is a cone-and-

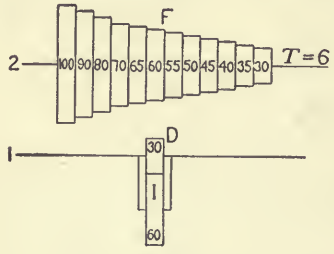

FIG. 259.—English.

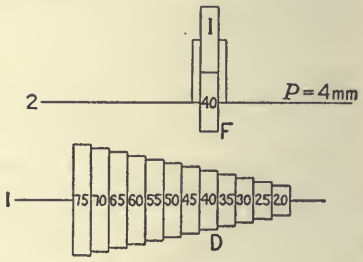

FIG. 260.—Metric.

Cone-and-tumbler arrangements for lead screws.

tumbler pair based on the arrangement in Fig. 257, while the secondary unit is a binary-series train which multiplies and divides the cone ratios by 2. Of the two wheels D and F in Fig. 257 one will serve as pinion (constant), the other will

become a cone (in effect a wheel of varying diameter). Whether F or D shall be expanded into a cone of gears depends upon the system of screw threads. In the English system t (threads per inch) is the primary quantity; and since t is proportional to F, as shown by Eq. (67), the follower will be the changing gear or the cone. In the metric system pitch p (measured in millimeters and simple binary fractions) is the given dimension, and Eq. (66) shows that the cone must now be driver.

For the typical examples outlined in Figs. 259 and 260 relations are as follows:

In Fig. 259,

$$t = T\frac{F}{D} = 6 \times \frac{F}{30} = \frac{F}{5}, \tag{68}$$

and with the speed of shaft 1 equal to that of the spindle the threads cut will range from 20 to 6.

In Fig. 260,

$$p = P\frac{D}{F} = 4 \times \frac{D}{40} = \frac{D}{10}, \tag{69}$$

so that on equal speed of cone and spindle the pitch ranges from 7.5 to 2 mm.

(*c*) **English and Metric Screw Threads.**—At this point it is appropriate to consider the cutting of metric threads with an inch-scale lead screw and the converse. Since 1 in. = 2.54 cm. almost exactly, the gear pair 50-127 will serve for conversion from one scale to the other. Suppose that $D = 50$ with $F = 127$ is used for one of the pairs in Fig. 258; then if the other pair has been chosen to cut t threads per inch (alone) it will cut t threads per centimeter when used with this translating pair. Thus $D = 30$ and $F = 75$ will cut a screw of 10 per inch with a 4-thread lead screw; but if the train be made $D_1 = 30$, $F_1 = 75$, $D_2 = 50$, $F_2 = 127$, there will be 10 threads per centimeter or a pitch of 1 mm.

(*d*) **Binary-series Trains.**—The train which accompanies the cone of Fig. 259 is shown in Fig. 261. Sleeve gear S turns exactly with the spindle and through the bevel reverse gearing at P drives the change train. Wheels A and B are mounted like change gears (Q being a regular radius arm or sector as in Fig. 257), and other wheels can be put in for getting special threads. The binary train includes gears D to K and gives the ratios 4, 1 and ¼. View (*b*) shows how it is folded into

compact shape and how gears F and G are withdrawn from mesh with D and E while making a shift: lever R must be swung to the lower position in order to get it from one guiding slot into another.

The binary train outlined in Fig. 262 has the similar effect of giving 4, 1 and ¼ as its principal ratios. Change wheels E-G = 40-80 and 80-40 are used for the main thread series, pair 80-80 only to get a few extra numbers. Ratio 1 can be made in two ways, either A-B with 40-80 or C-D with 80-40. The binary elements in the next two examples need not be separated from

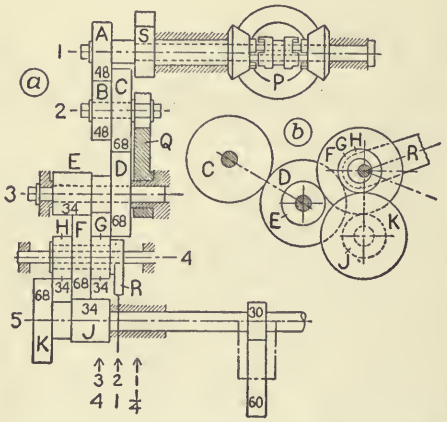

FIG. 261.—Hendey binary train.

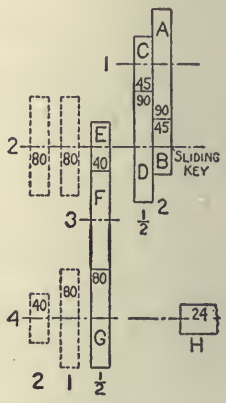

FIG. 262.—Brown & Sharpe train.

the rest of their trains: but the use of sliding gears and fact that the step ratio is 2 in each case are interesting general points. Figure 265 is given for the sake of its compound tumbler unit, which makes a very compact arrangement: there are three compound wheels A-B, C-D and E-F, with pair E-F turning four times as fast as C-D. From cone to lead screw there are the four D/F ratios 4, 2, 1 and ½.

(*e*) **Complete Lathe Trains.**—The tumbler cones that go with Figs. 261 and 262 have their largest steps nearly four times the size of the smallest, the ratios being 100 to 30 and 88 to 24 respectively. In Figs. 263 to 265 the cone range is less than 2 and the binary train conforms more strictly to its name. With the transmission pair 30-60 from stud 1 to shaft 3 in Fig. 263 the ratios are 8, 4, 2, 1 and ½, when taken in the sense or direction D/F. Notable features of this example are, the small tooth

numbers in the cone, the fact that the binary train is all below the cone (instead of preceding it as in the last two figures), and the use of direct-order open sliding cones in the binary train. With a 2-step and a 3-step cone pair there are six speeds, but an overlap in the middle of the series (like that in Fig. 262) makes the effective number five.

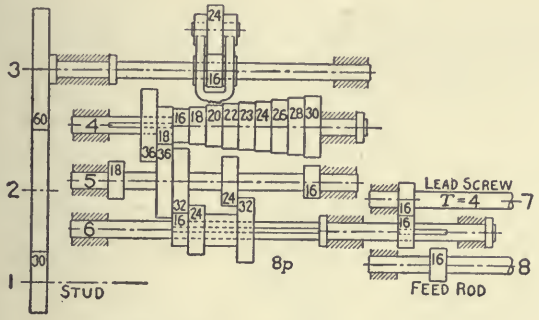

FIG. 263.—Morris feed train.

In Fig. 264 pinion shaft 1 is driven, through change gears, at either one-half or twice the speed of the spindle: the half speed gives 4 to 56 threads, the double speed 1 to 14. The binary train contains two back gears, A-B-C-D and E-F-G-H. Its *F/D* ratios, as produced by different clutch arrangements, are, M-P,

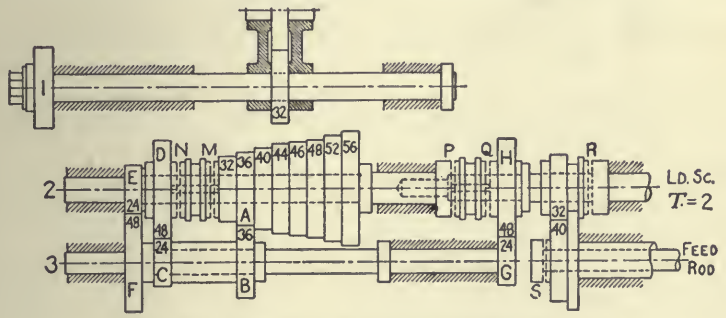

FIG. 264.—American Tool Works lathe feed.

1; N-P, 2; M-Q, 4; N-Q, 8. With the factors ½ and 2 from the transmission gearing above shaft 1 the complete ratio ranges from ½ to 16, with an overlap occurring on 2 and 4.

The binary train in Fig. 265 has two like compound gears C-D and E-F, carried on a swinging arm and driven by two-

step cone A-B on tumbler-cone shaft 4. These constitute a compound tumbler, any one of the four gears C, D, E and F being able to engage sliding gear G on axis 6.

Figure 266 is an almost unique exception to the general use of the tumbler cone. In its binary train there are two sliding keys, worked by collars P and Q. This train gives the F/D ratios 6, 3, $\frac{3}{2}$ and $\frac{3}{4}$ between shaft 6 and the spindle, the common factor 3 belonging to the scheme of the double gear cones. To get the idea of their layout put Eq. (67) into the form

$$t = TB\frac{F}{D} \text{ or } \frac{F}{D} = \frac{t}{TB}, \tag{70}$$

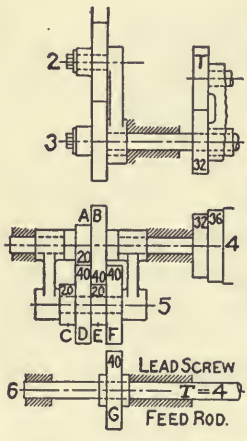

FIG. 265.—Lodge & Shipley binary train.

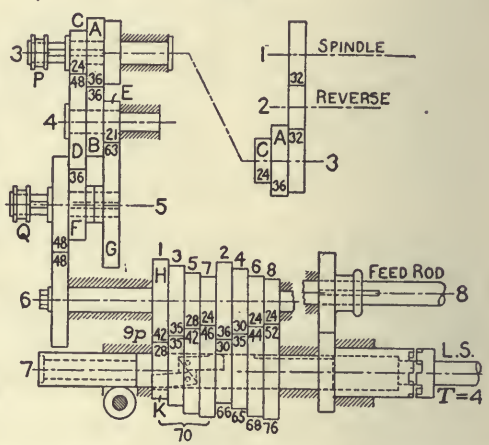

FIG. 266.—Whitcomb & Blaisdell lathe feed, with gear cones.

where B stands for the ratio above the cones (just given as 6, 3, etc.) and F/D belongs to the cone pair alone. With $T = 4$ use $B = 3$ and get the scheme of gearing set forth in Form 11.

FORM 11

No.	1	2	3	4	5	6	7	8	Line
$\frac{t}{BT}$ =	$\frac{8}{12}$	$\frac{10}{12}$	$\frac{12}{12}$	$\frac{14}{12}$	$\frac{18}{12}$	$\frac{22}{12}$	$\frac{23}{12}$	$\frac{26}{12}$	(1)
$\frac{F}{D}$ =	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{1}$	$\frac{7}{6}$	$\frac{3}{2}$	$\frac{11}{6}$	$\frac{23}{12}$	$\frac{13}{6}$	(2)
q $\propto$	5	11	2	13	5	17	35	19	(3)
q =	70	66	70	65	70	68	70	76	(4)

It is evident that in the devising of this train there must have been a good deal of ingenious cut-and-try work, in order to get the best combination of numbers. A distinctive feature is the use of special pitches for the gears in the cones which carry steps 2, 4, 6 and 8, so that with various tooth sums there shall yet be a constant axis distance.

As regards feeds for ordinary cutting, most quick-change lathes use the full lead-screw train to drive the feed rod also, as shown in Figs. 263 to 266. A good plan is to proportion the gearing in the tool carriage so that the cuts per inch will be some multiple of the screw threads, say five times as many.

(*f*) **Milling-machine Feed Trains.**—The most extensive feed-change trains not related to the production of screw threads are found in milling machines. Of the two examples in Figs. 267

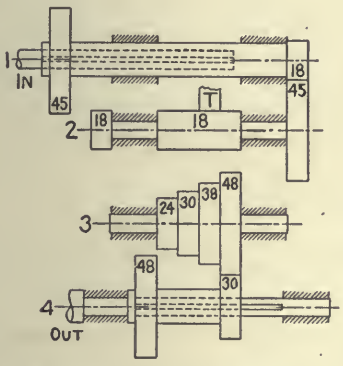

FIG. 267.—Brown & Sharpe.

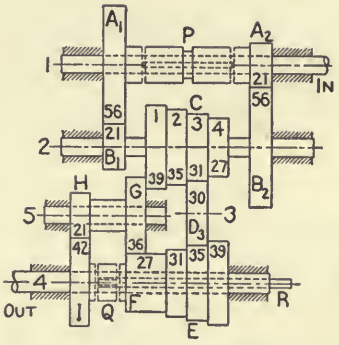

FIG. 268.—Kempsmith.

Milling-machine feed trains.

and 268 the first closely resembles the main train of its machine, Fig. 232, while the second has a unit like Fig. 143 (here laid out flat) for its small changes: the rotating or oscillating cage, which turns on axis 4, carries four D pinions and also the back-gear shaft 5. Each train gives sixteen speeds in geometrical series, the ratio of highest to lowest being 32 with Fig. 267 and nearly 40 with Fig. 268.

Two methods of feed-gear drive are used: in the older belt-cone machines (main drive) the feed motion is taken from the spindle (as in the lathe), and the feeds are stated in advance of work table per turn of spindle, as from 0.005 to 0.200 in. for Fig. 268. With the newer constant-speed drive, shaft 1 of the feed train is driven

from the main pulley at a constant speed (see beginning of drive on Fig. 232) and the feeds are then given in inches of table movement per minute, as 5/8 to 20 in. for Fig. 267. Since the circumferential speed of a milling cutter is what really counts, the second arrangement is the more logical and convenient.

Prob. 136. For a lathe with $T = 6$ make out a set of change wheels to cut 40, 36, 32, 28, 24, 20, 18, 16, 14, 12, 11, 10, 9 and 8 threads per inch. Have all the tooth numbers multiples of 5 and get along with as few wheels as possible. Give all the wheel combinations.

Prob. 137. For a lathe with $T = 2$ make out a set of change wheels to cut 12, 11, 10, 9, 8, 7½, 7, 6½, 6, 5½, 5, 4½, 4, 3½, 3¼ and 3 threads, again using multiples of 5.

Prob. 138. With a lead screw of $T = 4$ find the fewest change wheels that will give the American or Briggs pipe threads, which have 27, 18, 14, 11½ and 8 as values of t.

Prob. 139. Having the lead screw $T = 2$, select wheels to cut worms of 3/8, 5/8, 7/8, 0.4 and 1.4 in. pitch, also of 10, 15, 20, 25 and 30 mm. pitch.

Prob. 140. Find change gears to cut a screw of 12 threads per inch with a lead screw of 4 mm. pitch.

Solution. Twelve threads per centimeter would make the pitch $1\frac{9}{12} = \frac{5}{6}$ mm. Then by eq. (66) one pair must have the ratio

$$\frac{D}{F} = \frac{p}{P} = \frac{5}{24} = \frac{25}{120},$$

while the translating pair will be = 127/50. If desired the order may be changed from

$$\frac{25}{120} \times \frac{127}{50} \text{ to } \frac{25}{50} \times \frac{127}{120},$$

when any other 1:2 pair may be used in place of 25:50. Or it may be convenient to make the train

$$\frac{5}{24} \times \frac{127}{50} = \frac{5}{12} \times \frac{127}{100} = \frac{30}{72} \times \frac{127}{100}.$$

Prob. 141. With $T = 3$ per inch find change wheels to cut a thread of $p = 1.75$ mm.

Prob. 142. With $P = 4$ mm. find change wheels to cut a screw of $t = 11\frac{1}{2}$ per inch.

Prob. 143. Make out a table of all the threads that can be cut with the lead-screw train made up of Figs. 261 and 259. How can 11½ threads most conveniently be got?—With what change in pair A-B and with what setting of the other elements of the train?

In this and the following problems arrange the table as in Form 12, B standing for the binary-train factor, taken in the direction F/D. For each value of B state the active gear combination in the binary train.

FORM 12

B	Drive	Thread											
4	?	80	72	etc.									
1	?	20	18	16	14	13	12	11	10	9	8	7	6
¼	?	5	4½	etc.									

Prob. 144. The 24-tooth long pinion H in Fig. 262 drives a 12-step cone with 88, 80, 72, 64, 56, 48, 40, 36, 32, 28, 26 and 24 teeth. Make out a table of all the threads provided, finding what numbers are given only by the B ratios 2 and ½ from the change pair 80-80. What change in pair E-G and what setting of the cone will best give 11½ threads? Which of the two settings for $B = 1$ is more likely to be ready for use, considering the small size (16-in. swing) of the lathe?

Prob. 145. Make out a full thread table for Fig. 263 or Fig. 264, following Form 12.

Prob. 146. Write the thread table for Fig. 266, first with a line for each value of B, then with the threads in sequence from 52 down to 2. If the wheels in the first cone pair (with $q = 70$) have the diametral pitch = 9, find the exact pitch (to three places of decimals) for each of the wheel pairs in the second cone pair.

Prob. 147. Test fully the speed series of Fig. 267 or of Fig. 268.

SECTION 21. CONTINUOUS SPEED VARIATORS

(*a*) **General Principles.**—The large number of transmission devices that might be gathered together under this title are characterized by frictional drive and by speed change that can be continuous, or by any increment, rather than over a definite series of steps. In varying degree they suffer from the lack of positiveness inherent in transmission by friction. Beside the simple slip due to overload, such as may readily occur in wheel pairs like Figs. 51 and 52, there is frequently a secondary slip due to imperfect rolling, which occurs continually. Many devices of this class show inventiveness, concentrated on a narrow purpose, rather than broad mechanical sense; and many of them have never gotten beyond the stage of being patented and perhaps tried out. The examples here given are mostly from the more practical members of the group, with a few of less effective form that yet show a clear special principle.

(*b*) **The Disc-and-wheel Pair.**—One of the forms most used for light service is the combination of side disc A with wheel or edge disc B in Fig. 269, which has also the reversing effect already

set forth by Fig. 197. Wheel B may be splined to shaft 3 and shifted by a claw piece C as here, or it may be keyed to a shaft which moves endwise for change of speed. The full-line construction is rather lacking in rigidity, the dotted details suggest improvements in that respect: if reversal is not needed bearing E can be raised to E′, holding shaft 3 much more stiffly; and the insertion of a supporting roller at F will relieve shaft 3 of bending load and hold up disc A more rigidly to its contact with B.

(*c*) **Secondary Slip.**—This example serves excellently to illustrate the matter of secondary slip, by the magnified details in Fig. 270, each of which shows portions of discs A and B positioned

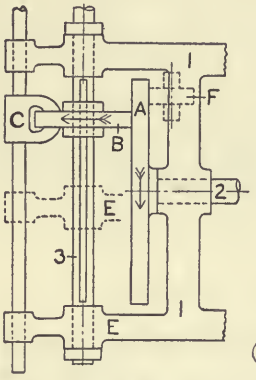

FIG. 269.—Friction disc drive.

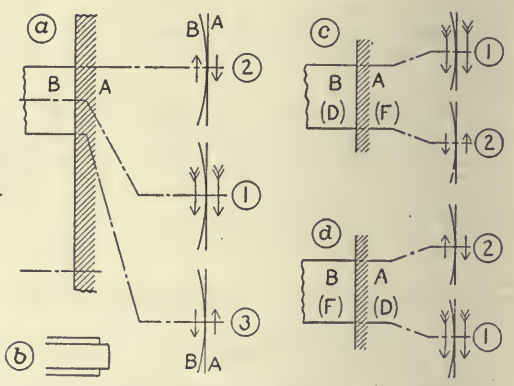

FIG. 270.—Analysis of slipping action.

as in Fig. 269, with little profile sections all taken as if viewed from the top. In case (*a*) there is supposed to be true rolling, without slip, at the middle of wheel B, as shown by section (1): the feathered arrows, equal and alike in direction, indicate coincident movement of elements of the two surfaces. All circular elements of cylinder B necessarily have the same circumferential velocity, but those on the side of disc A have velocities which vary as their radii. Consequently, A is moving faster than the outer or upper face of B, slower than the lower or inner face. The resultant slip is shown by pairs of short relative-motion arrows on profiles (2) and (3): A is slipping ahead in (2), behind in (3).

This unavoidable slip wastes a small portion of the power transmitted and tends to wear away the edges of the rim of disc B. In order to diminish its amount and effect that disc is commonly made as suggested at (*b*), with a slightly rounded face.

Further, it is usually of a softer material (than A) having a high coefficient of friction, held between metal plates. Suitable materials are, hardened rawhide, compressed and treated paper or other composition, and hard wood in segments which expose end grain.

The symmetrical distribution of slip shown by Fig. 270 (*a*) is probable only when the wheels roll without resistance. Under load the slip tends to be all in one direction, which will be such as to cause slower movement of the follower. With B as driver, case (*c*), true rolling at the outer edge means slower turning of A; but when A drives, case (*d*), slower movement of B corresponds with inner rolling and outer slip. Under heavy load there will be slipping clear across the line of contact, but greater at one end than the other.

(*d*) **Disc Drives with Axes Parallel.**—Figure 271 shows two simple ways in which the pair of Fig. 269 may be doubled so as

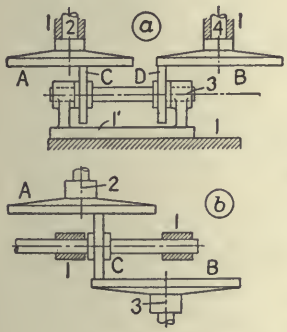

FIG. 271.

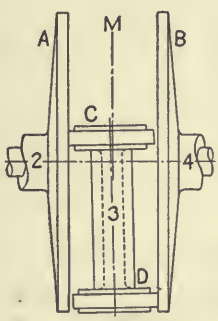

FIG. 272.

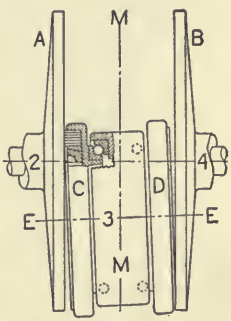

FIG. 273.

Various arrangements with parallel axes.

to give transmission between parallel axes. Both arrangements cause reversal of turning and are like stepped cones in that equal displacements of intermediate C-D or C (arithmetical progression) produce proportional changes in the speed ratio (geometrical progression). Turning in like direction, with similar manner of speed variation, is secured by the two combinations in Figs. 272 and 273. The slightly conical form of edge discs C and D in Fig. 272 increase the secondary slip, while the relative motion in Fig. 273 is such as to diminish that slip. The working faces of the intermediate in the second example are composed of rings of leather, securely clamped, and the

appropriate use of ball bearings is shown. Provision must of course be made for guided movement of support 3 along axis MM.

An interesting variation, compact and simple in the support and movement of the intermediate wheels, is shown by Fig. 274, while Fig. 275 gives the gear for control of the two wheels. Having circle-arc profiles, the working faces of side discs A and B can easily be generated in the lathe. The dotted outlines indicate pulley rims cast right on the discs, or one pulley might be on shaft 2, the other on long hub H. Spring pressure P will act through a thrust bearing, but by doubling together units of this type it is possible to eliminate external gripping forces,

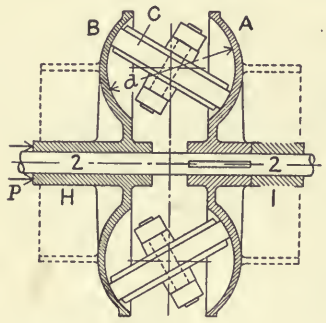

FIG. 274.

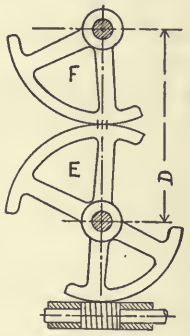

FIG. 275.

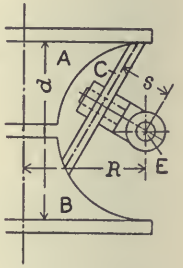

FIG. 276.

"Discs" of circular profile.

applied through running contacts. This is analogous to the change from "open grip" to "closed grip" with friction clutches —see Sect. 13 (*f*)—but is much more difficult of attainment.

The scheme in Fig. 276 has serious practical faults but shows interesting geometrical features. In all our previous examples secondary slip increases (toward excess) as the working radius is shortened on any side disc. Here such slip actually becomes less as an edge on C approaches its inner limit. Further, the manner of variation of speed ratio is considerably modified from that in the arrangements having cone-pair effect.

Prob. 148. Sketch out the following arrangement: Use two units like Fig. 274, splining the outer A discs to the shaft with springs outside of them abutting against collars fast on shaft; mount the inner B discs on a sleeve hub with belt pulley between them; and couple together the two rigs like Fig. 275 so as to insure proper coincident movement of the C-D wheels.

Prob. 149. In Fig. 274 let the middle or neutral position of wheels C and D be that in which their planes are parallel to axis 2-2; then with 45

deg. as the limit of swing in either direction, find the ratio of axis distance D (Fig. 275) to wheel diameter d that will make the maximum follower speed four times the minimum.

Prob. 150. (a) For Fig. 274, (b) for Fig. 276, compute and plot curves of velocity ratio as follows: On a base representing the developed profile of the contact surface of disc A or B lay off equal intervals to represent equally-spaced angular positions of disc C or D; then on each ordinate plot the logarithm of the turning ratio of follower to driver and draw curve, in order to see how it compares with the straight line of a geometrical progression. Reduce C or D to a mere plane disc and assume pure rolling. For (a) use the proportions found in the last problem; for (b) take $d = 14$, $R = 9\frac{1}{2}$, $s = 4$ inches.

(e) **Wedge Discs.**—An effective form of variable friction drive is the Sellers type of wedging or gripping discs in Fig. 277, where sectional views (a) and (b) show interchange of inner and outer

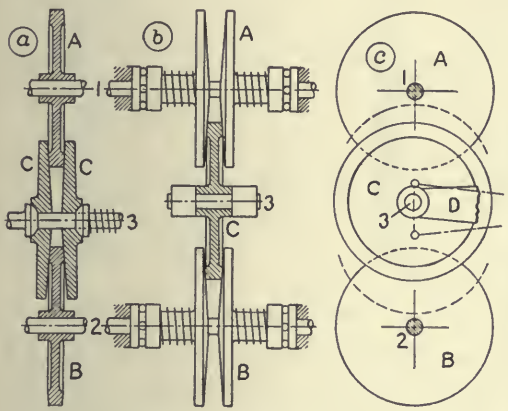

FIG. 277.—Three-axis, wedge-disc arrangements.

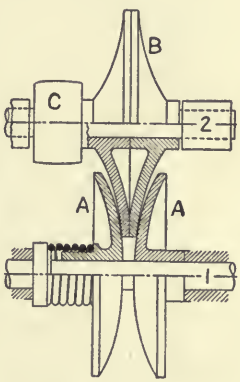

FIG. 278.—Short-range curved discs.

members. Axes 1 and 2 are fixed, while intermediate C is carried on the end of an arm D as indicated by side view (c). Note how the two C discs in (a) are held by ball-and-socket joints, so that they can swing out of parallelism—and in operation there is continual small movement at those joints. Arrangement (b) calls for a short sliding of the A and B discs, splined to shafts 1 and 2 and guided by long hubs, but only at the time when speed is being changed.

Sometimes only two axes are used in this combination, one disc swinging at the end of an arm after the manner of a tumbler gear; and then the discs may be multiplied as in the Weston type of

friction clutch, Fig. 176. Kinematically this is like the cone-and-tumbler pair in that only one radius is changed. A special form which with but two axes secures variation of both radii is shown in Fig. 278, where shaft 2 is at the end of an arm and a short belt connects pulley C with another pulley at the axis on which this arm swings. The profile of A has a little more curvature than that of B and a short movement of axis 2 transfers contact from one limit to the other.

(*f*) **Conical Pulleys.**—Since the name "cone pulley" is preempted for the commoner stepped cone, pulleys of actual cone form are called "conical." They must have small angle or long

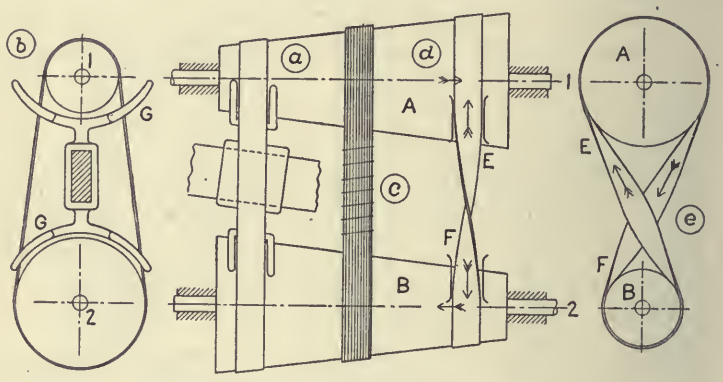

FIG. 279.—Conical pulleys with belt.

taper, the slope of 1 in 8 on radius, as shown in Fig. 279, being a maximum. Since they are usually placed at short axis distance, uniformity of tension with an open belt can be secured only by giving the pulley a slight barrel shape, or a slight swell at the middle—see Sect. 9 (*h*) and (*i*). The problem of belt guiding is troublesome, since the action analyzed in Sect. 9 (*e*) tends to cause the belt to "climb" the cone at each place of approach. Parts (*a*) and (*b*) of Fig. 279 show an elaborate guide, intended to control running in either direction.

Evidently a narrow thick belt will be better than a wide and thin one. If constructive difficulties can be overcome, a belt composed of narrow strips loosely connected by cross ties or pins, as suggested by Fig. 279 (*c*), will be ideal.

With a crossed belt, Fig. 279 (*d*) and (*e*), true or "straight" cones can be used. The direction of twist should be definitely and properly related to direction of running: here each approach-

ing "side" of the belt is (at the place of crossing) toward the big end of its cone, so that the tendency to climb will draw the "halves" of the belt apart rather than together. Note that the stretch is now at the same edge of the belt on both cones, instead of alternating as in case (*a*): this tends to make the belt a curved strip, and it is well to give such curvature, or extra stretch on one side, to a new belt. How will such a slight cuvature of the belt affect contact pressure at crossing?

An invention for securing simple belt action with conical pulleys is shown in Fig. 280, consisting of compensating bands carry-

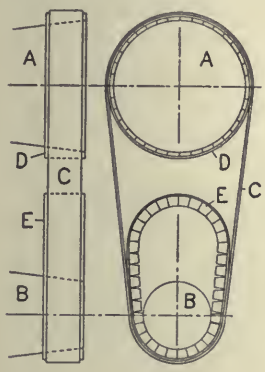

FIG. 280.—Use of compensating wedge band.

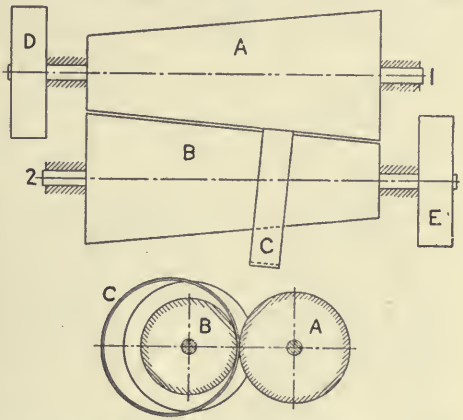

FIG. 281.—Conical pair with intermediate band, Evans type.

ing wedge segments. The secondary motion, which may become troublesome, is now that taking place as the blocks settle into and depart from their contact with the cone; and there is also a question as to the free running of the loose portion of the band, especially at high speed. There must, of course, be definite guiding, as complete as in Fig. 279 (*a*) and (*b*).

(*g*) **Cone Pair with Intermediate.**—The type of conical pair that is best for real transmission of power (as against a predominantly kinematic purpose) is presented by the sketch in Fig. 281. A short thick endless band, of leather or other suitable material, is gripped between cones that are set close together. The frictional action is that of a rolling pair, not a belt pair, but the contact is much more effective, and the transmissible force much greater, than with solid wheels. A complete countershaft unit of this type includes an easily manipulated guide and shifter

for the band; and the bearings of one cone should be adjustable, as by screws geared together, for easy regulation of pressure. As the band is made wider there is an increasing amount of secondary slip, with a peculiar twisting action upon the gripped portion of the band.

A number of designs have been proposed in which one or two rolling wheels are used between cones, functioning like the intermediate gear in Fig. 142; but such arrangements can be effective only for very light transmission.

(*h*) **Variable Pulleys.**—Returning to belt-and-pulley combinations, we consider next the idea of coincidently varying the diameters of a pair of pulleys, so as to get the effect of stepped cones

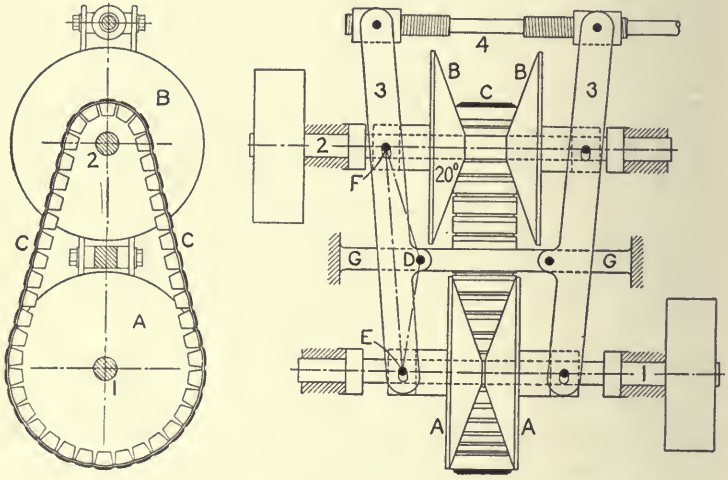

FIG. 282.—Adjustable pulleys with wedge belt, Reeves design.

without shifting the belt and without being limited to a few definite steps. One scheme is to use a wedge belt, as in Fig. 282: the pulley, instead of being a solid grooved wheel, is now composed of two cone discs which can be approached or separated, thus changing the running diameter. Figure 282 outlines a complete speed variator of this type. An important part of such a device is the rigging for control of cone movement, or for pulley adjustment. Naturally it takes the form of a pair of levers 3, 3; but a very interesting special point of this design is the placing of fulcrum D inside the center line EF. This secures that when separated (at B, B) the discs are moved a very little more rapidly

than when close together (at A, A); and the resulting change in sum of diameters is just what is needed for accommodation to constant belt length.

All devices of this general type effect change of diameter by means of a movement along the shaft axis; and the belt tension causes a reaction which must be received on thrust bearings. With the wedge belt heaviest collar pressures occur during speed change, at the cones which are being pushed together, because of the frictional resistance to squeezing outward a number of the wedge blocks. Obviously it will be better not to try to change speed under load. Here the blocks are of hard wood, fastened to a leather belt, with a thin metal plate outside of each block to serve as a broad washer for the bolts.

For light transmission the general arrangement of Fig. 282 is adapted by using a round belt with cones made of radial leaves, which intermesh so as to form grooved pulleys of variable diameter.

Another scheme is to form cylindrical pulleys with the rim in segments, which move in and out radially. Commonly the segments separate, so that the surface has gaps which widen as diameter increases; an attempt to use segments which overlap like (curved) shingles gives the surface more or less of a scalloped profile. A mechanism of cone-wedge type, along the general lines of Fig. 282, is simplest and most rugged, but is not easy to confine the segments or staves and keep them from flying outward under centrifugal force where the belt itself is not restraining them. Linkage arrangements overcome this difficulty but the necessity for a unit mechanism with each segment leads to a complex construction, with a multiplicity of small parts and poor rigidity.

Prob. 151. Look up the speed variator described in *American Machinist* for Sept. 22, 1904, Vol. 27, page 1281, and report a description of it.

REFERENCE

"Machinery's Encyclopedia," vol. VI, Variable-speed Mechanisms.

CHAPTER V

CYCLIC AND DIFFERENTIAL TRAINS

SECTION 22. CYCLIC GEARING

(*a*) **General Turning Relations.**—The subject now to be taken up is the class of gear arrangements in which one or more of the axes have themselves a motion of rotation about a fixed axis: except for the uncommon case of bevel gears, as seen in Figs. 302 to 305, these axes are and remain parallel to each other. One effect of the first step in the discussion will be to enlarge the idea of relative rotation and in some degree to relieve the concept of rotation or angular movement from dependence upon a fixed or particular axis. As heretofore, we are concerned with bodies

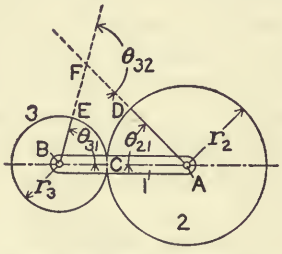

FIG. 283.—External pair.

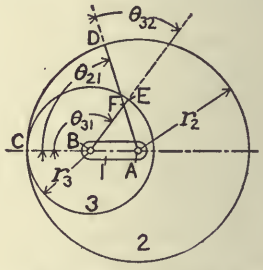

FIG. 284.—Annular pair.

Gear turning with frame bar fixed.

of plate, disc or bar form, which move in parallel planes; and this planar or coplanar motion can be represented by lines and point paths in a single plane of projection.

Consider the simple combination of frame bar 1 or AB and a pair of gears 2 and 3, as outlined (with the two types of contact) in Figs. 283 and 284. Let bar 1 be held fixed and define an initial or reference position of the mechanism by marking on discs 2 and 3 a pair of radii AC and BC, drawn to the contact point or pitch point C. Then turn wheel 2 to the right (clockwise) through an angle θ_{21},* the marked radius moving from AC to AD. This will

* Read "theta sub two-one" or "theta two-one," not "sub twenty-one."

make wheel 3 turn through the angle θ_{31}, its marked radius taking the position BE. The magnitude of θ_{31}, determined by equality of rolling arcs, is given by the relation

$$r_3\theta_{31} = r_2\theta_{21} \text{ or } \theta_{31} = \frac{r_2}{r_3}\theta_{21}. \tag{71}$$

Algebraic signs are to be used to indicate direction of rotation, plus for right-hand or clockwise, minus for left-hand turning.

The system of subscripts here introduced is intended to show relativity of motion. Thus θ_{21} stands for the angular movement of piece 2 on or relative to piece 1 and θ_{31} is the turning of 3 on 1. Similarly θ_{32} will be the angular displacement of radius BE, on piece 3, from its original coincidence with AD when both lay along AB. Written θ_{23} it would be the reversed angle of AD from BE.

FORM 13

	$\frac{r_2}{r_3}$	θ_{21}	θ_{31}	θ_{32}	θ_{23}	
Fig. 283.....	5/3	+45°	− 75°	−120°	+120°	(1)
Fig. 284.....	7/4	+72°	+126°	+ 54°	− 54°	(2)

Figures 283 and 284 are drawn with particular proportions, which can best be given the tabular statement in Form 13. From both diagrams and table it is apparent that θ_{32} is got by algebraic subtraction of θ_{21} from θ_{31}—which means addition when the directions are opposite, deduction when they are alike.

The relations among angles of turning hold equally among rates of turning or angular velocities. These are given either in angular measure (radians per second, symbol ω) or in revolutions per minute (symbol N), the latter being preferable for the present discussion of uniform velocities. The principle brought out by Figs. 283 and 284 may be stated in general terms as follows:

Let two bodies B and C, rotating in a plane, have with reference to a third body A the rates of turning N_{ba} and N_{ca} respectively. To get the rate of turning N_{bc} of B with reference to C, subtract N_{ca} from N_{ba} or the angular velocity of C from that of B. If piece A is held fixed N_{ba} and N_{ca} are absolute velocities, which might be designated simply as N_b and N_c. Then the

rule is, *To get the rate of turning of one rotating body relative to another, subtract the rotary speed of the second from that of the first.*

In Figs. 283 and 284 it is the obvious and convenient thing to draw radial reference lines on the two wheels; but the measuring of relative angle θ_{32} at the instantaneous intersection F is a first step away from dependence upon the fixed axes A and B. Further, any pair of lines drawn on planes fast to 2 and 3, originally in the same direction (coincident or parallel) then turned through θ_{21} and θ_{31}, would give the same angle θ_{32}.

Prob. 152. Draw Fig. 283 with $r_2 = 1.5$ in. and $r_3 = 1$ in.; in addition to the marked radii AC and BC draw on the two wheels vertical lines not passing through A and B. Turn wheel 2 through $\theta_{21} = -30$ deg. and besides ADFEB get a new configuration of the originally vertical lines; dimension each angle at the intersection of its lines and check the two constructions against each other.

(*b*) **Simple Cyclic Motion.**—Now take from Figs. 283 and 284 the three-body combination whose form or configuration is made

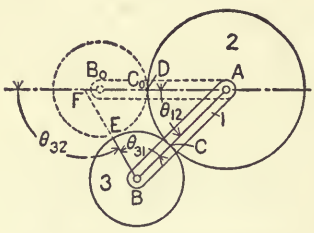

FIG. 285.—Epicyclic motion.

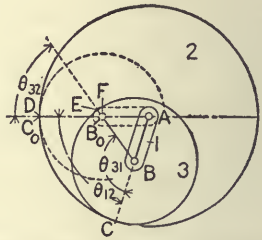

FIG. 286.—Hypocyclic motion.

definite by the (rigid) figure DABE, and turn it backward on center A through the angle θ_{21}, so that piece 2 returns to its original position, with AD on AC_0 in Figs. 285 and 286. The effect is the same as if bar AB had been turned through θ_{12} from AB_0, wheel 3 rolling on wheel 2 in **epicyclic** motion, Fig. 285, or **hypocyclic** motion, Fig. 286.

The rotations involved in cyclic or rolling motion of this kind can more conveniently be determined by making the operation just described an algebraic rather than a geometric one. It is always easy to write out the motions relative to the frame piece, when that is the member held fast, as in line 2 of Form 14; then the swing back through θ_{21} is got by subtracting that angle from all the values in the "Hold 1" line, and the "Hold 2" results check up with Figs. 285 and 286. Only movements relative to

FORM 14

	Figs. 283, 285			Figs. 284, 286			
Figs. 283, 284....	θ_1	θ_{21}	θ_{31}	θ_1	θ_{21}	θ_{31}	(1)
Hold 1.....	0°	+45°	− 75°	0°	+72°	+126°	(2)
Add $-\theta_{21}$.....	−45	−45	− 45	−72	−72	− 72	(3)
Hold 2.....	−45	0	−120	−72	0	+ 54	(4)
Figs. 285, 286....	θ_{12}	θ_2	θ_{32}	θ_{12}	θ_2	θ_{32}	(5)

the fixed member are here considered; and since the zero angle sufficiently indicates which member does not turn, there is really no present need for the second number in the double subscripts: a more general discussion may definitely require it.

(*c*) **Cyclic Trains.**—Application of this method to the finding of r.p.m. relations for the somewhat less simple train of Fig. 287 is

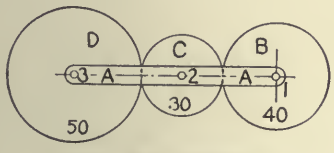

FIG. 287.—Simple.

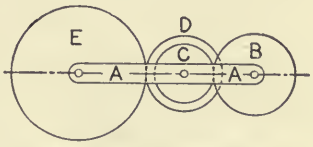

FIG. 288.—Compound.

Three-axis trains.

FORM 15

Fig. 287	N_a	N_b	N_c	N_d	
Case 1.....	0	−100	+133.3	− 80	(1)
Hold B.....	+100	0	+233.3	+ 20	(2)
Case 2.....	0	+ 75	−100	+ 60	(3)
Hold C.....	+100	+175	0	+160	(4)
Case 3.....	0	−125	+166.7	−100	(5)
Hold D.....	+100	− 25	+266.7	0	(6)

shown in Form 15. Let wheels B, C and D be held fast successively, in each case give bar A the speed of 100 r.p.m., and get the speeds of the other gears. As the problem is solved in Form

15, the first line of each case shows frame A held, a speed of −100 r.p.m. given to the wheel which is later to be held, and computed values of the other two speeds. The addition of +100 to each number in the first line brings A to rest and gives speeds for the new condition in the second line.

Instead of writing a different set of speeds for "bar held fast" in each case of Fig. 287, it is possible to pass from one case to another by a method which should be understood. In Form 16 line (1) shows the primary speeds for case 1, with −100 r.p.m. given to B. Subtract N_c, whatever it is, and get the speeds in line (2): this changes the reference piece from A to C and gives a set of relative speeds for that condition. The numerical division which brings N_a to the desired 100 r.p.m. does not change the ratios among speeds, but makes the results agree with those in line (4) of Form 15.

FORM 16

Case 2	N_a	N_b	N_c	N_d	
Hold frame.....	0	−100	+133.3	− 80	(1)
Subtract N_c.....	−133.3	−233.3	0	−213.5	(2)
Divide by 1.333.	−100	−175	0	−160	(3)

The simple intermediate C of Fig. 287 may be replaced by the compound wheel C-D of Fig. 288 with only a quantitative effect upon velocity relations. Any such three-axis train has its interesting cases when one of the end wheels B and D or B and E is held fast: holding C merely makes of it two epicyclics, with different-sized rolling wheels.

Some important special cases of simple epicyclic trains are outlined in Fig. 289. Diagram (*a*) has gears B and C equal; then while bar A makes one turn wheel C, rolling on fixed wheel B, makes two turns in the same direction. Putting in the idler D, at (*b*), reverses the turning of C on A, and now as A turns C does not rotate at all. In trains (*c*) and (*d*) gear C is half the size of B and in both cases the rotation of C is equal and opposite to that of A. These statements can be proved by the methods of Forms 14 and 15.

Figure 289 (*a*) is the simplest case of epicyclic or "sun-and-planet" motion. A common analysis of it is as follows: Let A

make +1 turn with B fixed; reversing relativity, it may be said that B is making −1 turn on A and causing C to turn +1 on A. The +1 turn of C on A added to the +1 of A on B gives +2 of C on A. This line of reasoning is helpful in fixing the concept of relative motion, but becomes exceedingly difficult to follow out clearly in the more complex trains.

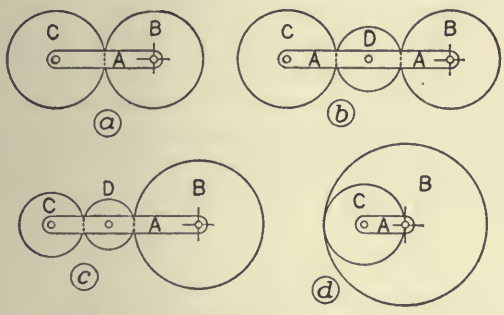

FIG. 289.—Special proportions.

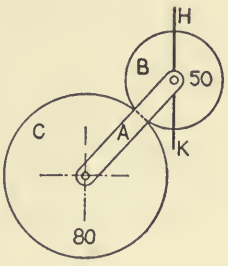

FIG. 290.—Planet wheel held.

Prob. 153. In Fig. 287 take the tooth numbers $b = 56$, $c = 35$, $d = 84$ and find speeds with 100 r.p.m. of arm A for the case of each wheel held fast. Compute as in Form 15, then work Form 16 for both N_d and N_c.

Prob. 154. In Fig. 288 take $b = 60$, $c = 42$, $d = 56$, e = 100 and get results as in the last problem.

Prob. 155. For Figs. 289 (*c*) and (*d*) prove that $N_c = -N_b$, as stated in text.

Prob. 156. In Fig. 290 line HK on gear B is kept always vertical, so that B does not have any angular movement as A revolves on axis 1. How much will C turn per revolution of A, and in what direction?

Prob. 157. What will be the result if an idler is put between B and C in Fig. 290? What if B and C are then made equal, as in Fig. 289 (*b*)? What if C is smaller than B?

(*d*) **The Reverted Train.**—Upon the train of Fig. 288 impose the special condition $b + c = d + e$, as instanced in Fig. 291; then with the resulting equality of center distances 1-2 and 2-3 double it back into the reverted form shown in Fig. 292, closely resembling the common back-gear train. Apply to the example the method of Form 15, so as to get the calculations in Form 17, where the case of C-D held is omitted. The most noteworthy showing is that when the smaller gear B (of B and E) is held the other gear E turns in the same direction with arm A; but

when the larger gear E is held the rotation of B is opposite to that of A.

FORM 17

Fig. 292	N_a	N_b	N_e	N_{cd}	
Case 1.....	0	−100	− 53.3	+133.3	(1)
Hold B.....	+100	0	+ 46.7	+233.3	(2)
Case 2.....	0	−187.5	−100	+250	(3)
Hold E.....	+100	− 87.5	0	+350	(4)

Down to this point numerical data and results have been given in all examples and problems, because they are simpler and more compact than algebraic expressions: but for the very important

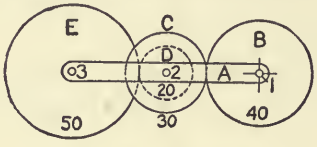

FIG. 291.

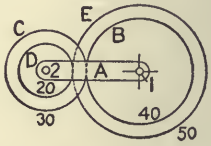

FIG. 292.

Derivation of the reverted train.

case presented by Fig. 292 the more general solution is desirable. Let *b*, *c*, *d*, and *e* stand for either diameters or tooth numbers and get Form 18 by the method of Form 17, leaving out N_{cd} and taking one turn of the driving member as basal speed or rotation. Since the back-gear train would speed down from B to E, N_e in line (2) is plus; and since it would speed up from E to B, N_b in line (4) is minus—all in accordance with the statement under Form 17 as to direct or reversed motion.

FORM 18

Fig. 292	N_a	N_b	N_e	
Case 1.....	0	−1	$-\frac{b}{c}\frac{d}{e}$	(1)
Hold B.....	+1	0	$+1-\frac{b}{c}\frac{d}{e}$	(2)
Case 2.....	0	$-\frac{c}{b}\frac{e}{d}$	−1	(3)
Hold E.....	+1	$+1-\frac{c}{b}\frac{e}{d}$	0	(4)

To get expressions useful for purposes of design, let $d = c - y$ and $e = b + y$; then substituting in the results of Form 18 and reducing, we have, for $N = 1$ in each case,

$$N_e = \frac{(b + c)y}{(b + y)c}, \quad (72) \qquad N_b = -\frac{(b + c)y}{(c - y)b}. \quad (73)$$

A numerical example will show the application of the formulas.

Example 12. Proportion a train like Fig. 292 that will make $N_e = 0.1N_a$. General relation, for $N_a = 1$,

$$N_e = \frac{(b + c)y}{(b + y)c} = \frac{1}{10}, \quad 10(b + c)y = (b + y)c.$$

Assume $y = 2$ and get $20(b + c) = bc + 2c$, $20b + 18c = bc$. Further let $b = c$, then $38b = b^2$, $b = 38$. The train will have the wheels B = 38, C = 38, D = 36, E = 40, and N_e will evaluate to exactly 0.1.

Again, with $y = 2$ try $c = b - 10$ and see how nearly $N_e = 0.1$ can be realized.

From $20b + 18c = bc$ get $20b + 18b - 180 = b^2 - 10b$, $b^2 - 48b = -180$, $b = 24 + 19.87 = 43.87$. Use $b = 44$, $c = 34$, and find that

$$N_e = \frac{78 \times 2}{46 \times 34} = \frac{39}{391} = \frac{1}{10.126}.$$

(*e*) **Trains with Internal Gears.**—These can be only of the reverted form, and typical examples are given in Figs. 293 and 294.

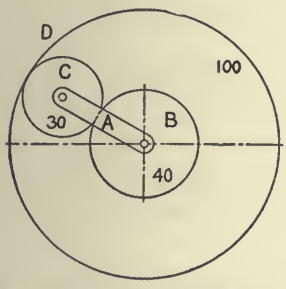

FIG. 293.—Single-annular.

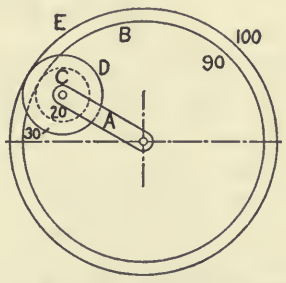

FIG. 294.—Double-annular.

Annular-gear reverted trains.

The intermediate C might be compounded in Fig. 293 but there must be a compound C-D in Fig. 294. Numerical solutions, for the particular proportions of the diagrams, are set forth in Form 19. The pure type, Fig. 294, can give much larger velocity ratios (between arm A and wheel D or wheel E) than can the mixed type in Fig. 293. Note as regards Fig. 294 that the larger gear must be held in order that the other may turn with arm A,

FORM 19

Fig. 293			Fig. 294		
N_a	N_b	N_d	N_a	N_b	N_e
0	−100	+ 40	0	−100	−135
+100	0	+140	+100	0	− 35
0	+250	−100	0	− 74.1	−100
+100	+350	0	+100	+ 25.9	0

while the smaller gear is held for reversal: this the opposite of the statement under Fig. 292. Figure 293 is always a direct drive.

Prob. 158. For Fig. 293 prove that with $N_a = 1$,

$$N_b = 2\left(1 + \frac{c}{b}\right) \quad \text{and} \quad N_d = 2\left(1 - \frac{c}{d}\right). \tag{74}$$

Prob. 159. Design two trains like Fig. 293, one with B held to make $N_a = 0.6N_d$, the other with D held to make $N_b = 4N_a$.

Prob. 160. For Fig. 294 work out a general algebraic solution as in Form 18 and show that the equations like (72) and (73) are,

$$N_e = -\frac{(b - c)y}{(b + y)c} \quad \text{and} \quad N_b = \frac{(b - c)y}{(c + y)b}. \tag{75, 76}$$

Prob. 161. Proportion two trains like Fig. 292 to make $\frac{N_e}{N_a} = \frac{1}{8}$ and $\frac{N_b}{N_a} = -\frac{1}{8}$. Try $c = b$ and $c = \frac{2}{3}b$ with $y = 3$, also $c = \frac{2}{3}b$ and $c = \frac{1}{2}b$ with $y = 4$.

Prob. 162. Design trains like Fig. 294 for the speeds $N_b = +0.01$ and $N_e = -0.01$, with $N_a = +1.00$. Try $c = \frac{1}{4}b$ and $c = \frac{1}{5}b$. Get exact solution or nearest approximation.

(*f*) **The Method of Levers.**—Using the idea of the virtual lever, from Sect. 3 (*d*), Figs. 41 to 43, together with the principle of the instant center, the matter of relative direction of turning in the reverted train can be made definite in perhaps a clearer and more convincing fashion than by algebraic signs alone. In Figs. 283 and 284 wheels 2 and 3 turn upon bar 1 at the fixed and permanent centers A and B. In Figs. 285 and 286 center B continues to be a permanent joint in the mechanism but is no longer fixed. Beside these unchanging centers (or axes), contact point C has, in the first two figures, the property that it is a point

of common velocity for the two rotating wheels; and when one wheel is held fast, Figs. 285 and 286, the other may be conceived as turning upon it, at and for the instant, with C as center of rotation. Since on each wheel center C occupies successive positions along the pitch circles, those curves (which roll upon each other without slip) are called centrodes or center paths.

The idea of the instant center has much fuller development in connection with linkage mechanisms, in Sect. 29. The explanation just given should enable the reader to use it intelligently for present purposes. The further fact that for points on a radial line (through a center of turning) the linear velocity (perpendicular to the radius) varies directly as the distance from the center is represented graphically by the inclined line NST in Fig. 295, etc.

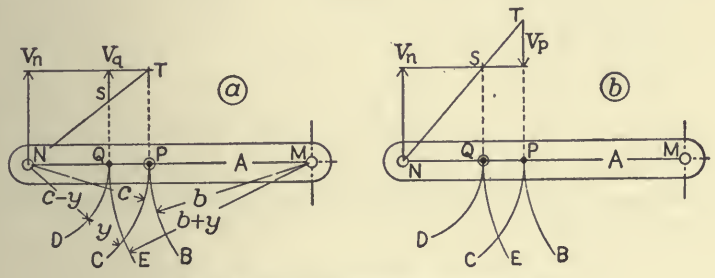

FIG. 295.—The instant-center method.

Consider now Fig. 295, which reproduces a portion of Fig. 292 for the respective case of B or E held fast, points M and N being the wheel centers. With B held, case (*a*), contact point P is the center on which, at the instant, compound wheel C-D may be conceived as turning. The instantaneously active part of this pair of wheels is the radius or bar PQN; and the velocities of points N and Q are proportional to the radii PN and PQ. Assuming the same pitch for all the gears, the same symbols may be used for radial dimensions in Fig. 295 as for tooth numbers in Eqs. (72) and (73).

Now V_n is produced by radius MN (of bar A) with the angular velocity ω_a and V_q (on wheel E) by MQ with ω_e; and since these velocities are at the same time produced by and related through PQN (turning on P), we get

$$V_q = \frac{y}{c}V_n, \quad \omega_a = \frac{V_n}{b+c}, \quad \omega_e = \frac{yV_n}{(b+y)c}.$$

Whence

$$\frac{\omega_e}{\omega_a} = \frac{N_e}{N_a} = \frac{(b + c)y}{(b + y)c}, \tag{77}$$

which agrees, of course, with Eq. (72).

Diagram (*b*) of Fig. 295 makes very clear the reversing effect of holding wheel E, or of putting the instant center Q between the active points N and P of the lever which represents rolling wheels C and D. In Fig. 296 similar diagrams are drawn for the hypocyclic train of Fig. 294, and these confirm the statements at the end of Art. (*e*) as to direct or reversed motion.

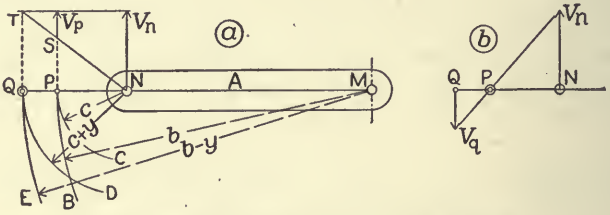

FIG. 296.—Application to annular gearing.

Prob. 163. For Fig. 295 (*b*) and for Fig. 296 as drawn and also to cover the other case of Fig. 294 (draw it) derive algebraic expressions like Eq. (77), getting results identical with Eqs. (73), (75), and (76).

SECTION 23. VARIOUS CYCLIC TRAINS

(*a*) **Compound Cyclic Trains.**—This name is appropriately given to arrangements of the type shown in Figs. 297 to 301: the essential characteristics are the presence of three active pairs of gears and the possibility of tracing two complete (although overlapping) reverted trains of the simpler type in Figs. 292 to 294. These features are self-evident in the first four examples, where train B-C-D-E drives arm A and then E-D-F-G determines the motion of wheel G: the really new matter here introduced is the use of the first of these trains in reversed direction, with drive from gear to arm. Wheel D of Figs. 299 and 300 disappears in Fig. 301 and the elementary trains are now B-C-E and E-C-F-G, and three active pairs being B-C, C-E and F-G.

In these figures the kinematic members or links of the mechanism are numbered in sequence, with 1 on the frame and gear E fast to it, 2 on arm A, 3 on the driving shaft and gear B, 4 on the planet-wheel group C-D-F or C-F, and 5 on gear G and the driven

shaft. Figures 297 and 298, likewise 299 and 300, show two arrangements of the same train, and if drawn in outline side view like Fig. 292 would be indistinguishable. The difference is an interchange of position and effect between pairs D-E and F-G.

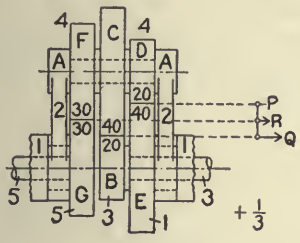

FIG. 297.

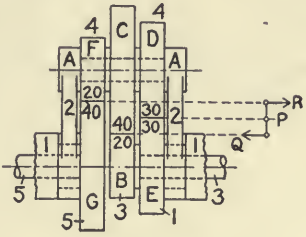

FIG. 298.

Compound trains with external gears only.

FORM 20

	N_a	N_b	N_e	N_g
Fig. 297.....	0	+100	+25	+50
	−25	+ 75	0	+25
Fig. 298.....	0	+100	+50	+25
	−50	+ 50	0	−25

The value of the train is now the ratio of N_g to N_b, which is $+\frac{1}{3}$ in Fig. 297 and $-\frac{1}{2}$ in Fig. 298, as determined in Form 20 and marked on the drawings. The manner of direct drive or of reversal is shown by the little lever diagrams at the right of each

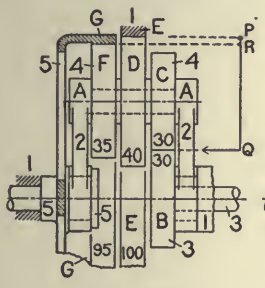

FIG. 299.

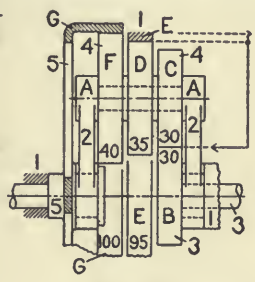

FIG. 300.

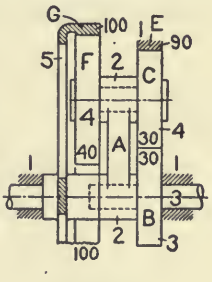

FIG. 301.

Compound trains with annular gears.

figure. The governing statement is that if standing gear E is larger than driven gear G the motion is direct, while if E is smaller than G there is reversal.

In Figs. 299 and 300 the internal gears E and G are but partially shown. The two cases are again differentiated by the interchange of dimensions in pairs D-E and F-G, with resulting direct or reversed drive. As shown by the calculated results in Form 21, the ratio of speed reduction is very great. This is apparent also from the lever diagrams, where the linear velocity derived for the pitch point belonging to F and G is so small as to be scarcely visible on the drawing.

FORM 21

	N_a	N_b	N_e	N_g	Ratio
Fig. 299	0	$+1$	$-\frac{40}{100}$	$-\frac{35}{95}$	$\frac{N_g}{N_b} = \frac{3}{95} \times \frac{10}{14}$
	$+\frac{4}{10}$	$+\frac{14}{10}$	0	$+\frac{3}{95}$	$= \frac{3}{133} = \frac{1}{44.3}$
Fig. 300	0	$+1$	$-\frac{35}{95}$	$-\frac{40}{100}$	$\frac{N_g}{N_b} = \frac{-3}{95} \times \frac{19}{26}$
	$+\frac{7}{19}$	$+\frac{26}{19}$	0	$-\frac{3}{95}$	$= \frac{-3}{130} = \frac{-1}{43.3}$

The trains of Figs. 299 and 300 are drawn in close analogy to those of Figs. 297 and 298, but for all practical purposes the simpler arrangement in Fig. 301 is sufficient and preferable. With F larger than C the turning of G is opposite to that of B, but for direct drive it is necessary only to make C the larger. The velocity relations are worked out in Form 22, and the final

FORM 22

Fig. 301	N_a	N_b	N_e	N_g	$N_g/N_b =$
Hold A...	0	$+\frac{e}{b}$	-1	$-\frac{e}{c}\frac{f}{g}$	$\frac{1-\frac{e}{c}\frac{f}{g}}{1+\frac{e}{b}}$ (78)
Hold B...	$+1$	$1+\frac{e}{b}$	0	$1-\frac{e}{c}\frac{f}{g}$	

important result is stated in Eq. (78). Compare the numerator of this velocity ratio with the expressions for N_e and N_b in Form 18.

Prob. 164. For Figs. 297 and 298 prove that the velocity relation similar to Eq. (78) is

$$\frac{N_g}{N_b} = \frac{\frac{d}{e} - \frac{f}{g}}{\frac{d}{e} - \frac{c}{b}}. \tag{79}$$

Incidentally, get an expression for N_f/N_b.

Prob. 165. Find the corresponding relation for Figs. 299 and 300, getting

$$\frac{N_g}{N_b} = \frac{\frac{d}{e} - \frac{f}{g}}{\frac{d}{e} + \frac{c}{b}}. \tag{80}$$

Show that for Fig. 301 this reduces to Eq. (78).

Prob. 166. Calculate numerical results for Fig. 301, evaluating Form 22 like Forms 20 and 21, then check principal results by Eq. (78).

Prob. 167. What ratio of reduction will be got with a train like Fig. 301, having the tooth numbers $b = 20$, $c = 51$, $e = 122$, $g = 121$?

Prob. 168. Try the effect of using a slightly distorted pitch in the last problem, making $g = 120$ but keeping all the other numbers unchanged, including f. Ratio = 1/2173.

Prob. 169. In Figs. 297 and 298 let $d = c - x$, $e = b + x$, $f = c - y$, $g = b + y$, and by substitution in Eq. (79) derive the simple relation

$$\frac{N_g}{N_b} = \frac{b(x - y)}{(b + y)x}. \tag{81}$$

Prob. 170. Elaborate the "lever" diagrams of Figs. 297 and 298 into a form like Fig. 295, then derive Eq. (81) from these diagrams after the manner of Eq. (77) and Prob. 163. This geometric proof shows, more strikingly than the disappearance of c during the algebraic deduction, how and why it is that not the whole diameters of the planet wheels but only the differences of those diameters have determinative influence.

Prob. 171. Investigating Figs. 299 and 300 by both the methods in the last two problems, prove that

$$\frac{N_g}{N_b} = \frac{b(x - y)}{(b + 2c + y)(2c + x)}. \tag{82}$$

As before, $x = d - c$, $y = f - c$.

Prob. 172. Letting $y = f - c = g - e$, prove from and for Fig. 301 that

$$\frac{N_g}{N_b} = -\frac{by}{2c(b + 2c + y)}. \tag{83}$$

Prob. 173. Design a train like Fig. 301 for speed reduction in the ratio 1,000 to 1, or for some other assigned ratio.

(*b*) **Bevel-gear Trains.**—Retracted cyclic trains, similar in effect to the spur-gear trains which contain internal wheels, can be made up from bevel gearing. Of the simple examples

first given, the one in Fig. 302 has no spur-gear equivalent as a cyclic, but when used as a differential gearing for automobiles (the most frequent application) it has the special equivalent shown in Fig. 331. If either B or D is held fast the other turns at twice the speed of arm A; and this similarity of motion for the two wheels is what cannot be got with the analogous spur-gear train in Fig. 293.

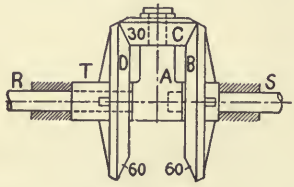

FIG. 302.—The "differential" bevel-gear train.

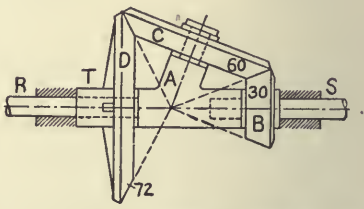

FIG. 303.—Simple bevel-gear cyclic train.

The more general form of this three-wheel combination shown in Fig. 303 is, however, very much like Fig. 293 in effect; and the velocity relations derived in Form 23 are just like those deter-

FORM 23

Fig. 303	N_a	N_b	N_c	N_d	
Case 1.....	0	-1	$\frac{b}{c}$	$+\frac{b}{d}$	(1)
Hold B.....	$+1$	0	$\frac{b}{c}$	$+\frac{b+d}{d}$	(2)
Case 2.....	0	$+\frac{d}{b}$	$\frac{d}{c}$	-1	(3)
Hold D.....	$+1$	$+\frac{b+d}{b}$	$\frac{d}{c}$	0	(4)

mined numerically in Form 19. Note in both figures that arm A is on shaft R and wheel B on shaft S, while wheel D has an extended hub T which can be held fast or from which motion can be taken. Since wheel C is not coplanar with the others, the device of giving the whole train an added rotation about axis RS cannot, by simple algebraic subtraction, determine the absolute motion of C. Fortunately this motion, a composition of rotations about two axes that intersect, is of no quantitative

interest from the present viewpoint of transmissive effect: and the mere turning on its own axis remains unchanged.

Figure 304 shows a train which corresponds with Fig. 301 in form and in lettering and is exactly represented by Form 22 and Eq. (78): E is held and slow motion is given to G. Proportions for forward and reversed drive are distinguished in

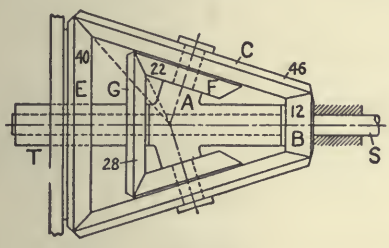

FIG. 304.

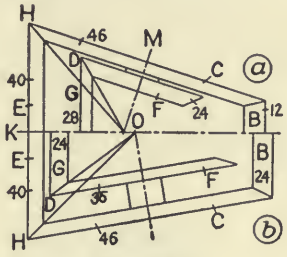

FIG. 305.

The Humpage speed-reducing train.

Fig. 305. As appears from Eq. (78), the question is whether $\frac{e}{c}\frac{f}{g}$ is less or greater than 1. Changing to $\frac{e}{c} \div \frac{g}{f}$, we see that in case (*a*) the divisor $\frac{g}{f}$ is greater, in case (*b*) it is less, than $\frac{e}{c}$. The method of levers can also be extended so as to cover this train: cone E being fixed, the instant axis on which double cone C-F turns as it rolls is the tangency line OH; and the radial plane through OH and OM corresponds with the virtual lever for spur gears. In case (*a*) line OD, between OH and OM, moves in the same direction as the latter or as gear B, and carries gear G with it. In case (*b*) line OD is on the other side of instant axis OH and therefore receives a reversed motion.

(*c*) **Simple Cyclic Transmissions.**—Among the five examples in Figs. 306 to 310 the clutching and holding devices claim a larger degree of ingenuity and of interest than do the simple gear trains. In Fig. 306 sliding piece 5 serves either to lock arm A to frame 1 by jaw clutch J or to make it fast to cone pulley 3 by pin K. The first of these engagements, with A held from turning, gives the effect of an ordinary back gearing (compact in form); the second locks the gear train, so that cone pulley and shaft turn together and the train is cyclic in appearance only, not in action. When clutch 6 of Fig. 307 is slid to the right it holds sleeve 5 and gear B from turning and there is true cyclic

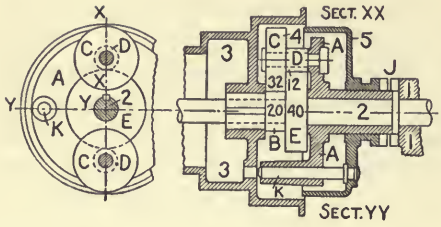

FIG. 306.

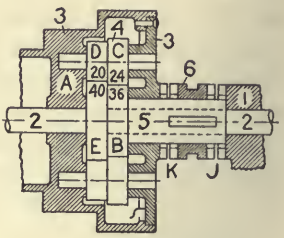

FIG. 307.

Cyclic back gears.

drive of shaft 2 at low speed. With K engaged B is fast to pulley 3, the train is locked, and shaft 2 turns at full speed.

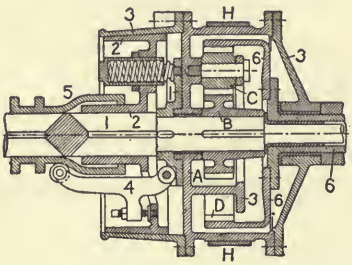

FIG. 308.—Thornycroft.

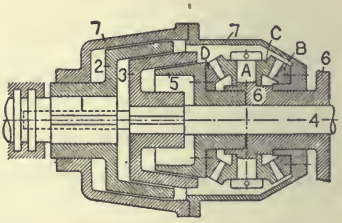

FIG. 309.—Hesse.

Reversing motor-boat drives.

The two reversing gears for motor boats in Figs. 308 and 309 are both false cyclics like Fig. 306; that is, there is driving motion through their gear trains only when arm A is held from turning. When clutch 2 in Fig. 308 is engaged all the turning parts are locked together and the drive is direct from engine shaft 1 to propeller shaft 6. This clutch, as commonly in motor cars, is held in engagement by springs, levers 4 and shifting "cone" 5 being used only to release it. A single lever first shifts 5 to the left, then (passing neutral) clamps brake band H to hold casing 3, a part of which serves as arm A; and now there is slower reversed drive through B-C-D. Three gears C are used, to give a better balance and distribution of load; but as in any such multiple arrangement, very accurate construction work is needed to make them take equal loads and run quietly.

The bevel gearing of Fig. 309 gives the same speed of propeller in either direction of running. Shaft 4 and the double cone 3 upon it are shifted endwise by a lever in order to change

drive, but the real gripping force is supplied by the push or pull of the propeller on shaft 4. Arm A on sleeve 6 is always held from turning, but can slide endwise enough to let casing 7 grip cone 2. For direct drive cone 3 is pushed into 2 and all the rest of the mechanism is free to move or stand as it will. For reversal shaft 4 and with it sleeve 6 are slid to the right by the control lever, and are then pulled in the same direction by the propeller: cone 2 grips 7 and cone 5 grips 3, clutching gear B to shaft 1 and gear D to shaft 4, so that motion is transmitted through the reversing gear train. A determining purpose in this design was to leave the engine entirely free, with no gears running to make a noise, when the lever is at neutral. How does Fig. 308 function in that respect?

(*d*) **A Cyclic Countershaft.**—A scheme which had some vogue just before the wide development of all-geared machine

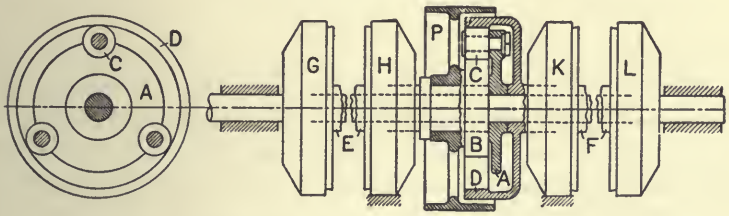

FIG. 310.—A cyclic countershaft.

tools was to put more or less of quick-change gearing on the countershaft. A good instance of the use of cyclic gearing in this service is illustrated by Fig. 310, where two speeds and reversal are got from one belt. The condensed description is as follows:

The driving belt is on pulley P, to which gear B is fast.

Sleeve E carries arm A and sleeve F carries gear D.

The several clutch engagements are, G locks E to shaft, H holds E and K holds F from turning, L locks F to shaft.

The three working conditions are,

Engage G and L, lock A and D to shaft, drive directly at full speed.

Engage G and K, hold gear D, and drive with cyclic action at reduced speed.

Engage H and L, hold arm A, and get reversal at reduced speed.

With four clutches there are six possible combinations of two

at a time, of which three are used. The other three, G-H, H-K and K-L, must be prevented by some interlocking device in the control system: what effect would each have?

(*e*) **Cyclic Hoists.**—Figure 311 shows the cyclic train of a much used form of hoist block. The hand-chain wheel is fast on shaft

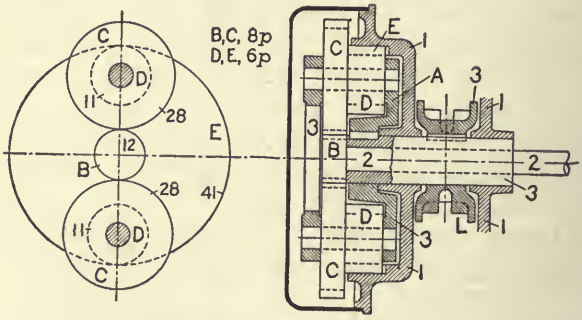

FIG. 311.—Triplex hoist block.

2 and with it there is a holding brake which will be shown when that subject is taken up, in Part II. Through the train from pinion B on shaft 2 to annular wheel E on the frame of the machine a slow driven motion is given to arm A and lifting wheel L, which are parts of member 3.

In the hoist of Fig. 312 driving shaft 2, on which is the hand-chain wheel, carries an eccentric with center E and radius arm OE. Spur gear 3 has a bearing on this eccentric and moves with it, but is kept from turning by crossed slide 4, on which it bears at lugs L, L. As shaft 2 rotates it rolls wheel 3 inside of 5 and gives to the latter a slow turning motion: the lifting-chain wheel is fast to gear 5.

This may be considered a cyclic train, with eccentric radius OE as arm A. Conditions are as in Fig. 290, angular-motion relations will not be affected by holding center O and letting E swing around it, and solution will be by the method of Form 14. Thus if gear 3 has 38 teeth and gear 5 has 40 teeth it can easily be shown that the turning ratio is 20 of the hand wheel to one of the lifting wheel.

Another simple analysis, which would place this mechanism with a class of differential gears discussed in the next section, Figs. 327 and 328, is reasoned thus: In a complete revolution of shaft 2 the 38 teeth of wheel 3 are matched with 38 of the teeth

of wheel 5, and the remaining 2 teeth (or their combined pitch) measure the amount by which wheel 5 has advanced or turned. At the rate of 2 in 40 per revolution, it will take 20 turns of shaft 2 to make wheel 5 turn once.

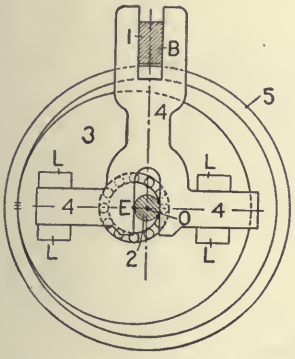

FIG. 312.—Eads hoist block.

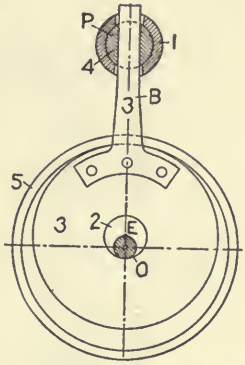

FIG. 313.—Bentall reducing motion.

The modification of this mechanism outlined in Fig. 313 can be used as a speed-reducing gear between a quick-running prime mover (engine or motor) and a slow-moving work machine such as a reciprocating pump: it differs from Fig. 312 only in the mechanically simpler but kinematically more complex way of holding wheel 3 from turning. With bar B bolted to the wheel and passing through guide block P which has its bearing in the frame, the whole member 3 receives a slight oscillation. Upon the slow progressive motion of wheel 5 this oscillation will have a relatively large effect, but one in no wise harmful for such service as driving a pump.

Going back to the title of this article, it may be remarked that the vertical-axis windlasses called capstans, used on board ship, very often have cyclic gear drive.

(*f*) **A Cyclic Gear Box.**—The motor-car transmission in Fig. 314 represents the class of cyclic trains in which greatest complexity is found. Its parts and their relations are as follows:

Shaft A, coming from the engine, drives through pinion a. Shaft B, going to the rear axle, is driven through pinion d.

Frame C, free to turn except when held by hand brake (or clutch) J, carries two compound planet gears, namely, b-c on one side and f-c-h on the other.

Frame D, similarly controlled by band K, projects inward as a sleeve to which gear g is fast. Frame E, which carries gear k in the same manner, can be held by band L or can be clutched to frame C by means of the coil clutch F, operated by lever G and cone H.

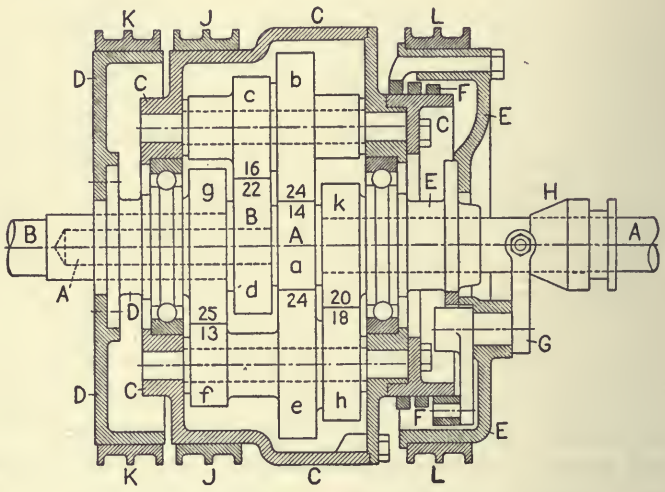

FIG. 314.—A cyclic gear box, three speeds and reverse.

In this one mechanism are combined the three trains of Figs. 292, 297 and 298, as will appear in the following description of its action. The four conditions of running are:

1. Engage F, lock whole train together, drive B at full speed of A.

2. Hold C, make active the simple reverted train a-b-c-d (like Fig. 292), and get middle speed for B.

3. Hold D, make active the compound train a-e-f-g plus a-b-c-d (like Fig. 297), and get low speed.

4. Hold E, make active train a-e-h-k plus a-b-c-d (like Fig. 298), and get reversal at low speed.

Numerical speed relations, for the tooth numbers on the figure, are worked out in Form 24. Line (1) gives general expressions for the case of frame C held and line (2) the corresponding numerical values. Line (4) for D held and line (6) for E held are got from line (2) by the usual method of subtracting the speed of D and of E. In lines (3), (5) and (7) the method of Form 16 is used to get all the speeds in terms of 1,000 r.p.m. of driving shaft A. The

most important result is the corresponding speed of B, which for the whole series has the four values 1,000, 424, 174 and −212. The speeds of the members that run free (or intermediate) are also of considerable interest.

FORM 24

Fig. 314	A	B	C.	D	E	
General.....	+1	$+\frac{a}{b}\frac{c}{d}$	0	$+\frac{a}{e}\frac{f}{g}$	$+\frac{a}{e}\frac{h}{k}$	(1)
Hold C.....	+1.0000	+0.4243	0	+0.3033	+0.5250	(2)
Middle speed.	+1,000	**+424**	0	+303	+525	(3)
Hold D.....	+0.6967	+0.1209	−0.3033	0	+0.2217	(4)
Low speed....	+1,000	**+174**	−435	0	+318	(5)
Hold E.....	+0.4750	−0.1008	−0.5250	−0.2217	0	(6)
Reverse.....	+1,000	**−212**	−1,105	−467	0	(7)

(*g*) **A Belt-and-pulley Cyclic.**—Figure 315 is given in order to show the possibility of arranging a retracted cyclic train with belt gearing as well as with toothed gears. The primary problem of holding wheel B or E and finding the relative speeds of arm A

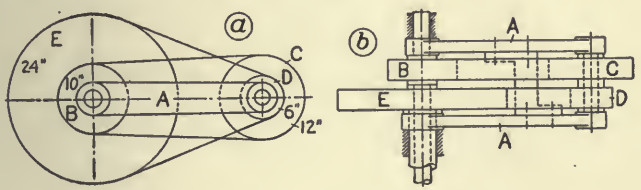

FIG. 315.—A belt-and-pulley cyclic train.

and wheel E or B is solved in the regular manner, as shown by Form 25.

This example will serve to introduce a new type of problem, in which definite driving motions (presumably independent) are given to two members of a cyclic train. Such problems might be grouped with one class of the differential actions discussed in the next section; but since they are solved by the methods of the present discussion it is more convenient to consider them here.

(*h*) **Aggregate Motion.**—Suppose that with Fig. 315 there are two given speeds, say +20 r.p.m. for arm A and −20 r.p.m. for pulley B. To solve, hold A and give to B the combined speed $N_b - N_a$; then add N_a and get N_e as in Form 26.

FORM 25

N_a	N_b	N_e
0	+1	$+\frac{5}{24}$
−1	0	$-\frac{19}{24}$

FORM 26

N_a	N_b	N_e
0	−40	− 8.33
+20	−20	+11.67

Again, let the problem be to find how arm A will turn when certain chosen, independent speeds are given to the two pulleys B and E on the fixed axis. For example, let $N_b = +100$ and $N_e = +25$, or $N_b = 4N_e$. Write line (1) of Form 27 in the usual way. Add the unknown speed x which arm A will have. The value of x is determined by the requirement

$$x + 1 = 4(x + 5/24), \tag{84}$$

whence $x = 1/18$. Introduced into line (3) this gives proportional speeds, showing that A turns one-nineteenth as fast as B. In line (4) the speeds are brought to the given r.p.m.'s.

FORM 27

	N_a	N_b	N_e	
Hold A.....	0	+1	$+\frac{5}{24}$	(1)
Add speed x:.....	x	$x + 1$	$x + \frac{5}{24}$	(2)
$x = \frac{1}{18}$	$+\frac{1}{18}$	$+\frac{19}{18}$	$+\frac{19}{72}$	(3)
Speeds.....	+5.26	+100	+25	(4)

Prob. 174. Calculate the speed ratio for the two bevel-gear trains represented by Fig. 305.

Prob. 175. For 100 r.p.m. of the belt cone find the reduced speed of shaft 2 in Fig. 306 and in Fig. 307.

Prob. 176. Discuss the gear arrangement of Fig. 310 as follows: Assume $b/d = 0.2$, 0.3, etc., to 0.7, and for each ratio find the slow forward speed and the backward speed, expressed in terms of pulley speed as unity. Find also the ratio of reversed to slow forward speed.

Prob. 177. If the diameter of the hand-chain wheel is 3.15 times that of the lifting-chain wheel in Fig. 311, what is the velocity ratio of hand pull to load?

Prob. 178. Apply to Fig. 314 the relation expressed by Eq. (81), deriving the low and reversed speeds already found in Form 24. Gear a, having 14 teeth, is b in the formula. Note that kinematically gear c might equally well be on axis f-e-h, and relations will be a little more clearly apparent if it is so taken.

Prob. 179. Starting with $a = 16$, proportion a train like Fig. 314 to give, as nearly as possible, the speed series 1, 0.4, 0.16 and -0.16.

Work for 0.16 first, then for -0.16, after which the design of a plain back gearing for 0.4 will fix the whole diameters of the planet wheels. All the ratios can be got exactly except -0.16.

Prob. 180. In a problem like the last, two cases: (1) $a = 15$, speed series 1, 0.5, 0.3, -0.3; (2) $a = 14$, speed series 1, 0.45, 0.2, -0.25.

Prob. 181. (*a*) In Fig. 292 let $N_b = +100$ r.p.m. and find N_e when $N_a = +25$ and -25. (*b*) Find N_e for a series of values of N_a from $+100$ to -100 and plot a curve of N_e on N_a as base.

Prob. 182. Discuss Fig. 293 in the same fashion, taking the data, (*a*) $N_b = +100$, $N_a = +15$ and -15; (*b*) N_a from $+60$ to -60, get N_d.

Prob. 183. For Fig. 294 get an expression giving N_e in terms of $N_b = 100$ and of N_a, then calculate values for N_a varying from $+200$ to -200 by 40 at a time.

Prob. 184. Apply the method of Eq. (84) to Fig. 292, finding N_a for $N_b = +100$ and $N_e = +80, +40, +20, -20$ and -40. First $N_a = 57.14$.

Prob. 185. For the retracted train like Fig. 292 let t be the value of the train, from B to E, when A is fixed, equal to $+8/15$ in that figure; and in the condition covered by Form 27 and Eq. (84) let n be the ratio N_e/N_b between the given speeds of the two wheels. Prove that the accompanying motion of arm A is given by the expression

$$\frac{N_a}{N_b} = \frac{n - t}{1 - t}, \tag{85}$$

then try out this formula against the results in the last problem.

Prob. 186. Get an expression like Eq. (85) for the train of Fig. 293 or of Fig. 294, then check it against a few calculations made by the extended method of Prob. 184.

SECTION 24. DIFFERENTIAL MECHANISMS

The common feature of the devices gathered into this section is that in some way or other two component or contributing motions, variously originated and transmitted, enter into the production of a final effect. The examples are grouped according to the way in which this idea works itself out: and each group will be introduced by a brief statement of the principle of combined action that is illustrated.

I. Two simple motions of like kind are derived from the same source within the mechanism and very directly substracted or added.

(*a*) **Differential Cord Motion.**—The two simple hoisting machines in Figs. 316 and 317 have their load pulley C suspended in a loop or "bight" of rope or chain, which is shortened by

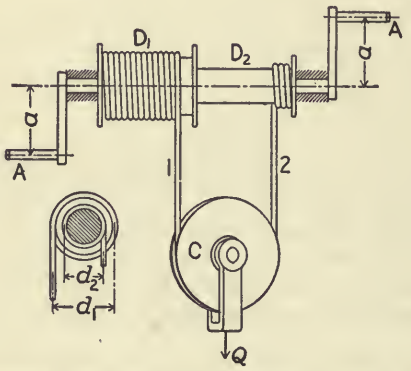

FIG. 316.—The differential windlass.

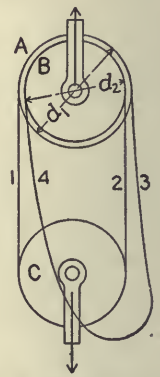

FIG. 317.—Western differential hoist.

winding up side 1 but at the same time paying out side 2 less rapidly. The plain hand windlass cannot have a high velocity (or load-to-force) ratio because the drum may not be so small as to bend the rope too sharply. The differential windlass gets over this difficulty, but is limited in height of lift because so much rope must be handled. The hoist in Fig. 317, made only in moderate lifting capacities (say up to 3,000 lb.), has a single endless chain, of which one bight carries the load while the other serves as hand chain, the pull being applied to side 3 for lifting. The difference between d_1 and d_2 is so small that the friction of the machine sustains the load, and a light pull on side 4 is needed for lowering. Wheels A and B are, of course, compounded or in one piece.

Using r_v for the input-to-output velocity ratio, as in Sect. 8 (*c*), its values are,

Fig. 316, $$r_v = \frac{2\pi a \times 2}{\pi(d_1 - d_2)} = \frac{4a}{d_1 - d_2}. \tag{86}$$

Fig. 317, $$r_v = \frac{2\pi d_1}{\pi(d_1 - d_2)} = \frac{2d_1}{d_1 - d_2}. \tag{87}$$

The factor 2, really a divisor into the difference $(d_1 - d_2)$, takes account of the double suspension of pulley C.

(*b*) **The Differential Screw.**—This name describes exactly the arrangement typically represented by Fig. 318 (*a*), but may be generalized to cover also the right-and-left screw at (*b*). Letting p stand for the pitch of an equivalent single screw, or for the distance by which the two nuts approach or separate from each other for one turn of the screw, the statement

$$p = p_1 - p_2 \tag{88}$$

governs in both cases, remembering that in the second p_2 is negative with respect to p_1.

An interesting application of the differential screw is seen in Fig. 319, which shows a special form of the type of chuck used on

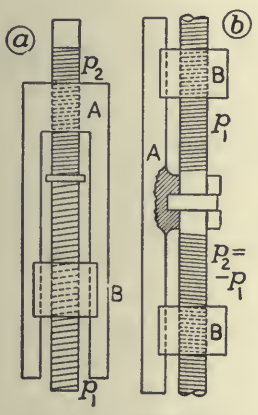

FIG. 318.—Elementary differential screw.

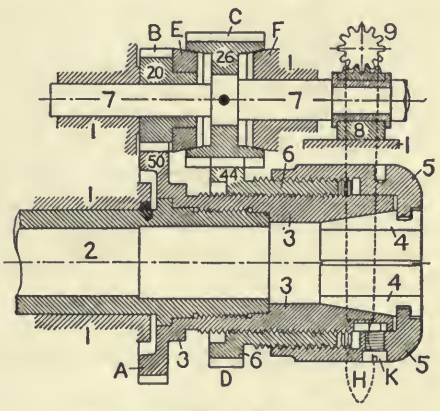

FIG. 319.—Herbert bar chuck with differential screw.

turret lathes for holding a "stock" bar that comes through the hollow spindle. This spindle 2 has its bearing in frame 1 and upon it is screwed fast the chuck body 3, in the front end of which are the cone-wedge jaws 4 (three pieces). Nose piece 5, compelled to turn with 3 by key pin K, moves the jaws in and out, being itself moved endwise on 3 by the differential-screw sleeve 6; this has both threads right-hand, but of different pitch, the outer one being the coarser. Screwing 6 to the right on 3 will draw in or close the jaws, screwing it to the left will open them. The operating mechanism consists of gear A formed on piece 3 and

turning with the spindle, gear D on piece 6, the auxiliary or side gears B and C, friction cones E and F, and the control handle H.

The lathe spindle turns in the usual direction, anti-clockwise when viewed from the right. If handle H is swung to the right cone F will hold gears C and D from turning; then 3 screws into 6, drawing it to the left, while 5 screws off it more rapidly and opens the jaws. If cone E is engaged the back-gear train A-B-C-D makes 6 turn faster than 3 and 5, screwing it toward the right and slowly drawing 5 to the left on 3. This chuck has a larger range of accommodation to irregularity in diameter of stock than is provided in the simpler jaw-clamping devices that are generally used.

Prob. 187. In a windlass like Fig. 316 let the diameters be 8 in. and 12 in. for the drums and 1 in. for the rope, with a 16-in. crank arm. What is the velocity ratio r_V? With the same crank and this velocity ratio what would be the necessary diameter of drum for an equivalent simple windlass? Remember that diameters must be taken to the middle of the rope.

If this differential windlass is to have a lift capacity of 20 ft., how long must the drums be in order to avoid overwinding, or the winding of the rope in a second layer? How much would the velocity ratio be affected by the overwinding of both ropes? Of either one?

Prob. 188. If wheel A of Fig. 317 has 20 link pockets in its periphery to 19 on wheel B, what is the velocity ratio of hand pull to load?

Prob. 189. In Fig. 319 let screw pair 3-6 have 5 threads and pair 6-5 have 6 threads per inch, the tooth numbers be $a = 50$, $b = 20$, $c = 26$, $d = 44$, and the taper of the clutch jaws be 1 in 3 on diameter. By what amount will the jaws be opened and closed per turn of spindle.

II. In a screw pair both screw and nut are turned, in the same direction but at different rates, so as to get a slow advance.

(*c*) **Differential Motion of Screw and Nut.**—Figures 320 to 323 show variations of the scheme whereby a strong screw thread of coarse pitch is made to give the motion effect of a very fine screw. The old-fashioned screw jack in Fig. 320 has both screw S and nut N turned by bevel gearing from the crank shaft. Wheels B and C are equal, wheels D and E are as 3 to 4, and sleeve G is splined to the screw. For quick motion handle K is swung upward, rotating the eccentric bushing H and bringing D into mesh with F; friction is then expected to hold nut N from turning.

The small drill of which the mechanism is shown in Fig. 321 is driven by a hand crank on shaft 2, through the miter gears 3 and 4. The feed train consists of gear A splined to spindle S, gears B and C fast on side shaft 5, and gear D on sleeve nut N.

With the tooth numbers given the nut makes 95 turns to 96 of the screw, so that the feed is very slow. For quick raising and lowering of the spindle, shaft 4 is pulled upward, by a little knob

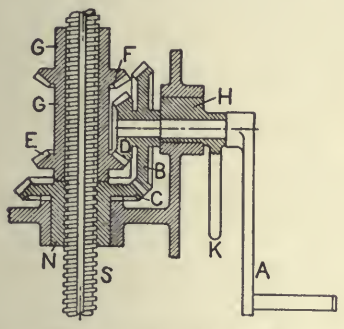

FIG. 320.—A differential screw jack.

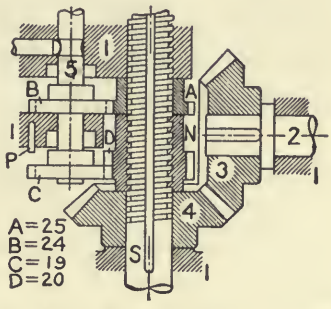

FIG. 321.—Feed gearing for a hand-power drill.

at its top end, withdrawing B from A and catching C on pin P, which slips between the teeth of C and thus holds nut N from turning.

(*d*) **Differential Boring-bar Feed.**—In some large boring machines the feed motion is got, not by slowly advancing the whole boring bar endwise, but by moving on the bar a tool head represented by ring 6 in Fig. 322. Of the differential-screw arrangements that have been used for this purpose typical exam-

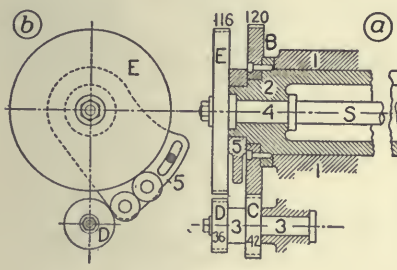

FIG. 322.—Frame arm held.

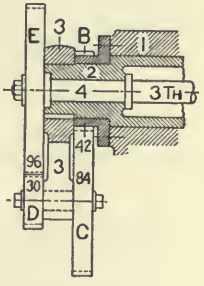

FIG. 323.—Arm turns.

Differential feed gearing for boring bar.

ples are shown by Figs. 322 and 323. In the first, compound wheel C-D is on a fixed axis and the gear train is so proportioned that gear E (fast to the feed screw) has a speed only a little different from that of bar 2 on which gear B is fast. The plain retracted train is here modified by putting two idlers into pair D-E,

carrying them on radius arm 5: then by changing only small wheel D the rate of feed may be varied.

In Fig. 323 there is a regular cyclic train, arm 3 being keyed to boring bar 2. The train must be so proportioned that if run as a back gearing it would give a big reduction of speed, for from line (2) of Form 28 it appears that the value of this train, $t = \frac{bd}{ce}$, is the difference between N_3 and N_e that turns the screw in its nut.

FORM 28

Fig. 323	N_3	N_b	N_e	
Hold 3.....	0	−1	$-\frac{b}{c}\frac{d}{e}$	(1)
Hold B.....	+1	0	$+1-\frac{b}{c}\frac{d}{e}$	(2)

In Fig. 324 the same gear train is used, but now wheels C-D turn on a fixed axis, nut N has a bearing in the frame of the machine, and screw 3 is splined in the frame to keep it from turning: in fact, line (1) of Form 28 is exactly embodied. Since the screw does not now turn we may no longer call this a differ-

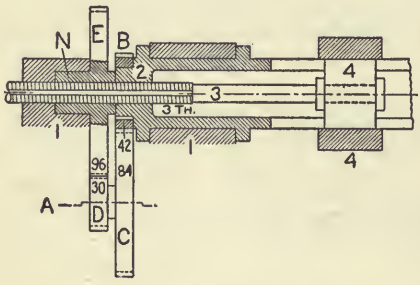

FIG. 324.—Inversion of Fig. 323, with screw held.

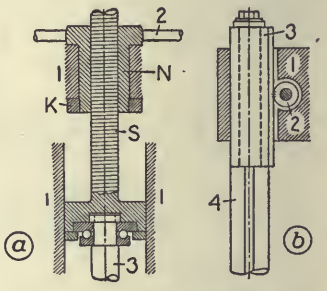

FIG. 325.—Methods of feeding drill spindle.

ential device; but the change from Fig. 323 to Fig. 324 is a good example of the *inversion of a mechanism*, or change in the member held fast.

The screw arrangement in Fig. 324 is essentially equivalent to the screw feed sometimes used on drilling-machine spindles. The example sketched in Fig. 325 (*a*) has screw S held from turning by a square head between guides 1, 1. Nut N in the

sleeve hub of gear 2 is slowly turned by a feed train a good deal more extensive and more flexible than the one in Fig. 324. The commoner rack-and-pinion spindle feed is outlined in Fig. 325 (*b*). Sleeve 3, with a rack cut in or fastened to it, does not turn; spindle 4, reduced in diameter, runs through 3 and turns within it. Occasionally a worm is used instead of a pinion to move the rack, its axis making an acute angle with the axis of the spindle.

Prob. 190. With two threads per inch on the screw of Fig. 320, what is the lift per turn of crank? If the crank arm is 15 in. long, what is the velocity ratio of hand force to load?

Prob. 191. In Fig. 321 the screw has 6 threads per inch. Show that the rate of feed is 576 turns per inch of penetration, first proving the statement in the text as to relative turning.

Prob. 192. For Fig. 322 find the rate of feed, in turns of bar per inch of tool traverse, with the dimensions on the diagram. Then make a table of feeds for a series of change wheels D having 30, 32, 34, 36, 38 and 40 teeth.

If the bar is turning like a lathe spindle, clockwise in view (*b*), what will be the direction of feed (to right or left) with the right-hand screw drawn? How could reversal of feed be secured, keeping the same screw?

Prob. 193. Find the rate of feed per turn of boring bar in Fig. 323 and in Fig. 324. What is the direction of feed in Fig. 324 and which "hand" screw must be used with Fig. 323 to get the same direction?

III. By means of two gear wheels and a rolling pinion, in what is essentially a special form of the cyclic gearing in Fig. 292 or Fig. 294, get a differential motion of what may be called the vernier type.

(*e*) **The Planet Differential.**—Consider first Fig. 326, where A and B are supposed to be bars carrying uniform scales in the form of V notches, but with 31 divisions of B equal to 30 of A. Use a cross bar M to match any pair of notches, as at No. 0 in sketch

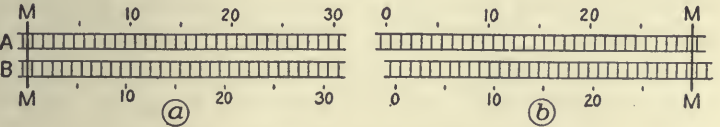

FIG. 326.—Differential motion of vernier type.

(*a*). Move M forward one notch at a time, at each transfer pushing B forward just enough to secure a true matching. When No. 30 is reached, sketch (*b*), bar B has one notch left over from the matching process, or has been pushed ahead by the length of one of its own divisions.

With toothed wheels instead of straight bars this scheme is made practical: pinion C of double width, in either Fig. 327 or Fig. 328, takes the function of matching-bar M, and as it rolls around gives to wheels B a steady slow advance, A being held fixed. Smoothness of action results from the fact that gear B,

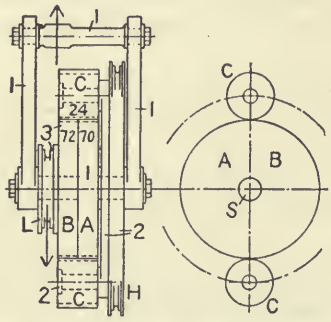

FIG. 327.—Plant differential applied in hoist

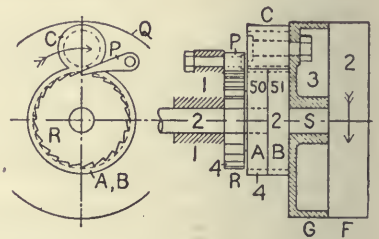

FIG. 328.—Two-speed drive for cam shaft of screw machine.

apparently of the same diameter as A, really has a larger pitch circle and a shorter addendum: see Sect. 38 (*s*) for the gear-tooth problem involved.

The hoist in Fig. 327 has wheel A fast to its frame, and with a tooth difference of 2 it can use two rolling wheels and thus get a better balancing of forces. The ratio of turning of H to L is $r_n = 36$. A long-used device for slowly turning the cam shaft or drum of an automatic lathe or "screw machine" is sketched in Fig. 328. Pulley F and gear B are keyed to shaft S, on which is a worm to drive the cam shaft. Pulley G, carrying pinion C, is loose on S. When the belt is on G gear A is held fast (by pawl P on ratchet R) as an abutment against which the teeth of C push while slowly driving B forward. At the end of a cut the belt is shifted to pulley F and the whole train (including gear A and its ratchet) turns with shaft S at full speed, to make the quick change between cuts.

The mechanism shown in Fig. 329 is a train with a very large ratio of speed reduction, used to give the slow continuous motion required by a chain-grate stoker. The differential unit, repeated in gear groups A-B-C and D-E-F, varies from that of the last two examples in having internal gears. Driving pulley P is keyed to sleeve S on which is formed an eccentric K_1; as it turns the latter rolls gear C around inside of fixed gear A and

driven gear B, turning this in the ratio of 1 to 51. Gear B is combined with eccentric K_2 to form member 4, and by its rotation gear E, keyed to shaft 6, is driven in the ratio of 1 to 71. The whole speed ratio is then $r_n = 51 \times 71 = 3{,}621$.

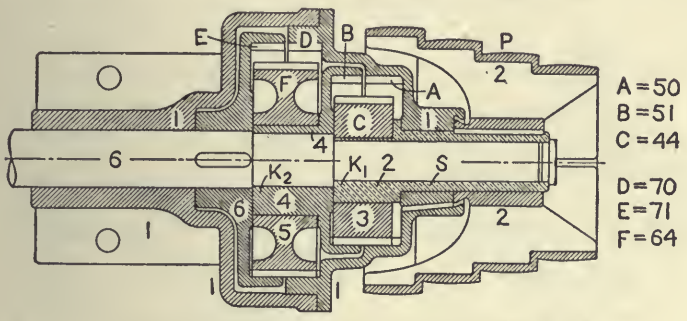

Fig. 329.—Differential drive for Coxe chain grate.

In all the examples thus far presented the differential mechanism has been, as regards external relations, just like other speed-changing trains: there is one input speed and one output speed, any complexity in the relation between them being wholly internal to the mechanism. It is possible, however, to have more than one partial or component motion at either end of the train, and we now take up some devices in which this idea appears.

IV. The output or delivered speed may be divided between two driven members in a proportion not determined by the driving train.

(*f*) **Differential Axle Drives.**—If any vehicle having two wheels on an axle is to roll freely on a curved path, it is necessary that these two wheels be free to turn at slightly different simultaneous rates. In common railway vehicles this condition is not met, with the consequent requirement of a strong guiding action by rail on flange, to overcome the friction that tends to hold the locomotive or car truck to a straight path. In self-propelled road vehicles the desired freedom within the main motion of progression is got by means of the devices in Figs. 330 and 331. The bevel-gear box train of Fig. 302 has been most used. In Fig. 133 we see at N the location of gear A of Fig. 330, which is driven by a bevel pinion on the transmission shaft from the gear-change box. Gear A is fast on the revolving casing B, in

which axle C for planet wheels D, D is secured. On a straight track there is no motion within this combination of gears, wheels E and G with their half axles F and H turning exactly together. On a curve the outer wheel turns a little faster, the inner just as much slower, than driver A.

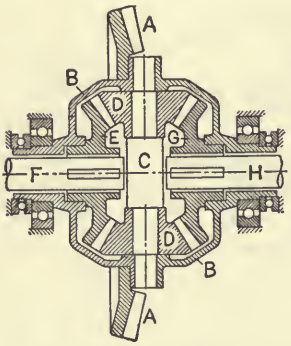

FIG. 330.—Bevel-gear type.

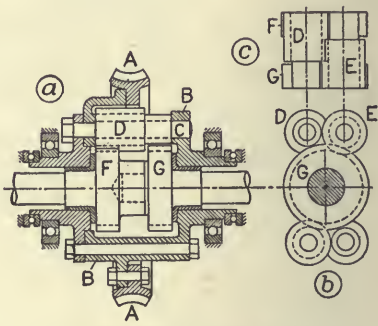

FIG. 331.—Spur-gear type.

Automobile axle differentials.

The spur-gear differential in Fig. 331 is an alternate to Fig. 330, equivalent in effect. Instead of one bevel pinion D there is now a pair of overlapping long spur pinions D and E, which mesh with each other between F and G, as best shown by the partial top view (*c*). This arrangement is more complex and costly than the bevel-gear box, but it eliminates end thrust on wheels E and G (of Fig. 330) and the incorrect toth action that will result from maladjustment of the end or side bearings that hold the bevel gears in place.

Since movements within the differential train are small and slow, there is no occasion at all for ball bearings—a fact which greatly favors compactness and rigidity in this unit.

From another viewpoint—that of force action—these might well be called equalizing devices, because their function is to deliver equal driving efforts to the two half axles, irrespective of their relative turning, or without restraining them to a particular relative motion.

The lack of full constraint of motion between driving shaft and wheels is sometimes a serious fault: if one wheel gets on a slippery spot or into a mud hole it may take all the motion, leaving the other wheel standing and the car stalled. This difficulty can be overcome in part by introducing extra friction into the

differential box (with some loss of freedom in ordinary action); and can be fully met, although at a high cost of ingenuity and complexity, by adding some device for locking the differential at will.

V. Two independent driving motions combine to produce a definite output effect.

(*g*) **Speed-difference Indicators.**—The two instruments in Figs. 332 and 333 have the same function, namely, to show whether two marine engines or turbines are running at equal speed. Consider the first: spindles 1 and 2 are positively driven by the two machines; if their speeds are exactly equal (but opposite) planet wheel G and arm H do not turn at all; if No. 1 runs

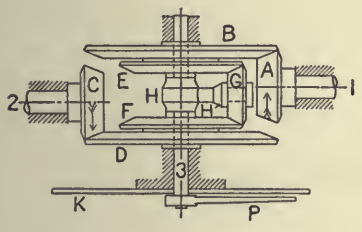

FIG. 332.—Kilroy's relative speed indicator.

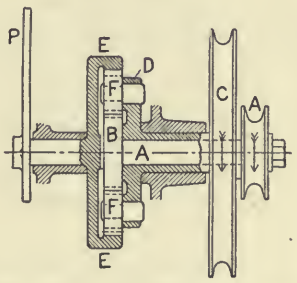

FIG. 333.—Yarrow synchronizer for turbines.

faster pointer P turns clockwise at half the excess of speed of wheel E over wheel F, if No. 2 runs faster the pointer goes the other way.

The use of the spur-gear differential of Fig. 293 introduces into Fig. 333 the lack of similarity, between what are here input motions, upon which comment was made under Fig. 302. With driving pulleys of the same size and speed (at the two machines) it is necessary to use very different receiving pulleys A and C on the instrument in order to make pointer P stand still. This dissimilarity suggests some interesting questions, which may be put into the form of problems.

Prob. 194. If the tooth numbers in Fig. 333 are 26, 14 and 54, find the exact ratio of diameters of cord wheels A and C. How will the working of the apparatus be affected by change in the thickness of cord used?

Keeping the same tooth numbers, redesign the train so as to use the planet-wheel arm (here piece D) as indicating member; sketch arrangement, then get the relative size and turning direction of the cord pulleys.

Prob. 195. Suppose that the equal cord velocities coming to Fig. 333 give a speed of 200 r.p.m. to the larger wheel C. How rapidly will the indicator turn if either machine (alone) runs 5 per cent above this normal speed, and in which direction? Answer for either (*a*) original or (*b*) new design.

One way to solve is to use x for the unknown movement of the indicating member as in Eq. (84), Sect. 23 (*h*). Another is to treat the added movement as something separate from the original balanced movement, working as if the cord wheel whose speed does not change were standing still.

Prob. 196. Using algebraic symbols get a general expression for the r.p.m. of pointer P, Fig. 333, due to an excess of speed in either machine which is equivalent to one r.p.m. of the larger cord wheel: either (*a*) or (*b*) case.

(*h*) **Double-input Mechanisms.**—The mechanisms just described are applications of the idea suggested in Sect. 23 (*h*) and made quantitative in Prob. 181 to 186, namely, of giving definite and independent speeds to two elements of a cyclic train and getting a resultant speed of the third member. The scheme is attractive kinematically, offering wide possibilities of speed variation and reduction. Its practical availability suffers, however, when considerations of power transmission and friction loss are taken into account, as will appear from the study of a simple example.

The arrangement outlined in Fig. 334 might be used as the main train of a lathe. Single-speed belt pulley A is fast on sleeve 2 along with gears B and E, and is supposed to deliver its main driving effort through gears C, C to the arm fast on output shaft 3. Through the shunt train E-F-G-H-K, into which speed-varying elements might easily be incorporated, bevel wheel D is given a rotation opposite to and not much different from that of wheel B. With the definite numbers on the figure, +300 r.p.m. for B and −280 r.p.m. for D, shaft 3 turns at only 10 r.p.m. With a very simple mechanism, especially as to number of axes, a very great reduction of speed has been secured.

In a first view of force and work conditions disregard frictional losses. Of the total work delivered by wheel B a certain portion is applied to the driving of shaft 3, while the rest is transmitted around the shunt train from D through K and G to E, where it helps to drive sleeve 2. Then the belt has only to supply an effort equal to the excess of B torque (resisting) over E torque driving); and it is by virtue of this differential force action that a big multiplication of working force from the pulley to wheel B is possible. On the question of the division, between shaft

3 and wheel D, of the work delivered by wheel B, remember that there are two factors in work rate, force P and velocity V as in Sect. 7 (*e*): with the proportions in Fig. 334 wheel D is moving much the faster, hence takes most of the work.

Now comes the vital matter of frictional loss. In a gear train of normal type velocity V gets smaller as force P grows larger.

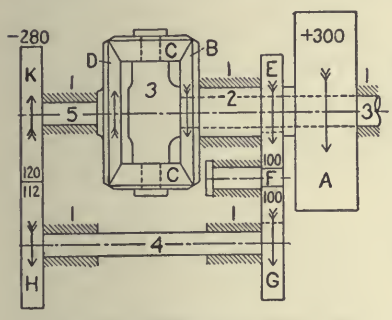

FIG. 334.—Differential drive with shunt train.

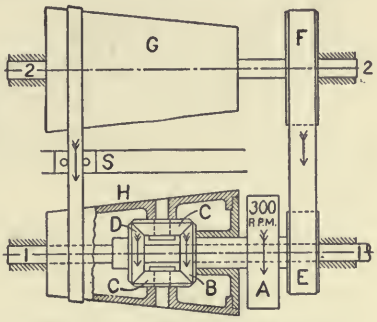

FIG. 335.—A wide-range speed variator.

Here there are big contact pressures at teeth and bearings throughout the train, associated with high velocities. Then of the work received by wheel D a large part is absorbed by friction and a diminished amount is returned to E; therefore the belt must deliver a much larger driving effort relative to the tooth pressures at B, and both the pulling capacity and the mechanical efficiency of the mechanism are greatly reduced. One way of summing up conclusions is to say that the direct transmission of work from pulley A to shaft 3 is accompanied by the circulation of a larger amount through the rest of the train: and the frictional toll upon the whole work handled has to be borne by the slender current in the main channel.

When, however, forces are light and speed variation is the thing sought, this type of differential train may be very useful. Thus in Fig. 335, where gear B and pulley E are fast on a sleeve with driver A, belt cone H carries planet wheels C, C, and gear D is on output shaft 1, a very wide range of delivered speed is readily secured. By varying N_h from N_b to $0.5N_b$, this output speed is varied from N_b down to zero.

Prob. 197. Prove the statement as to speed range in Fig. 335, then for the effect of varying N_h from N_b to $0.5N_b$ find the ratio between the end diameters of the like cones G and H and between pulleys F and E.

Prob. 198. In Fig. 335 let pulley A make 300 r.p.m., then for the sixteen speeds 0, 20, 40, . . . 280, 300 of shaft 1 determine the positions of belt guide S, expressing them as fractions of the length between extreme positions.

SECTION 25. INDEXING AND MATCHING

(*a*) **Scope of Subject.**—Under this title we shall consider certain specific problems in machine-tool kinematics, grouped here because they involve principles that have been developed in the proceeding portion of this chapter. Mostly they are concerned with the production and action of toothed wheels, but not in such a way as to go into the question of tooth shape or profile.

By indexing, as a machine-shop term, is meant the operation of dividing the circle exactly into a whole number of equal parts. Strictly, what is divided is not so much a circle as the unit turn or 360 degrees; but in almost any shop operation there is a pretty definite circumference to which the angle division is applied, with the effect of measuring off the sides of a regular polygon.

The problems in matching of gear teeth and of screw threads form a brief supplement to the body of the section, in Arts. (*g*) to (*i*).

(*b*) **Plain Indexing.**—Indexing is done mostly on the milling machine and on gear cutting machines. For general service a

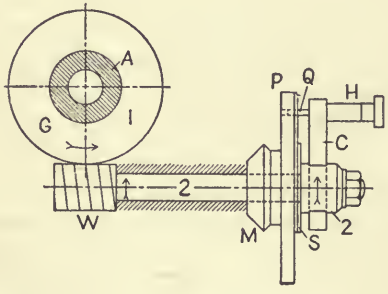

FIG. 336.—Mechanism of index head.

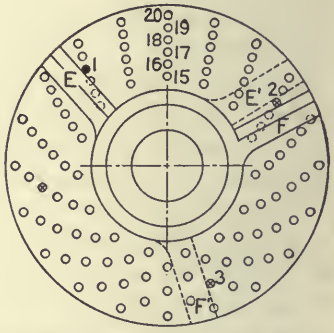

FIG. 337.—Index plate, with sector.

distinct appliance called the index head is used, having the mechanism outlined in Fig. 336. Spindle A has its bearings in a small headstock, with which comes a companion tailstock; then work can be held either between centers or in a chuck (or on a

stub mandrel) attached to the spindle. On the spindle is a worm gear G, always made with 40 teeth, turned by an accurately fitting single-thread right-hand worm W. At the other end of the worm shaft is the index crank C, with handle H and plunger pin Q which can be let into any hole in index plate P.

This plate, more fully shown by Fig. 337, carries the essential dividing element of the apparatus, in the form of a number of circles of equally spaced holes. Plate P is fast on miter gear M, which for purposes of plain indexing is clamped so that it will not turn. The quantities involved in a dividing operation are,

n = number of divisions per unit turn desired at spindle A;
h = number of holes or spaces in the circle to whose radius pin Q is set;
c = number of spaces passed over the pin in an operation, or of holes beyond its initial position.

For $1/n$ turn of the spindle worm shaft 2 turns $40/n$; and in the passing of pin Q over c spaces crank C turns c/h; then the equation of relation is

$$\frac{c}{h} = \frac{40}{n}. \tag{89}$$

If n is less than 40 and c therefore greater than h, crank C must make more than a full turn. Thus for 13 divisions,

$$\frac{c}{h} = \frac{40}{13} = 3\frac{1}{13} = 3\frac{3}{39},$$

the fractional portion of the quotient or ratio being brought to terms which make its denominator agree with one of the circles on the index plates. In all our examples and problems we shall keep within the limits of selection set by Table 5, where 39 is the only circle number h having 13 as a factor. The conclusion is that in order to turn spindle and work piece through one-thirteenth of a revolution handle H must make three complete turns and advance 3 holes in the 39 circle. Again for $n = 56$,

$$\frac{c}{h} = \frac{40}{56} = \frac{5}{7} = \frac{15}{21} \text{ or } \frac{35}{49};$$

less than a full turn is needed and there is choice of two circles, with less chance of being obliged to change the plate on the machine.

TABLE 5.—PARTICULARS OF THE BROWN & SHARPE INDEX HEAD

Plate	Holes in circles	Regular change gears	Special change gears
1	15, 16, 17, 18, 19, 20	24, 24, 28, 32	46, 47, 52
2	21, 23, 27, 29, 31, 33	40, 44, 48, 56	58, 68, 70
3	37, 39, 41, 43, 47, 49	64, 72, 86, 100	75, 84

With the "regular" change gears any number up to 382 can be indexed, with the special gears any number up to 1,008.

The index sector, marked S on Fig. 336 and shown on the plate in Fig. 337, when once set for the correct turn of the index crank, obviates the need of counting holes at each shift of handle H. Not only can the two arms be set and clamped together at any angle, but the ring carrying one arm is graduated, so that by setting it according to the table of instructions the risk of error in counting holes the first time is eliminated. The use of the sector, first to govern the advance from hole 1 to hole 2, then turned into the dotted position for the next advance to hole 3, is obvious. It turns freely, yet (like the joint of a draftsman's compass) with enough friction to hold it pretty firmly in any position.

For quick indexing in simple numbers it is usual to provide a direct method by putting on spindle A itself a plate having one circle commonly of 24 holes or notches, with a special latch pin on the frame of the index head. The worm-shaft bearing has an eccentric throwout, like the one in Fig. 126 (*g*); and when the worm is clear the spindle with its face plate can be turned by hand.

Prob. 199. With the index plates in Table 5 make out an indexing table like Form 29 for all the division numbers up to 50. It is convenient to be able to index a number with any one of the plates; therefore give, so far as possible, a direction for each plate. If there is choice on a plate use the larger circle. See whether any circle is unnecessary for this range of numbers.

FORM 29

Number n	Plate 1	2	3	n	1	2	3
2	20	20	20	25	$1\,1\frac{2}{20}$	.....	.....
3	$13\frac{6}{18}$	$13\,1\frac{1}{33}$	$13\,1\frac{3}{39}$	26	.....	.....	$1\,2\frac{1}{39}$
11	.....	$3\,2\frac{1}{33}$	.....	..	etc.		

(*c*) **Compound Indexing.**—The test or criterion of ability to index any number n is expressed by putting Eq. (89) into the form

$$c = \frac{40 \times h}{n}, \tag{90}$$

Since c must be an integer, the evident requirement is that 40 and h must together contain all the factors of n.

Consider now the second half-century of numbers, from 51 to 100, in connection with Table 5. At once the prime numbers 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97 are ruled out; but there is also a group of composite numbers, 51, 57, 63, 69, 77, 81, 87, 91, 93 and 96 which cannot be made to satisfy Eq. (90) in the requisite fashion. Typical examples are,

$$\frac{40 \times 17}{51} = 13\tfrac{1}{3}\ (a),$$

$$\frac{40 \times 33}{77} = 17\tfrac{1}{7} \text{ or } \frac{40 \times 21}{77} = 10\tfrac{10}{11}\ (b); \tag{91}$$

all of the other numbers except 91 being like (*a*) in that 3 is the excess factor which does not cancel.

Change the statement of example (*a*) back to the form of Eq. (89), getting

$$\frac{40}{51} = \frac{c}{17} = \frac{13}{17} + \frac{1}{51}, \tag{92}$$

and showing that it is a question of measuring off the small additional fraction $\frac{1}{51}$ of a turn for the index crank. A unit for such small measurement is found in the difference between the angles covered by respective spaces in two different index circles. Suppose, for instance, that the crank is advanced $\frac{1}{17}$ of a turn and that plate and crank together are then brought back through $\frac{1}{18}$: the net effect is an advance of $\frac{1}{17} - \frac{1}{18} = \frac{1}{17 \times 18} = \frac{1}{51 \times 6}$ for the crank. To make it $\frac{1}{51}$ we have only to advance $\frac{6}{17}$ or 6 on 17, then return $\frac{6}{18}$; the whole operation will be $+1\frac{9}{17} - \frac{6}{18}$. This example introduces the principle of **compound indexing,** which is made possible mechanically by using an index pin in an arm fast to the frame, working on the back of the plate.

The general scheme is to select a circle h carrying a high factor of n, then on the back select another circle whose number h' is a multiple of n/h. This choice and the required movements

must be worked out and checked up in detail. In our example h' must have 3 as a factor, and we might use 15 instead of 18: then we should have the difference $\frac{1}{17} - \frac{1}{15} = \frac{-2}{51 \times 5}$, the minus sign indicating that to go forward $\frac{1}{17}$ and back $\frac{1}{15}$ will return rather than advance the crank: so that for an advance it must be $+\frac{1}{15} - \frac{1}{17}$. Now $\frac{5}{2}$ spaces cannot be indexed; but if we go back and change the original requirement to $40\frac{5}{51} = 12\frac{3}{17} + 4\frac{5}{51}$, we then have $4 \div \frac{2}{5} = 10$ as the number of spaces in the compound operation, which added to the primary $12\frac{3}{17}$ gives

$$\frac{12}{17} + \frac{10}{15} - \frac{10}{17} = \frac{2}{17} + \frac{10}{15}$$

This one example, with the practice problem following, will sufficiently illustrate the idea and possibilities of the method. It requires an excessive amount of arithmetical agility; and even with fully worked out instruction tables the double operation is slow and subject to mistake. Further, no provision is made for prime numbers higher than those on the index plates.

Prob. 200. (*a*) Get a correct compound movement for one of the numbers 57, 63, 69, 77 or 87. (*b*) Try 81, 91, 93 or 96.

(*d*) **Differential Indexing.**—The range of the index head is very greatly increased in a purely mechanical way, without any added complexity of routine manipulation, by the extension of its mechanism that is outlined in Fig. 338. The first change gear D or D_1 is fast on the back end of spindle A, and through either a simple or a compound change train drives shaft 3 and turns the index plate. As before, let $1/n$ turn of the spindle be the unit movement: then crank C turns $40/n$ and at the same time plate P turns t/n, letting t represent d/f or d_1d_2/f_1f_2, the value of the change-gear train. The net movement of crank on plate is therefore $(40-t)/n$, also measured by c/h so that the equation of relation becomes,

$$\frac{c}{h} = \frac{40 - t}{n}. \tag{93}$$

In manipulating this equation the procedure is to choose for c/h a value close to $40/n$, but having as simple a number as possible for its denominator. Then, directly,

$$t = 40 - n\frac{c}{h}; \tag{94}$$

or, if $c/h = 40/n'$ which means that n' is a number close to n that would be given by plain indexing, we may write

$$\frac{40}{n'} = \frac{40 - t}{n}, \text{ and get } t = \frac{40(n' - n)}{n'}. \qquad (95)$$

Try our example $n = 51$ from the last article, using Eq. (94);

if $\frac{c}{h} = \frac{13}{17}$, $t = 40 - 51 \times \frac{13}{17} = 40 - 39 = +1$;

if $\frac{c}{h} = \frac{14}{17}$, $t = 40 - 51 \times \frac{14}{17} = 40 - 42 = -2$.

Either of these t's can easily be got with the gears in Table 5, arranging the train as in Fig. 338 (*a*) for a plus value of t, putting in another idler for a minus value.

Examples of the use of Eq. (95), the more generally serviceable method, are given in Form 30. With $n = 51$ the first

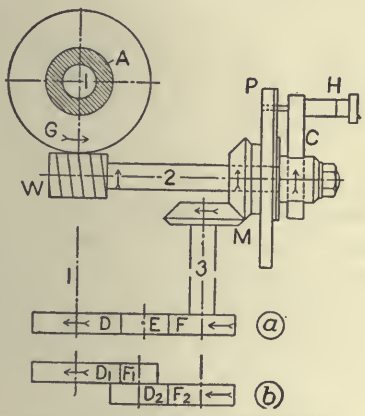

FIG. 338.—Train for differential indexing.

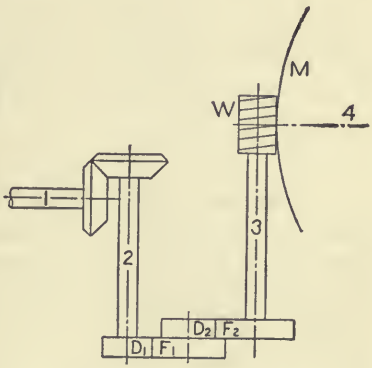

FIG. 339.—Indexing train of gear cutter.

choice of $n' = 52$ brings in the prime factor 13 which is not found among the "regular" change wheels in Table 5; but with $n' = 50$ a workable result is secured. The three solutions for $n = 53$, all good, give an idea of the flexibility of the scheme. With the high number 187 the first three trials show how easy it is to go outside of the range of change wheels provided; in the fourth n' has the high factor 11 in common with n, and this factor then cancels from $(n' - n)$; the last shows the obvious use of the nearest simple value of n'.

FORM 30

n	n'	$\frac{c}{h}$	t	$\frac{d}{f}$	Line
51	52	$\frac{40}{52} = \frac{30}{39}$	$\frac{40}{52} = \frac{10}{13}$	not	1
51	50	$\frac{40}{50} = \frac{16}{20}$	$-\frac{40}{50} = -\frac{4}{5}$	$-\frac{32}{40}$	2
53	54	$\frac{40}{54} = \frac{20}{27}$	$\frac{40}{54} = \frac{10}{27}$	$\frac{40\ 32}{72\ 48}$	3
53	55	$\frac{40}{55} = \frac{24}{33}$	$\frac{80}{55} = \frac{16}{11}$	$\frac{64}{44}$	4
53	56	$\frac{40}{56} = \frac{35}{49}$	$\frac{120}{56} = \frac{15}{7}$	$\frac{72\ 40}{56\ 24}$	5
187	188	$\frac{40}{188} = \frac{10}{47}$	$\frac{40}{188} = \frac{10}{47}$	not	6
187	190	$\frac{40}{190} = \frac{4}{19}$	$\frac{120}{190} = \frac{12}{19}$	not	7
187	195	$\frac{40}{195} = \frac{8}{39}$	$\frac{320}{195} = \frac{64}{39}$	not	8
187	220	$\frac{40}{220} = \frac{6}{33}$	$\frac{40 \times 33}{220} = 6$	$\frac{48\ 72}{24\ 24}$	9
187	180	$\frac{40}{180} = \frac{4}{18}$	$-\frac{280}{180} = -\frac{14}{9}$	$-\frac{56\ 48}{24\ 72}$	10

(*e*) **Indexing on the Gear Cutter.**—On a machine for automatically cutting teeth by means of the simple circular milling cutter—see Fig. 590 for form of operation—the cutter slide is stopped at the end of a cut and quickly run back to the initial position. When nearly back it sets going the indexing train, of which the primary counting unit has been shown in Fig. 207. The gear train proper is outlined in Fig. 339, where shaft 1 is No. 3 of Fig. 207, receiving 1, 2, 4 or 8 turns. Through a change-gear train this movement is carried over to worm W and applied to the master worm wheel M, which is on the work spindle.

Let m = teeth on wheel M, with a single-thread worm;
q = number of turns given to shaft 1;
n = number of teeth in gear to be cut;

then

$$\frac{1}{n} = q\frac{d}{f}\frac{1}{m}, \quad \text{whence } \frac{d}{f} \text{ or } t = \frac{m}{qn}. \tag{96}$$

For example, if $m = 180$ and $q = 2$, the change-train ratio for

a gear of 56 teeth will be $t = \frac{180}{(56 \times 2)} = \frac{45}{28}$. Since anything of the nature of differential indexing is impracticable, if not impossible, such a machine must have a large number of change wheels, carrying all the prime numbers or factors that it is to be capable of indexing.

Prob. 201. Arranging computations as in Form 30, determine indexing directions for one of the following groups of numbers, keeping to the "regular" change wheels in Table 5.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
73	93	63	87	79	67	77	69	59	83
117	167	143	149	131	114	123	184	126	195
211	231	172	259	188	245	291	288	221	229
296	344	318	336	361	317	376	377	328	308

Prob. 202. Similarly determine directions for one of the following groups, using "special" change gears when necessary.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
241	189	123	247	199	276	153	203	231	169
348	361	255	369	323	349	368	293	287	289
411	433	447	388	487	457	401	503	471	484
523	569	581	541	568	512	597	545	584	587

Prob. 203. Determine how to use the index head in order to measure off degrees, also halves, quarters, fifths and sixths of degrees.

Prob. 204. Look up a full description of the Brown & Sharpe index head and make a sketch of the modification of its gear train that permits the swing of the spindle to an inclined position without breaking the differential-indexing connection. Good drawings will be found in *Machinery's Encyclopedia*, under *Indexing*.

Prob. 205. In the Brown & Sharpe instruction book look up the directions for indexing $n = 377$ and express them as a special application of Eq. (93).

(*f*) **Generating the Helix.**—In the common lathe operations of turning and facing, carried on with uniform longitudinal or cross feed, the tool point is really following either a helical or a spiral path on the work piece. Generally this fact is only incidental, but in screw cutting the production of a helix exact in form and dimension becomes a matter of first importance. Beside the regular lathe method of repeated cuts with the ordinary tool of chisel type, there is the thread-milling process, for which is used the same mechanism of change-gear train and lead screw. The tool carriage is modified into a suitable support for

the arbor of a formed disc cutter, which has an independent drive and can be set to the slant of the thread. Both the turning of the work piece and the traverse of the carriage are now slow feed motions.

The screw thread is a helix of very short lead relative to its diameter: by lead is meant the advance along the axis in one turn, identical with pitch only on a single-thread screw. For long leads or big slants the milling process is the one commonly used, with a few cases of planing, as in the rifling of a gun. The change in magnitude of lead ratio is accompanied by a sort of inversion of the gear train: on a lathe or thread miller rotation

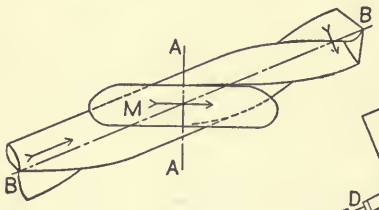

FIG. 340.—Cutter and drill.

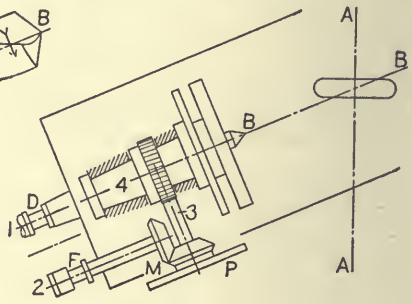

FIG. 341.—Index head.

Helical or "spiral" milling.

is the larger component of helical motion, and the smaller component of axial traverse is got from it by means of a secondary change train: but in a milling-machine operation of the type in Fig. 340 axial traverse comes first and from it is derived the rotation component.

Milling cutter M in Fig. 340 simply rotates in a fixed position; then the drill blank or other work piece must advance along its axis BB and at the same time turn slowly. On the "universal" milling machine the work table is an upper slide that can be swivelled to a desired angle and yet be mechanically driven by its own longitudinal feed screw. Then on the extension of this screw, at 1 in Fig. 341, is mounted the first wheel D or D_1 of a change train that drives shaft 2 of the index head and so turns the spindle, axis 4. Of course the index pin must be engaged, so as to lock its miter gear to worm shaft 3.

If l is the lead of the helix the requirement is that the work piece shall turn once while the table advances l inches. Let s = threads per inch of the feed screw (commonly 4), so that ls

turns are needed; as before use t for the value of the change train, whence the whole turning ratio from screw 1 to spindle 4 is $t/40$; then the determining relation is

$$ls\frac{t}{40} = 1,\ t = \frac{d}{f} = \frac{40}{ls} = \frac{10}{l} \text{ (generally).} \qquad (97)$$

Prob. 206. For a helix of l in. lead on a cylinder of d in. diameter select change gears from Table 5 to meet Eq. (97) and calculate the angle to which table of milling machine must be swung. Use one set of data:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
l.....	6	7	8	9	10	12	14	16	18	20
d.....	1.6	1.5	2.5	3	3	2.5	3.5	3.5	4	4

The simple relations in Fig. 341 are no more than an introduction to the large extension of this subject in connection with the generation of helical gear teeth by means of the hob or helical cutter. The matter might be carried forward here, but to make it a part of the study of the mechanism of gear tooth production seems the better course: see Sect. 41. Turning then to the "matching" problems, the first one concerns frequency of contact between the same teeth in a pair of gears.

(*g*) **Distribution of Tooth Contacts.**—A question of interest and of some importance as regards uniformity of tooth wear may be expressed as follows: Fixing attention upon a certain tooth of one wheel, what is the interval between its contacts

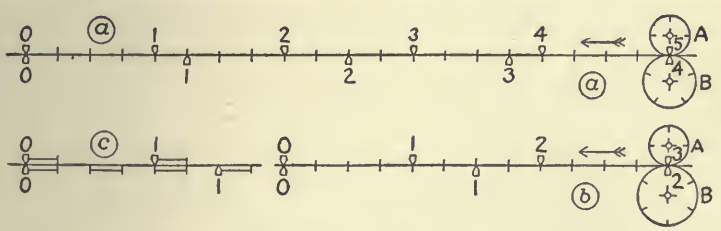

FIG. 342.—Circumferential relations of pitch circles.

with a particular tooth of the other wheel; and during the cycle of action thus set off how many of the teeth of the second wheel does this primary tooth touch?

Reducing the problem to simplest terms, the teeth may be sufficiently represented by mere graduation marks on the pitch circles, as in Fig. 342, where for further simplicity the numbers of divisions are taken smaller than on actual wheels. Imagine that a straight line rolls between the circles, on which each one graduates

its own series of divisions. Then the common linear distance between like contacts, really measured round and round the respective pitch circles, here appears in a common development. Evidently this distance must be a common multiple of the numbers of pitches on the two wheels; and the shortest distance will be the least common multiple. With the relatively prime numbers 4 and 5 in case (*a*), the interval is measured by 5 turns of A and 4 of B. With the common factor 2 in 4 and 6, case (*b*), the turns are cut down to 3 and 2.

As to total contacts, it is apparent that when the numbers are prime to each other a tooth of either wheel touches every tooth of the other wheel. The effect of a common factor is to divide the teeth or pitches into groups—pairs in Fig. 342 (*c*)—then the successive contacts are between undivided groups.

If now tooth numbers a and b have a common factor f, the relation for numbers of turns between like contacts is

$$\frac{a \div f}{b \div f} = \frac{n_b}{n_a}; \tag{98}$$

while during each cycle (and continuously) a tooth of either wheel touches $1/f$ of the teeth of the other wheel. The set of teeth which it thus touches is unchangeable, being spaced at intervals of f pitches around the second wheel.

The practical bearing of this matter is that if the tooth has an initial irregularity of form or some flaw of surface or material tending to cause irregular wear, uniformity will be promoted by a wide distribution of contacts. This is secured by making the tooth numbers either prime to each other or with a low common factor. When a certain exact velocity ratio is required this may not be feasible; but most cases of heavy power transmission do not call for precise ratios, and it is a good thing to design even cut gears of high grade material so as to vary the tooth contacts as widely as possible.

Prob. 207. For each of the three pairs in one of the following groups find the length of contact cycle in turns of both wheels, then determine the sequence of contacts of one tooth on smaller wheel A within a group of a teeth on wheel B, working according to the scheme of Form 31, which is left to your own study and interpretation.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
24–56	28–96	30–80	32–72	36–80	40–72
35–63	45–65	42–75	25–95	21–48	33–84
17–24	18–43	13–34	20–67	15–38	16–39

FORM 31

Exact turns A....	2	5	7	9	12	14	16	19	21 to 33
Approx. turns B..	1	2	3	4	5	6	7	8	9 to 14
Pitches on A.....	28	70	98	126	168	196	224	266	294 to 462
Pitches on B.....	33	66	99	132	165	198	231	264	297 to 462
Difference.....	−5	+4	−1	−6	+3	−2	−7	+2	−3 to 0

This is for a pair with 14 and 33 teeth. Numbers in the "difference" line locate teeth on wheel B, from the primary tooth as zero.

(*h*) **Matching Pitches.**—Consider the question of shifting a solid gear cone with what is called "locked shift" in Sect. 16 (*d*). The chances are that when a change is attempted the teeth of the new pair will not match, and the question arises, How far is it likely to be necessary to turn the cones in order to secure entrance into the new engagement? To see the relation involved we take the two-step cones in Fig. 343, where A-C and B-D are both com-

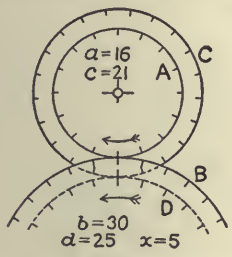

FIG. 343.—Sliding-cone gear pairs.

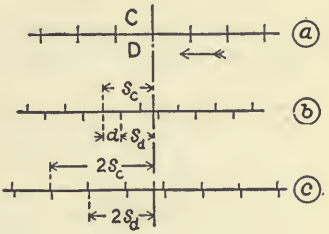

FIG. 344.—Pitch-circle relations.

pound or rigidly connected gears. Pair A-B is now engaged, pair C-D is matched, and we wish to know how far it is to the next angular position where C and D will again match.

Take one pitch of the engaged pair A-B as the unit of movement and express the corresponding movements of the pitch circles of C and D, from the initial agreement shown in Fig. 344 (*a*). Letting s_c and s_d be the respective advances and using both general symbols and particular numbers, we have

$$s_c = \frac{a+x}{a}p = \left(1+\frac{x}{a}\right)p = \left(1+\frac{5}{16}\right)p. \qquad (99)$$

$$s_d = \frac{b-x}{b}p = \left(1-\frac{x}{b}\right)p = \left(1-\frac{5}{30}\right)p.$$

These are shown in Fig. 344 (*b*) and it is evident that the difference

$$d = \left(\frac{x}{a}+\frac{x}{b}\right)p = \left(\frac{5}{16}+\frac{5}{30}\right)p = \frac{23}{48}p \qquad (100)$$

is an important quantity; for it is when the cones have turned far enough to bring this difference up to unity that the teeth of C and D will again match. At 23/48 per pitch at contact point A-B the movement required is evidently $48/23 = 2.09$ such pitches, as seen in Fig. 344 (*c*).

From this typical example we can infer that the angular interval between matching positions is generally small. The same conclusion applies to the act of throwing in the back gear of a lathe when the cone pulley is latched to the spindle, which is sometimes done in order to lock the spindle, as for screwing a face plate or chuck on or off.

(*i*) **Matching Screw Threads.**—A problem which requires the actual matching of scales having unequal units of division is

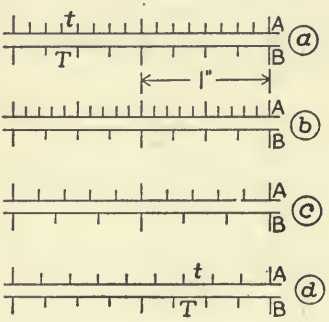

FIG. 345.—Matching screw threads.

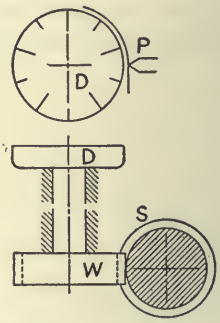

FIG. 346.—Threading indicator on lathe.

presented by one method of cutting screw threads on the lathe. Most frequently the machine is reversed and run backward after each cut; but in many cases time can be saved by opening the lead-screw nut, running the carriage back by hand, then closing the nut for the next cut. The question is how to insure that the tool will properly enter the thread that it is cutting.

In each diagram of Fig. 345 are shown two scales, the lower one standing for the lead screw of T threads per inch, the upper for the new screw of t threads: initial agreement at A-B represents the condition at end of cut, where the lead screw was released. The simplest case (*a*) has t a multiple of T, so that every division of scale B coincides with one of A and the nut can be closed at random. With T and t both even numbers there is agreement at every half inch, case (*b*); if either or both are odd agreement comes at the full inch, case (*c*); while if t contains a fraction, as does $4\frac{1}{2}$

at (*d*), the matching interval is a number of inches equal to the denominator of the fraction.

The scheme of releasing and catching the lead screw is made effective by means of the threading indicator outlined in Fig. 346. On a short vertical spindle with its bearing in the tool carriage are a worm wheel W and a graduated dial D. The wheel meshes with lead screw S, functioning as pinion on rack when the carriage is not moving with the screw. The scale on the dial represents inches and half-inches of carriage traverse with reference to screw. Then if pointer P (on the body of carriage) has a slight range of movement, so that it can be set to a mark of the dial before the nut is opened, the latter can be reclosed just as the pointer comes to a proper graduation, without stopping the lathe.

CHAPTER VI

LINKAGE MECHANISMS AND MOVEMENTS

This chapter takes up motion that is generally nonuniform, reciprocating and cyclical—compare Fig. 28—dealing principally with movements or displacements. Certain simple relations and the beginnings of methods for velocity are introduced, but their development and application goes over into Part II. Selection and grouping of mechanisms is largely from the viewpoint of kind of motion problem presented.

We begin with the simple and much used engine mechanism, for which geometrical (not plotted) diagrams can be drawn, and upon it base practically all the kinds of diagrams that are useful in showing movement. Then in Sect. 27 these methods are applied to the problem of the eccentric-driven slide valve, with determination of movements and of their results in valve action.

Section 28 presents some fundamental principles of mechanism, first considering the inversion, equivalences and relationships of simple linkages, then going into the general problem of constrained motion and the evolution of more complex linkages. The next section deals similarly with the principles of planar motion, presenting translation and rotation (singly and as components), the velocity triangle, instant centers as to determination and use, and centrodes. The subject of velocity is thus introduced because there is immediate need of certain relations and methods in what follows.

The various planar mechanisms in Sect. 30 are predominantly concerned with the production of some simple geometrical form of movement, mostly along desired point paths. In some cases geometrical interest is a good deal stronger than utilitarian. Section 31 follows with a selection of the chamber trains used in rotary pumps and motors and closes the chapter the small group of nonplanar linkages.

SECTION 26. DIAGRAMS OF SLIDE MOVEMENT

(*a*) **General Ideas.**—Coplanar motion has been described in Sect. 2 (*e*). If the parts of a mechanism all move in parallel planes it can be sufficiently represented by a flat outline or geometrical skeleton, lying in a single plane of projection. The latter will be perpendicular to the axes of all turning pairs and parallel to the axes of sliding pairs. We may consider either that the frame or reference member has a fixed position in this plane or that the plane is fast to that member—the second concept covering the case when the whole machine has a motion of progression, as does the locomotive.

From the ideas set forth in Sect. 2 (*h*) we now proceed to a study of the motions of linkage members, with especial interest in those of the nonuniform, reciprocating and commonly cyclical type described in Sect. 2 (*k*).

In general the position of a body in a plane is made determinate by fixing the location of two of its points: and its movement can be fully represented by drawing the paths of these points and marking upon them a series of successive coincident positions. The centers of turning joints are commonly chosen for such master points, because of their natural prominence and because they belong to two members at once.

There are two very important simple cases in which the motion of but one point of a body need be found. The first is rectilinear sliding, where all points have the same motion. The second is rotation on a permanent axis or center, the latter serving as one of the master points: the other may be located on its circular path or the radius to it may be drawn and located on a fixed angle scale.

The problems first to be taken up are of this one-point kind, and have another element of simplicity in that the whole cycle of movement can be shown and determined by a geometrical diagram, without need of successive plotting of positions. Specifically, they are concerned with the relative motion of crank and slide in the engine mechanism and its variations—see Figs. 4 and 5 for the most common form.

(*b*) **Slide Motion from Crank Rotation.**—In machines of the crank-and-piston type, the crank is commonly constrained (by the action of a flywheel) to a nearly uniform rotation. Then no matter which the direction of drive, whether from piston to

crank or from crank to plunger, it is usual and logical to take the crank as *kinematically* the primary or driving member.

For the simplest case of slide motion and approaching the problem distinctly from the geometrical side, let point C in Fig. 347 be travelling in a circle path or swinging at the end of rotating radius OC, and let it be continually projected on a diameter AB at D. Then if the motion (velocity) of C is uniform, point D will have what is called **harmonic motion** back and forth in

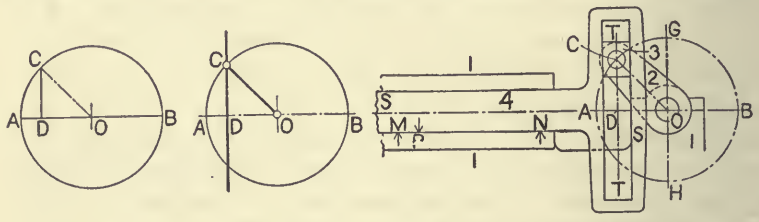

FIG. 347. FIG. 348. FIG. 349.—Crossed slidercrank.
Evolution of harmonic-motion mechanism.

its path AB. At present we are quantitatively interested in relative positions only, and the velocity specification may be left in reserve.

As the first step toward a mechanism, emphasize radius arm OC in Fig. 348 (as against circle path) and let a moving line TT, through C, be kept always at right angles with AB. Now about this skeleton build the mechanism in Fig. 349, embodying the right-angle requirement in cross-slotted slide SS or member 4. The sketch suggests that some other point P on the slide (perhaps a piston face) may be of more immediate interest than projection D. The useful fact, from the viewpoint of motion graphics, is that the movement of P on its path or stroke line MN is identical with that of D on AB.

This **crossed slider-crank** mechanism has occasionally been used in small engines. Arranged as in Fig. 350 it gives a very compact means of introducing flywheel control into a steam pump or compressor; and a crossed slide of this form is sometimes called a Scotch yoke. Proportioned as in Fig. 351, with big slide blocks surrounding an eccentric of small throw or radius, the mechanism is a common drive for punching and shearing machines.

For use with a crank, as in Figs. 349 and 350, the crossed slide is an awkward machine member, and the rubbing surfaces of pair 34 are very hard to keep in proper lubrication. Naturally,

practice has turned to the connecting rod and wrist pin as a better means of transmission between crank and slide, so that the usual mechanism has the form in Figs. 1 and 3, which is called the plain **slider-crank.**

(*c*) **The Slider-crank Mechanism.**—In this regular engine mechanism, Fig. 352, the ends of stroke MN for pin W are determined by making AM = BN = rod-length CW, or by putting

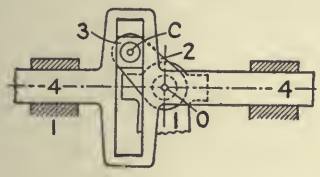

FIG. 350.—The "Scotch yoke."

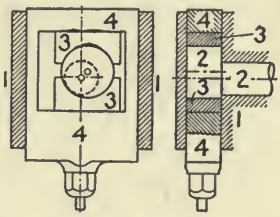

FIG. 351.—Punch drive.

the crank on the respective dead centers shown in Fig. 353. Characterized by the fact that rod and crank are in line, these not only fix the limits of piston movement but are also the places where a force along the rod exerts no turning influence on the crank shaft. In terms belonging to piston in cylinder, the dead centers are called *head-end* for extreme left (away from shaft) and *crank-end* for extreme right (toward shaft).

If the connecting rod could remain horizontal or parallel to slide axis MO, so as to locate a vertical line TT in Fig. 352, it

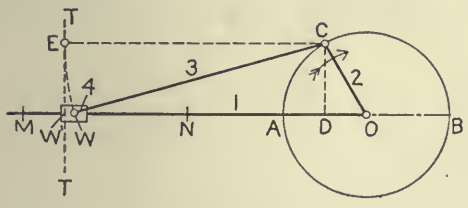

FIG. 352.—The slider-crank.

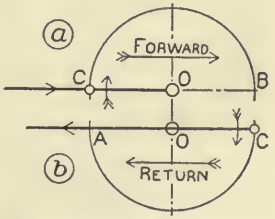

FIG. 353.—Dead centers.

would produce the same kinematic effect as block 3 in Fig. 349, making MW′ = AD. Actually it swings to an angle with the axis, shortening its projected length or the horizontal distance from pin C to pin W. The result is an added-displacement W′W, always toward the shaft. For a given angle of crank from dead center, this causes the piston displacement or travel to

be longer if from M in forward stroke, shorter if from N in return stroke.

Since a connecting rod imagined to be of infinite length would remain parallel to the stroke line, it would transmit harmonic motion to the slide. For this reason the crossed slider-crank is sometimes said to be (or to be equivalent to) an engine mechanism with infinite connecting rod.

(*d*) **Direct Diagrams of Slide Movement.**—For the crossed slider-crank draw straight tangents AA′ and BB′ in Fig. 354,

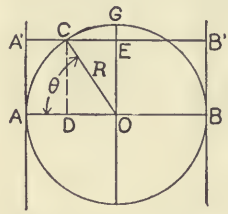

FIG. 354.—Harmonic.

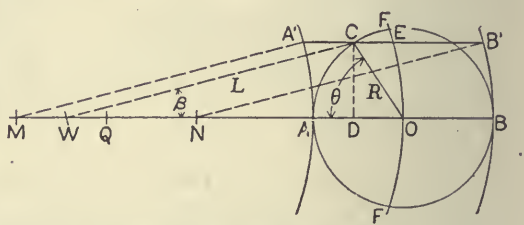

FIG. 355.—Actual engine.

Position diagrams for piston slide.

perpendicular to stroke line AB, then through any position of C pass a parallel A′B′. Obviously C is located on A′B′ just as D is on AB, but the necessity of projection to D is avoided. The proper conception of the diagram, making it mobile, is to think of line A′B′ as moving up and down with C, while C moves on this line just like the slide on its stroke. Letting R = arm or radius of crank, s = slide travel A′C from end of stroke and t = travel EC from middle of stroke, this diagram expresses the two equations,

$$s = R(1 - \cos\theta), \quad t = R\cos\theta. \tag{101}$$

For the engine mechanism, Fig. 355, the stroke-end tangents are arcs struck from M and N with rod length L as radius. Slide position W being located by the same radius from C, figure MA′CW must be a parallelogram because A′C and MW are parallel and the other two sides are equal. This makes A′C always equal to MW or s. The mid-stroke reference line is a similar arc from Q as center, through O.

To get a formula for this s, note that MO = $R + L$, DO = $R\cos\theta$ and WD = $L\cos\beta$: then

$$s = R(1 - \cos\theta) + L(1 - \cos\beta). \tag{102}$$

Evidently the second term, $L(1 - \cos \beta)$, is the extra displacement due to rod swing, W'W in Fig. 352.

(*e*) **Derived Diagrams.**—In Fig. 356 the two diagrams of Figs. 354 and 355 are first drawn together; then with the unrolled or "developed" crank circle as base and A'C or EC as ordinate the curves in Fig. 357 are plotted. When this is done we get a

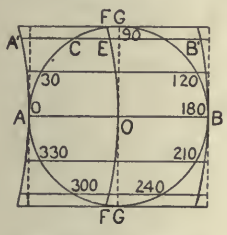

FIG. 356.—Circular.

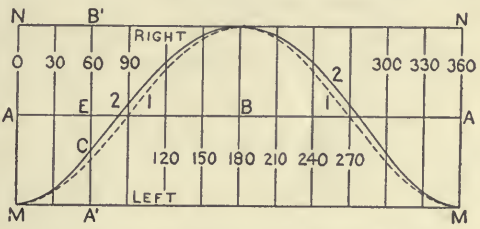

FIG. 357.—Developed, on unrolled circle.

Position diagrams on base of crank angle.

mobile diagram by thinking of total ordinate MN or A'B' as moving across the figure while curve-intersection C travels up and down on it. Since to choose a scale of so many degrees to the inch is more convenient than to lay off and divide the exact circumference of circle AGBG, the base is generally crank angle rather than exact travel of crank pin. Further, if uniform rotation is imposed the diagram will really be on a time base.

Dotted curve 1 for the crossed slide is a symmetrical sine curve or sinusoid—this name applying equally whether the function plotted is sine or cosine. Full-line curve 2, for the engine mechanism, shows very clearly that with actual connecting rod the piston is aways to the right (except at ends of stroke) of the position that it would occupy in harmonic motion.

The last diagram to be considered is of the polar type, slide movement being shown on the rotating crank arm, prolonged into a diameter. For the crank position OC, Fig. 358, take distance DC and lay it off as CK, or take EC and measure it out from the center as OK. The diametrically opposite positions OC', with C'K = D'C' or OK = E'C', gives the same point K but locates the slide in right-hand half of stroke. For this simple movement the locus of K is a circle, on OA as diameter; and during a full revolution point K goes twice around this circle. In proof of the circle form, triangles AKO and OEC are equal because AO = OC, OK = EC and angle AOK = angle OCE; therefore AKO is a right angle and curve AKO a semicircle.

For the engine mechanism, Fig. 359, the polar curve must be plotted point by point: now its two halves or the two rounds of point K are separate, the parts which belong to successive quadrants of crank motion being indicated by like numbering. Since crank positions OF, OF correspond with slide at middle of stroke, these lines must be tangent to curves 1-2 and 3-4 at center O.

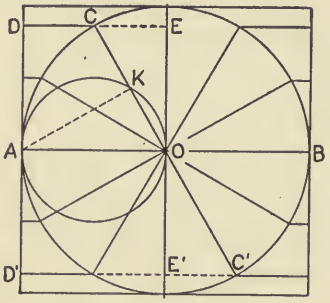

Fig. 358.—Harmonic motion.

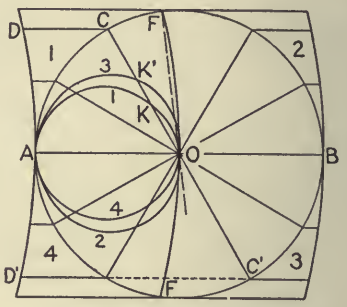

Fig. 359.—Actual engine.

Polar diagrams of slide movement.

Now in Fig. 359 lengths CK and C′K′ are distances of slide from left-hand end of stroke, for the crank at OC and OC′ respectively; that is, on the rotating diameter point C represents the head end of the stroke. If displacement is measured from center O instead of end C the distinction of direction is that if the intercept is direct (or K between O and C) the slide is to the

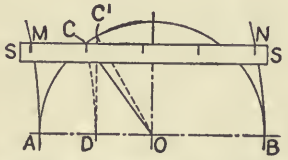

Fig. 360.—On circular diagram.

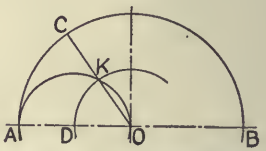

Fig. 361.—On polar diagram.

Finding crank angle for a given slide position.

left, while if it is indirect (with K on CO extended through O) the slide is to the right of midposition.

The simple polar circle, Fig. 358, has wide use in valve diagrams, but the plotted curve, Fig. 359, involves too much layout work to be of much practical utility: it is given here chiefly to show the full development of the scheme.

(*f*) **Crank Angle for Slide Position.**—Thus far, in both layout and use of diagrams, we have worked from crank position to

slide position. Sometimes occasion arises to use the diagram the other way. In Figs. 360 and 361, for instance, point D is at one-fourth of the stroke and it is a question of finding the corresponding crank angle. With infinite rod a simple perpendicular from D up to C′ is enough . For the actual mechanism, desiring to avoid the need of striking arc DC with a beam compass from a remote center, the practical scheme is to hold a scale against the T-square and find by trial the place where MC equals AD. On the polar diagram, Fig. 361, all that is needed is a circle DK from center O.

Prob. 208. Slide Displacements. Draw the two diagrams of Figs. 356 and 357 with the dimensions $R = 3.5''$, $L = 10.5''$, using the angle scale 1 in. = 20 deg. for the developed diagram. Make determination at every 15 deg., numbering at every 30.

Prob. 209. Slide and Crank Positions. (*a*) On a diagram like Fig. 356 locate crank pin for slide at each tenth of stroke and at the middle of the first and last tenths. In dotted lines show also the corresponding angles for infinite rod.

(*b*) Plot polar curve as in Fig. 359, getting points every 15 deg., and dot in circle from Fig. 358: then locate crank for slide at each eighth of stroke, for both forms of mechanism.

Use $R = 4''$, $L/R = 3$.

In order to get a sense of the mobile value of the polar diagram, make a paper template of line AOB, rotate it on O, and note how intersection K travels out and back upon it.

Prob. 210. Letting s_o = distance of slide from mid-position when crank is at 90 deg., S = length of stroke and n = ratio L/R show that

$$s_o = \tfrac{1}{2}(n - \sqrt{n^2 - 1})S; \qquad (103)$$

then find value of s_o/S for $n = 3, 4, 5, 6, 7, 8$ and 10.

Note that in stationary engines n is commonly 6, in marine and automotive engines it is sometimes as low as 4, in locomotives it may range up to 8.

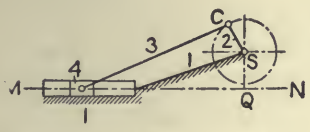

FIG. 362.—Offset slider-crank.

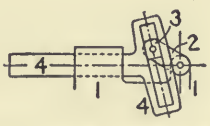

FIG. 363.—Oblique crossed slider crank.

(*g*) **Oblique and Offset Slider-cranks.**—Unsymmetrical forms of the two slider-crank mechanisms are shown in Figs. 362 and 363. The one with offset stroke line is sometimes used in single-acting engines and has also in some degree the effect of a quick-return motion. The oblique crossed slider-crank is of

considerable geometrical interest and underlies the action of an important type of valve gear, the "link motion."

In Fig. 364 let crank OC have its pin center C projected obliquely upon stroke line MN at D. Squarely across the oblique lines draw an auxiliary diameter KL, on which C projects at E. It is evident that the extremes of travel are reached when C is at K and L, stroke MN being now greater than diameter AB: also that the movement of D on MN is exactly similar to that of E on KL, but larger in the ratio of OM to OK or OC.

On the enlarged diagram in Fig. 365 let OK and OM be two cranks which are parts of one rigid member, as indicated by the shaded triangle OKM. Turn this member on O to any position OCC′, projecting C obliquely and C′ squarely to D. For a proof of agreement at D, derive OD first from OC, then from OC′, as follows:

From OC, projection OE = $R \cos \theta$ and OD = OE sec γ = $R \cos \theta \sec \gamma$. Radius OC′ = $R \sec \gamma$ and OD = OC′ $\cos \theta$ = $R \cos \theta \sec \gamma$, which establishes the identity of the two OD's.

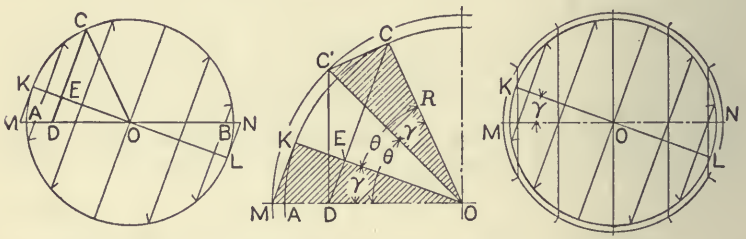

FIG. 364. FIG. 365. FIG. 366.

Equivalence of oblique and square crossed slider-crank.

Equivalence between a square crossed slide on crank OC′ and an oblique slide on crank OC is further shown by Fig. 366, where the two diagrams are drawn together. The fact is made clear that there must be two distinct angle scales, one having its zero at M, the other at K.

(*h*) **The Offset Engine Mechanism.**—An engine mechanism with offset stroke line (or offset crank shaft, if the viewpoint be changed) is outlined in Fig. 367. The extreme wrist-pin positions M and N are now got by striking from O the lengths OM = $R + L$ and ON = $R - L$: dead centers OK and OL of the crank are no longer in line or on the same diameter. During forward stroke MN the crank turns through longer arc KGL, during return

stroke NM through shorter arc LHK; then with uniform rotation the average slide velocity is less in forward than in return stroke.

The slide-position diagram is got just as for the symmetrical mechanism, by striking tangent arcs with radius L from M and N. Proof that P′CR′ is the same as MWN is indicated by drawing the (apparent) parallelogram MWCP′, then arguing as for Fig. 355.

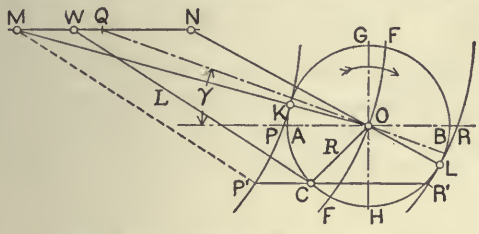

FIG. 367.—Mechanism layout.

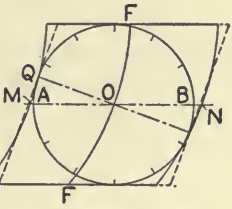

FIG. 368.—Diagram.

Slide position in the offset slider-crank.

Point Q, at distance L from center O, is the logical midstroke reference point; and in the circle diagram this point is represented by arc FF through O. It is evident, most clearly perhaps from Fig. 368, that Q is not exactly midway between M and N, or that the extreme travels QM = OP and QN = OR are no longer equal.

The position diagram is drawn by itself in Fig. 368, with the addition of straight tangents for the associated oblique crossed slider-crank: obliquity is determined by line OQ of Fig. 367.

Prob. 211.—The Offset Stroke Line. (*a*) Use the same dimensions as in Prob. 208, with stroke line offset 3.5″ upward or by amount R. Combine features of Figs. 367 and 368, getting M, Q and N, then producing a complete diagram for both forms of mechanism, with determinations at every 15 deg. beginning at OQ as zero of crank angle. Draw horizontals, not clear across the diagram as in Fig. 356, but from arc FF outward through each C-point.

(*b*) Get the equivalent straight slider-crank by using enlarged radius OM as in Fig. 365 and with it an enlarged rod length L in the same ratio of 3 to 1. For these dimensions draw a diagram like Fig. 356, dividing it at every15 deg. from OA as zero. Since this diagram is symmetrical vertically, space may be saved by constructing only its top half.

Prob. 212.—Slide Movements Compared. From the two diagrams just drawn plot curves like those in Fig. 357, on a *common* angle base to the scale 1 in. = 20 deg. Get three curves as follows:

(*a*) Harmonic motion, dotted line, from the straight tangents in Fig. 368, or from equivalent vertical tangents on the diagram of Prob. 211 (*b*): try a few points both ways, to see that the determinations agree.

(*b*) Actual motion, from arc tangents in Prob. 211 (*a*), like those of Fig. 367. Draw curve in full line.

(*c*) Comparison curve from Prob. 211 (*b*), for straight mechanism with connecting rod. Curve in dash line.

In every case put center O, represented by Q or by arc FF, on base ABA, and draw lines MM, NN, as in Fig. 357, at true crank radius R from the base line, so that excess travel due to obliquity will be clearly apparent. In lay-out it is better to measure from middle than from end of stroke.

SECTION 27. VALVE DIAGRAMS

(*a*) **Slide-valve Mechanism and Movement.**—The simplest form of valve gear used on steam engines adds to the main slider-crank OCW, Fig. 369, a secondary mechanism OEV of the same kind, constituting a kinematic extension of the type in Fig. 19.

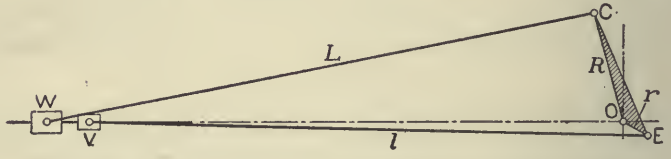

FIG. 369.—Main mechanism and valve drive in steam engine.

Eccentric OE—see Fig. 37 for its true shape—is fast to the shaft, forming with the crank a rigid member. Whereas main-rod length L is commonly from 5 to 7 times crank arm R eccentric

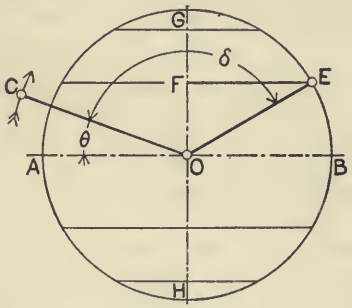

FIG. 370.—Direct diagram.

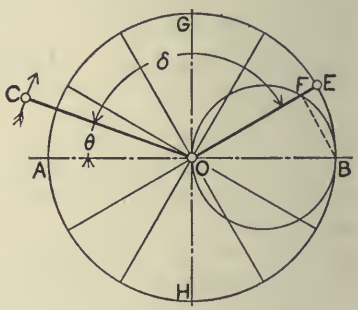

FIG. 371.—Polar diagram

Valve position related to eccentric position.

rod l ranges from 20 to 40 times radius r. For this reason it is usual to neglect the disturbance of valve movement caused by swing of rod, except in cases of particularly close analysis.

In terms of eccentric position, valve position can easily be found and shown by the simple graphical methods developed for the crossed slider-crank. Figures 370 and 371 are direct applica-

tions of Figs. 354 and 358, with small modifications. One difference is due to the fact that distance from midposition (not from end of stroke) is now the useful ordinate: this is given by FE in Fig. 370 and there is no occasion for the stroke-end tangents. Another is that travel toward the right—with cylinder at left as in Fig. 1 or Fig. 369—is now taken as plus, to be shown by a direct intercept OF cut from radius OE; this throws the polar circle to the right, instead of the left as in Fig. 358.

(*b*) **Valve Movement in Terms of the Crank.**—The crank-eccentric member COE being a rigid body, with a fixed angle δ between its arms, we might measure this angle forward from OC

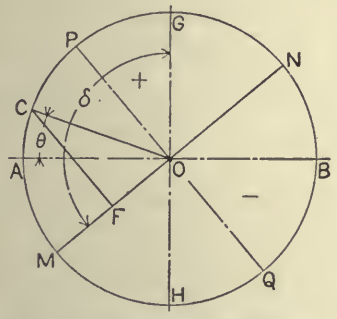

FIG. 372.—Reuleaux diagram.

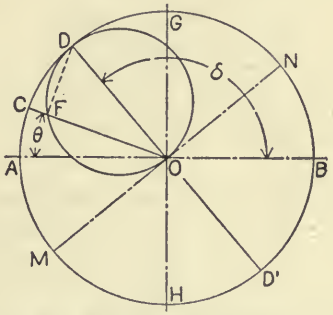

FIG. 373.—Zeuner diagram.

Valve position related to crank position.

to OE or backward from OE to OC in order to get a relation between position of main crank and of valve slide. Such an operation would be very inconvenient and bothersome in routine. To avoid it, to eliminate angle δ and to relate valve directly to crank, we take the determinative part of each diagram and turn it backward through δ until OE falls on OC, at the same time reducing OC to length-equality with OE.

The determinative part of Fig. 370 is fixed reference line GH with arm OE and travel ordinate FE: in Fig. 372 this changes to fixed MN, arm OC and ordinate FC. At first it is well to keep the original diagram, Fig. 370, somewhat in mind, chiefly for a sense of travel direction as indicated by + and − signs on Fig. 372: with use the final form gains increasingly complete meaning. In the change from Fig. 371 to Fig. 373, basal diameter OB comes to a fixed position OD and the two OF's are seen to be the same.

Turning to the layout of the whole engine, Fig. 1, it appears that what we now call plus valve travel is toward the crank shaft.

Further, when the eccentric is at dead center the valve diagram should show maximum travel, which is OP in Fig. 372, OD in Fig. 373. Then a general rule easy to remember is, **Put the eccentric on plus dead center and measure backward the eccentric angle δ so as to locate the corresponding position of crank.**

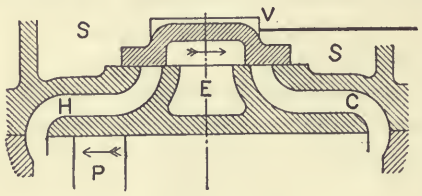

FIG. 374.—D slide valve.
Direct arrangement.

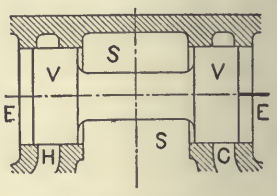

FIG. 375.—Piston valve.
Indirect.

For the Reuleaux diagram base line MN is perpendicular to this crank position, while for the Zeuner diagram diameter OD lies upon it.

(*c*) **The Valve and Its Action.**—Having established methods for completely representing and determining the movement of the valve as a simple slide, our next step is to consider its form and the action that results from this movement. Figure 374 shows a common flat valve at middle position on its seat. As the valve moves to right and left it alternately lets fresh steam from

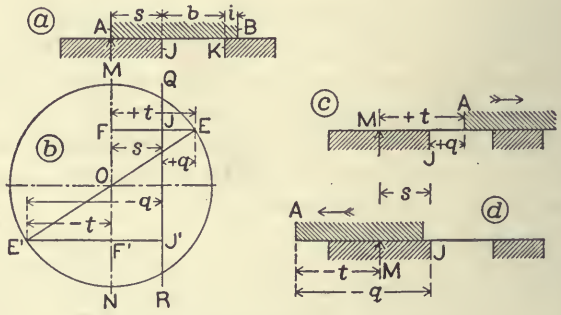

FIG. 376.—Movement of valve edge.

space S into "ports" H and C and lets used or exhausted steam from these passages into exhaust port E. Figure 375 shows a piston valve, equivalent in effect but with a change of arrangement, in that steam S is now in the middle, exhaust E at the ends.

In order to admit steam that will drive the piston toward the right, the valve in Fig. 374 must itself first move to the right,

hence it is called a *direct* valve. Arranged as in Fig. 375 the valve must go to the left in order to send the piston to the right, and this is called an *indirect* valve. Note that a valve face is a good deal wider than the port which it covers, with decidedly greater overlap on steam side S than on exhaust side E.

At (*a*) of Fig. 376 is given an enlarged detail of the valve face on left-end port H, from Fig. 374, with steam lap *s* and exhaust lap *i* marked as dimensions. Diagram (*b*), of the primary form in Fig. 370, is drawn beneath (*a*) in the proper relative position

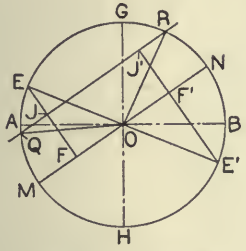

FIG. 377.—Reuleaux.

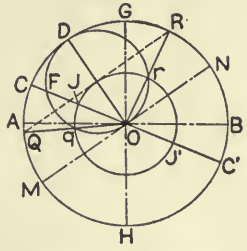

FIG. 378.—Zeuner.

Lap lines and port opening.

to show the movement of valve edge A with respect to its mid-position M. Obviously, however, the matter of real interest is the movement of edge A with respect to port edge J. This is determined by drawing a new reference line QR, parallel to MN at distance *s*: then JE is edge distance *q* just as FE is travel *t*. Random but opposite eccentric positions OE and OE′, locating the valve as shown at (*c*) and (*d*), illustrate the general statement that $q = t - s$. Then besides calling QR a new reference line we may say that it makes a continual subtraction of *s* from *t*, complete for the whole rotation of the eccentric.

The rotated or Reuleaux diagram like Fig. 372 is shown in Fig. 377; and it is now made clear that valve opens port when crank is at OQ, closes it when crank is at OR. In the Zeuner diagram, Fig. 378, a new reference point J on diameter CC′ is supplied by drawing a lap circle with radius OJ = *s*; and this circle makes a continual subtraction of *s* from *t* so as to give JF = *q*.

(*d*) **Illustration of Valve Action.**—The complete cycle of valve movement and its effect at all four edges can most effectively be shown by means of the angle-base diagram in Fig. 379. For the valve at (*a*) and the eccentric setting at (*b*) main drawing (*c*) is plotted after the type of Fig. 357. It is further related to the

Reuleaux diagram (*d*) and Zeuner diagram (*e*): as will be more fully explained presently, these are the real service forms, whose meaning is to be made clearer by drawing (*c*).

In Fig. 379 (*c*) the vertical scale is degrees of rotation, marked for the crank at the edges of the diagram. Curves A, B, V, C

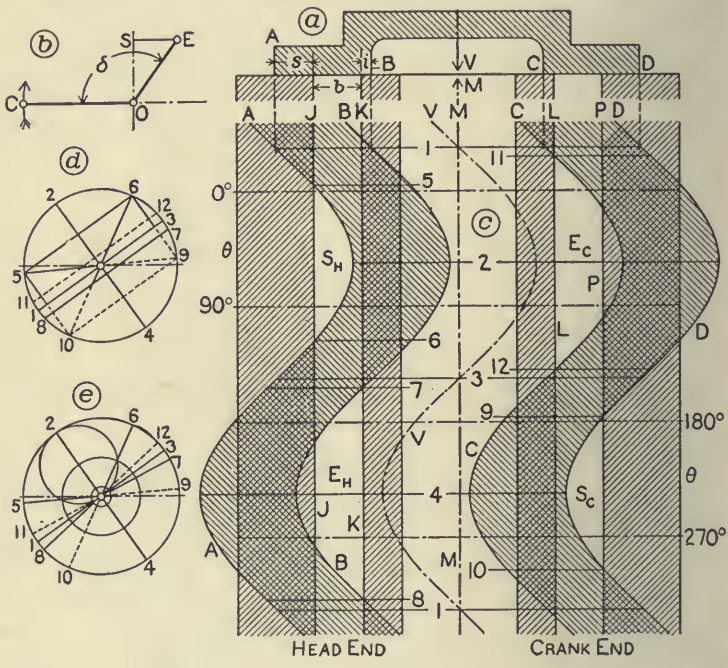

FIG. 379.—Complete showing of valve action.

and D are all alike or horizontally equidistant, but V is the only one referred to its middle M or center of symmetrical travel. Valve edge A is referred to port edge J, B to K, and so on. For any position of the mechanism the valve is located by drawing a horizontal at the particular crank angle. The scheme of marking special positions is, 1, 2, 3, 4, middle and extreme positions of the valve, or 90, 180, 270 and 0 deg. of eccentric travel; 5, 6, 7, 8, opening and closing of the head-end port H; 9, 10, 11, 12, opening and closing of crank-end port C; these will be more fully considered in the next article. Clear areas S_h and S_c show the openings for steam, E_h and E_c the openings for exhaust.

For the one purpose of showing valve movement as related to shaft rotation, the straight-base plot in Fig. 379 (*c*) is the best

and clearest diagram that can be made. With the more complex mechanisms that have to be plotted point by point it is the best diagram to use. But with the simple eccentric drive, for which the whole cycle of movement can be at once determined by a comprehensive geometrical construction, the circle diagrams thus got are naturally the more effective and useful forms. They contain all the information conveyed by Fig. 379 (*c*), are more compact and flexible, and are much easier to draw.

(*e*) **Complete Service Diagrams.**—Corresponding with the proportions in Fig. 379, complete diagrams of the two forms are given in Figs. 380 and 381. The Reuleaux diagram has two sets

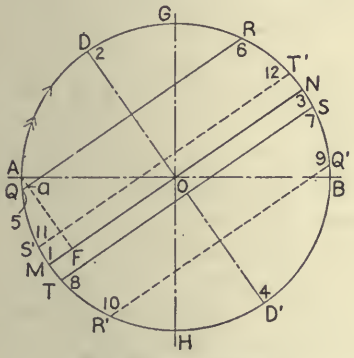

FIG. 380.—Reuleaux.

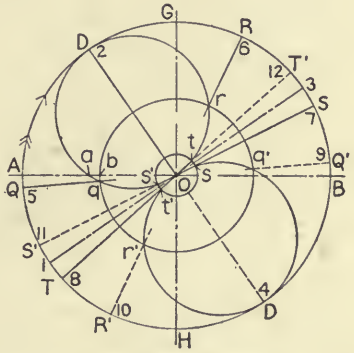

FIG. 381.—Zeuner.

Complete valve diagrams.

of lap lines, full for head end, dotted for crank end. On the Zeuner diagram, unless the laps are not the same for the two ends, one circle shows both steam laps, another both exhaust laps. A second circle on diameter OD′ serves to give left-hand travel by direct ordinates on the crank radius.

Starting at M, with valve in mid-position, go around both diagrams in clockwise order, tracing action for the head-end port and comparing with Fig. 379, as follows:

At Q the travel $t = s$ puts the valve and port edge-on-edge; since valve is moving out or to the right (because ordinate t increases as crank advances), it opens the port or begins *admission*.

At dead center A the port is open by a small amount aA or ba called the *lead*, with symbol e. The particular travel FA = Oa = $s + e$ is what really determines angle δ, as seen in Fig. 379 (*b*).

Passing extreme travel at D the valve returns, *cutting off* steam at R where it is again edge-on-edge, but with right-hand ordinate t now decreasing.

In order to open the port for exhaust or *release* the valve must be at distance i to the left and moving out: obvious in Fig. 380, this condition is determined on the Zeuner diagram by intersection s′ on the upper circle or s on the lower.

If desired the overtravel of exhaust edge of valve past the far edge of port can be shown, by drawing a parallel to ST at distance b in Fig. 380 or by another circle at radius $i + b$ in Fig. 381.

Exhaust is shut off at T, and because steam remaining in the cylinder will be compressed by the returning piston this is called the beginning of *compression*.

In brief, the four events Q, R, S and T or 5, 6, 7, and 8 are called admission, cut-off, release and compression. The chief quantitative functions of a valve diagram are to give the timing of these events and to show the amount of port opening, especially on the steam side where large lap cuts down this opening.

Since the relation of valve action to piston movement is what really determines steam action, piston movement as caused by the crank must continually be kept in mind. In Fig. 390 (following) the piston diagram from Fig. 355 is shown in close combination with the valve diagram; while in Fig. 391 a plotted relation curve for valve and piston is given.

(*f*) **Geometrical Considerations.**—Between the two forms of valve diagram there is little ground for preference either way. The Reuleaux is perhaps a little clearer and more exact for showing a single eccentric action, as in all the diagrams thus far drawn. When the effect of change of eccentric is to be shown, by a combination of several diagrams in one, the Zeuner form is a little clearer and easier to manipulate.

It is best to combine the properties of the two constructions, using them together as most convenient. Thus the Zeuner diagram is inaccurate for short travels or near mid-position, center O with intersection s or t giving a poor determination of crank position OS or OT in Fig. 381: consequently the Reuleaux lap line, tangent to the i-circle, should be used as a check or as the real determinant, after the manner of Fig. 382.

Given r, s and e, with δ required, vertical FD is the obvious means of solution, whichever kind of diagram is to be drawn.

Again, with r, e and cut-off OR as data, the fact that lap line RQ is at distance e from dead center A is the useful property.

A third form of diagram, which has been a good deal used and should be understood, is given in Fig. 383. This takes the far end N of the Reuleaux base line for a pole, giving travel t by the perpendicular distance NF from N to the crank diameter. The laps are represented by circles from N as center, determining

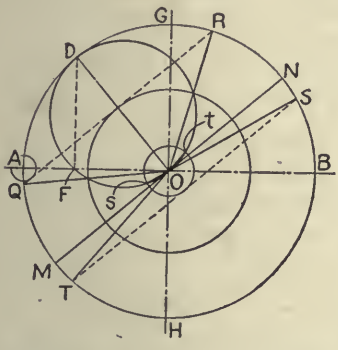

FIG. 382.—Geometrical relations.

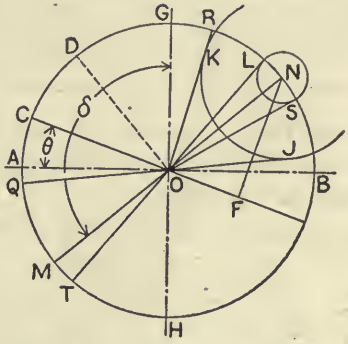

FIG. 383.—The Bilgram diagram.

the four events by tangencies JOQ, OKR, OS and LOT. Note that the locus of F is a circle on ON as diameter. Formulate your own rule as to the directional meaning (valve to right or left) of polar ordinate NF. The diagram has no particular advantages to compensate for diminished simplicity of representation, except perhaps that it is more accurate than the Zeuner diagram for short travels.

All of the diagrams thus far given are for the direct valve, Fig. 374, with eccentric OE in the second (moving) quadrant ahead of crank and the Zeuner circle in the first quadrant of the fixed diagram. Use of an indirect valve or the interposition of a reversing rocker arm—see Fig. 38—requires a diametrical reversal of the eccentric, into the quadrant following the crank; the valve circle then goes into the third quadrant, or the second, dotted circle of Fig. 381 becomes the primary circle. In the Reuleaux diagram there an interchange of lap lines, as between head and crank ends.

The events of valve action can be located either by crank angle or by position of piston on stroke line. Unless otherwise specified, in problems like the following get angle from piston position

on the assumption of harmonic motion, or as if for a crossed slider-crank.

Prob. 213. In Fig. 370 it is obvious that travel t is given by the equation

$$t = -r \cos (\theta + \delta). \qquad (104)$$

By juggling angles show that this same formula is true for FC in Fig. 372, OF in Fig. 373 and NF in Fig. 383.

Prob. 214. Given r, δ, e, i and b, draw a diagram of each kind with the completeness of Figs. 380 and 381. Draw for a vertical engine, with $\delta = 135°$ and $r = 2''$, $e = \frac{3}{16}''$, $i = \frac{5}{16}''$ and $b = 1\frac{1}{4}''$ as sample data.

Prob. 215. Given r, s and e, find δ and cut-off.

Prob. 216. Given r, e, cut-off and compression, find δ, laps and release.

Prob. 217. Given s, e and cut-off, find r and δ.

(*g*) **Varying Action of Valve.**—A very important item of valve action is the length of time during which the port is open. This depends upon the relative size of valve lap and eccentric radius. If there were zero lap the port would be open through just one

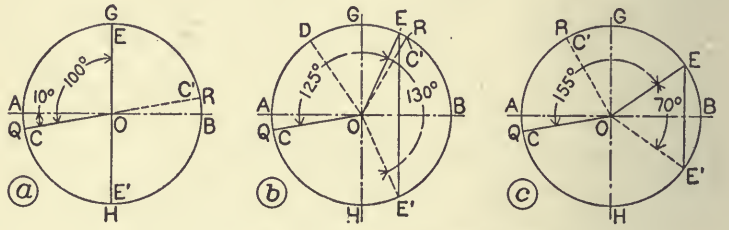

FIG. 384.—Vary lap to vary period of opening.

half of the revolution, closed through the other half. Such a condition is shown by Fig. 384 (*a*), which is a primary diagram for the eccentric itself, like Fig. 370. The arc of opening is GBH for the eccentric, then the introduction of lead QOA makes it QGR for the crank.

The period of opening is shortened by putting lap on the valve, with accompanying change in setting angle δ so as to keep the timing right, especially for admission; and length of opening might be controlled by varying the amount of lap.

In Fig. 384 (*b*) the requirement is, cut-off at three-fourths of stroke, or at 120 deg., with 10 deg. of lead. This means that the time of opening, angle EOE′ for the eccentric or QOR for the crank, must be 130 deg.; then bisecting QOR so as to locate Zeuner diameter OD we find AOD = 55 deg. or $\delta = 125$ deg. At (*c*) is shown the result of a similar adjustment to cut-off at

one-fourth of the stroke, with OR at 60 deg., requiring a large increase of steam lap.

Now to construct a four-function slide valve that can vary its dimensions while in service is impracticable. The natural alternative is to keep lap constant and vary radius r. Thus in

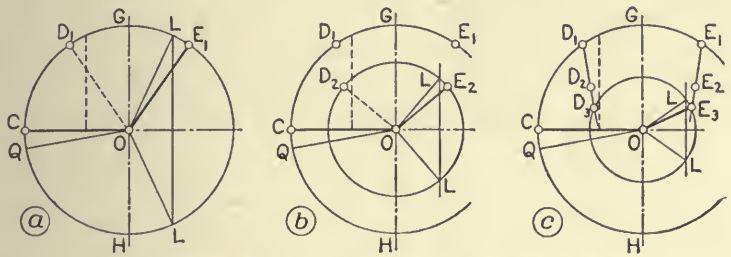

FIG. 385.—Vary and adjust radius to vary cut-off.

Fig. 385 (*a*) we start with the three-fourths cut-off case of Fig. 384 (*b*), showing OD_1 and placing the crank on dead center so as to locate OE_1 in a position symmetrical with OD_1. Case (*b*) is for half-stroke cut-off, or for an opening of 90 + 10 deg.: with a fixed location for lap line LL this value of 100 deg. for angle LOL determines radius OE_2 and makes $\delta = 140$ deg. Case (*c*) corresponds with (*c*) of Fig. 384, having the radius shortened to OE_3 but the angles unchanged. Either center E or point D has a definite path or locus of change.

The example in Figs. 384 and 385 is worked for a constant angle of lead. Linear lead itself can be kept constant by shifting E

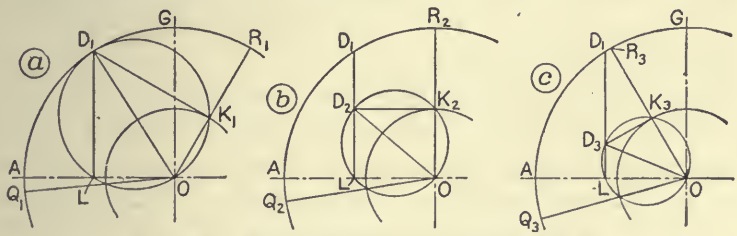

FIG. 386.—Zeuner diagrams for variable cut-off.

along a straight line at right angles to the crank arm or D along a vertical D_1L in Fig. 386. Here we pass to valve diagrams, preferring the Zeuner form, and work for the same three cut-offs as in Fig. 385. Having a D-locus the determinant for a particular D-point is the tangent KD from cut-off intersection K (radius

with lap circle). Usually the several diagrams here separated for clearness are combined in a single figure. Note the considerable variation of lead angle.

In a shaft or flywheel governor—so called because it is mounted on the shaft or in the flywheel of an engine—the geometrical effect is a controlled movement of eccentric center E along an arc path E_1E_0, Fig. 387. Point P is pivot fixed in the rotating plane of the wheel or crank shaft. The data needed for a valve-action problem are the coordinates a and b of point P, length Q of the eccentric "pendulum" PE and the limits of locus

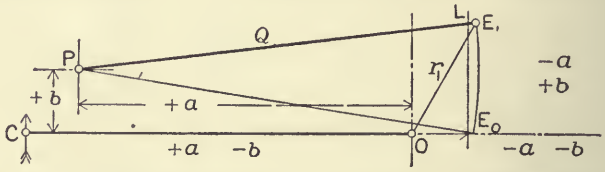

FIG. 387.—Dimension of eccentric "pendulum" in shaft governor.

E_1E_0: for the latter we give maximum radius r_1 and assume that E_0 is on the crank arm CO. At present we are not concerned with the form of the governor mechanism or with the force action whereby a small variation of speed accommodates engine power to imposed load.

The manner of lead variation, at once made apparent by drawing lap line LL on Fig. 387, is of minor importance, and a discussion of its effect would involve questions of steam engineering. Generally, whether in a shaft governor or in one of the less simple linkage valve gears which we shall take up later, a variable eccentric center has a locus which approximates a straight line perpendicular to the crank arm.

Prob. 218. The following data are from actual engines:

(*a*) Direct valve, $a = +14''$, $b = 0$, $Q = 15''$, $r_1 = 1\frac{3}{4}''$, $s = 1\frac{3}{16}''$, $i = 0$.

(*b*) Indirect valve, $a = -1\frac{1}{2}''$, $b = -\frac{7}{8}''$, $Q = 2\frac{3}{4}''$, $r_1 = 1\frac{7}{8}''$, $s = 1\frac{1}{16}''$, $i = \frac{1}{8}''$.

(*c*) Direct valve, $a = +5\frac{1}{4}''$, $b = +\frac{3}{4}''$, $Q = 6\frac{1}{4}''$, $r_1 = 1\frac{5}{8}''$, $s = 1''$, $i = \frac{1}{4}''$.

For one of these valve gears draw diagrams for latest cut-off (with OE_1), for earliest cut-off (with OE_0) and for cut-off at one-third of stroke. Make both Zeuner and Reuleaux diagrams. Show and discuss variation of the exhaust events.

(*h*) **Exact Valve Movement.**—The Reuleaux diagram lends itself readily to a determination of the effect of swing of the

eccentric rod upon valve movement. In Fig. 388, which is in the same terms as Fig. 370, a reference arc for mid-position is drawn, similar to FF in Fig. 355. Rod length l is taken at twenty times radius r, as a representative ratio. The arc, however, is not passed through center O but through the lead points F_1 and F_2, which correspond with the two dead centers of crank. The effect is to place mid-position of the valve halfway between the

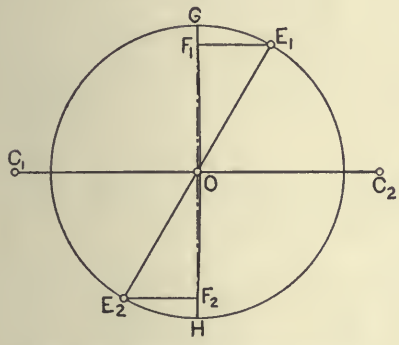

FIG. 388.—Effect of eccentric rod.

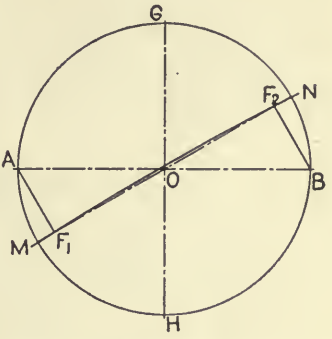

FIG. 389.—Modified Reuleaux diagram.

lead positions rather than halfway between ends of stroke: to represent the latter point arc FF would go through center O. In the figure the greatest difference between straight line and curve is a little over two per cent of eccentric radius r, this affecting valve positions near ends of stroke.

Figure 389 shows reference line GH and its replacing arc swung into the Reuleaux position MN. It can be seen that if the valve is set with equal leads the width of port opening will be a little less for head end than for crank end. Questions suggest themselves as to the effect of change to indirect eccentric setting and of the movement of a variable eccentric center. Investigation of such minor matters belongs to a higher degree of specialization in the particular mechanism than we wish to undertake here.

(*i*) **Relative Movement of Piston and Valve.**—All of the movement diagrams thus far presented have had the function of relating the position of a single slide to that of a crank—either the crank that actually drives it or another crank on the same shaft. Now comes the problem of relating two such slides, when driven by rigidly connected cranks. One type of relation is well

exemplified by the piston and valve of the steam engine, where it is a question of showing coincident absolute movements.

In Fig. 390 the piston diagram of Fig. 355 and a Reuleaux valve diagram are drawn together, on the same axes. At any crank position CC′ piston travel is s = M′C′ and valve travel

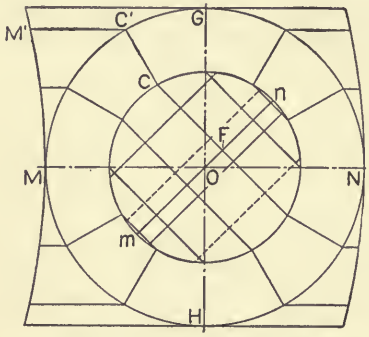

FIG. 390.—Combined circle diagrams for piston and valve.

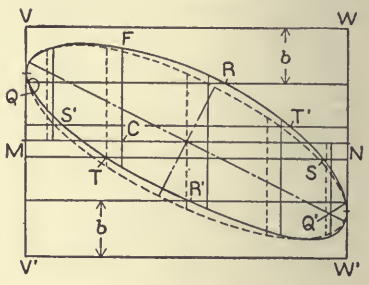

FIG. 391.—Plotted diagram relating valve movement to piston movement.

is t = FC. In the plotted diagram of Fig. 391 these two coordinates are laid out as MC = s and CF = t, giving point F on the curve of relation. With harmonic motion for piston as well as crank the curve thus obtained would be an ellipse, which can be proven as follows:

Use middle of stroke for origin, so that coordinates of a point on the curve will be $x = R \cos \theta$ and $y = -r \cos (\delta + \theta)$: then in $y = r \sin \delta \sin \theta - r \cos \delta \cos \theta$ substitute $\cos \theta = x/R$ and $\sin \theta = \sqrt{R^2 - x^2}/R$ and after reduction get

$$R^2y^2 + 2rR \cos \delta \, xy + r^2x^2 - r^2R^2 \sin^2 \delta = 0, \qquad (105)$$

which is the equation of an ellipse with its center at the origin and its major axis at the inclination γ given by

$$\tan 2\gamma = \frac{2rR \cos \delta}{R^2 - r^2}. \qquad (106)$$

The pushing of t-ordinates toward the right, due to swing of connecting rod, swells the upper part of the curve and flattens the lower part. The most striking result is the marked inequality of cut-off lengths QR and Q′R′ that are secured with equal laps—here somewhat exaggerated because L/R is taken at the low value 4. An attempt to make cut-offs equal by increasing lap on head-end side and decreasing it on crank-end side would unbalance the amount of port opening over the whole range in

both ends. Again we have come to a special problem, belonging to steam engineering rather than to mechanism.

Prob. 219. For the valve-gear data in Prob. 214 and with $L/R = 5$ for the engine, plot diagram like Fig. 391. Change to horizontal stroke line and use 8-in. base to represent stroke of piston.

(*j*) **The Double Slide Valve.**—The valve arrangement sectioned in Fig. 392 will serve as a typical case of two slides in such close combination that their immediate relative motion, one with respect to the other, is the thing sought. Main valve V_1 controls admission and the whole exhaust operation. Its ends are extended so as to enclose ports P, P, and cut-off is effected by the

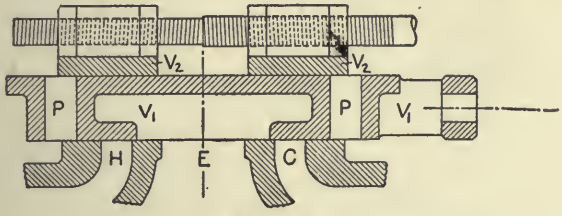

FIG. 392.—The Meyer double slide valve.

second or riding valve V_2, which closes P before this port is shut off from H or C. Here the second valve is driven by a fixed eccentric, and for hand-control of cut-off the laps are varied by means of the right-and-left screw on the valve rod. Sometimes the cut-off valve is solid and is driven by a shaft governor. Always there is a fixed eccentric for the main valve. Note that the cut-off valve has negative laps, failing to cover its ports when in mid-position on the first valve. Connected to their eccentrics, the two valves never get both into mid-position at the same time as here shown.

(*k*) **Actual and Virtual Eccentrics.**—In Fig. 393 the eccentric radii are shown in relation to crank OC, OE_1 for main valve V_1 and OE_2 for cut-off valve V_2. The respective travels, from a middle mark on the seat, are $F_1E_2 = t_1$ and $F_2E_2 = t_2$. But the matter of interest regarding the second valve is not so much its absolute motion on the seat as its relative motion on the first valve. If lines always vertical are passed through the moving eccentric centers, MN through E_1 and QR through E_2, it is evident that the horizontal distance between these lines will be the displacement of V_2 from mid-position on V_1.

In Fig. 394 this distance is shown at F_2E_2, and is seen to be the horizontal projection of E_1E_2 just as F_1E_1 is that of OE_1. Since E_1E_2 is fixed in the plane of OC and OE_1, it rotates exactly as do both of those lines, except that one end of it is not a fixed center of turning. But the fact that E_1 moves on and with MN does not influence the rotational effect of radius E_1E_2 upon the distance between MN and QR.

Now for purposes of simple graphical representation by diagram it is necessary to have a radius turning on a fixed center. This is got by completing the parallelogram of eccentrics $OE_1E_2E_3$,

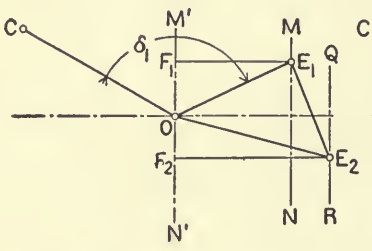

FIG. 393.—Relative eccentric.

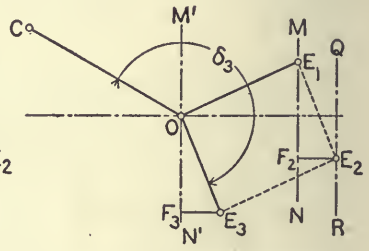

FIG. 394.—Virtual eccentric.

Analysis for movement of second valve on first.

and using the *virtual* eccentric OE_3. The latter is equal and always parallel to E_1E_2, and would give to a valve on a fixed seat the same movement that the relative eccentric E_1E_2 gives to the riding valve on its moving seat. The only geometrical proof needed is the evident equality of F_3E_3 and F_2E_2.

(*l*) **Diagrams for the Double Valve.**—Zeuner diagrams, both showing absolute movement, are drawn in Fig. 395 (*a*) and beneath is a sketch of the valves for crank position OC. Arrows on blocks V_1 and V_2 show directions of motion and also, by their lengths, give an approximate idea of comparative magnitude of velocities. Both travel intercepts, $OF_1 = t_1$ and $OF_2 = t_2$, are positive or to the right. Differences in slant of arc at the places where the two valve circles cross the crank radius show that V_1 is moving out more rapidly than V_2 is moving in.

Final valve diagrams are given at (*c*) of Fig. 395. The main valve circle on OD_1 is the same as above, and its lap circle AB determines admission at OQ_1 and cut-off at OR_1. The virtual circle on OD_3 is drawn in proper relation to OE_3 and lap circle GH fixes cut-off at OR_2 and reopening of port P at OQ_2. In order to cut off valve V_2 must be to the left on V_1 and going to the

left: intercept OH, increasing as the crank advances, meets these requirements; while the decreasing intercept OG corresponds with reopening. Having negative lap, the cut-off valve keeps its port closed during less than half of the revolution. This is immaterial, provided only that it does not reopen before the main valve cuts off.

For the same purpose of showing laps, Fig. 396 (*a*) is drawn like Fig. 392, with both valves in mid-position, even though this

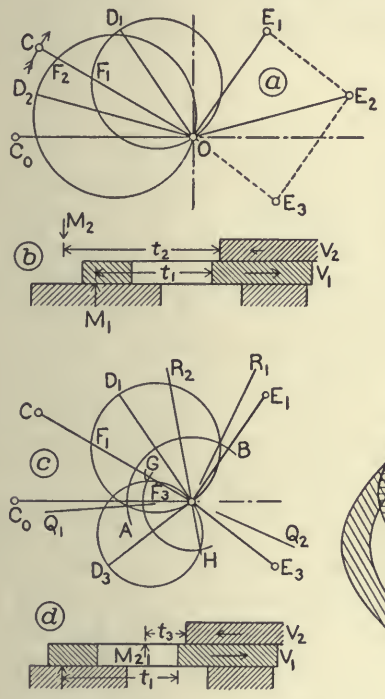

FIG. 395.—Absolute and relative movements.

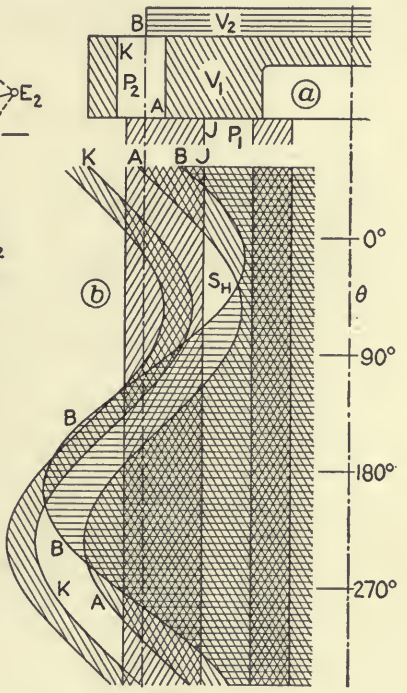

FIG. 396.—Developed diagram for both valves.

does not correspond with any crank position on the unrolled diagram beneath. For the main valve this second diagram is laid out just like Fig. 379 (*c*) (except for omission of exhaust edge), from the main valve circle on OD_1. For the cut-off valve curve B is plotted from circle OD_2 of Fig. 395 (*a*), since absolute movement on the plane of the paper is what is wanted.

The advantage of this type of valve gear is shown by the shape of clear area S_h. With short opening there is yet wide

opening and rapid movement of valve edges in both admission and cut-off. The reason is that both events take place with a valve comparatively near mid-stroke, hence moving rapidly.

Prob. 220. For the main valve let $r_m = 2''$, $\delta_m = 120°$, $s_m = \frac{7}{8}''$, $i_m = \frac{1}{4}''$; for the cut-off valve, virtual eccentric $r_v = 1\frac{1}{2}''$ and its angle $\delta_v = 210°$, with lap $s_v = -\frac{3}{4}''$.

First draw the two diagrams separately, determining full action of main valve and both cut-off and reopening by the riding valve. Then draw the valve circles together as in Fig. 395 (*c*) and for crank at 0, 30, 90, 180 and 270 deg. make sketch diagrams like Fig. 395 (*b*) or (*d*). Show position of valve and by arrows indicate direction of movement of each, both absolute and (for the riding valve) relative.

SECTION 28. PRINCIPLES OF MECHANISM

This section will carry forward, elaborate and round out the group of ideas introduced in Sect. 2. The key words for that group are **kinematic pair, kinematic chain** and **mechanism,** stopping short of the complete machine with its full design for function and for strength. We now go on to the variation and equivalence of mechanisms, continuing to use common examples, then take up more fully the question of constraint of motion.

(*a*) **Inversion** is the name for a kind of interchange of form or of function that can take place in pairs, in mechanisms and in kinematic chains.

Generally a lower pair consists of a solid and a hollow body in close surface contact, as stated at the end of Sect. 2 (*b*). Frequently a prevailing arrangement as to outer and inner parts has over against it a considerable group of exceptions. Thus a shaft carrying pulleys or gears commonly runs in hollow, fixed bearings; but the loose pulley on a countershaft, Figs. 103 to 106, or the main driving gear of a press, Figs. 166 to 170, turns upon a stationary shaft when running idle; and the quill shaft in a lathe backgearing, F in Fig. 126, is a conspicuous example of a hollow shaft turning upon a fixed solid shaft in active service. Again, the wrist pin of an engine is commonly fast in the crosshead and the connecting rod oscillates upon it; but occasionally the pin is fast in rod and has bearing in the crosshead.

One scheme of mechanism analysis separates a machine into driver, connecting or transmitting system, and follower. From this viewpoint a hoisting train such as Fig. 2 suffers functional inversion every time it changes from lifting to lowering under

load; for in reversed running the drum becomes driver and the motor shaft follower, unless we consider the intermediate shaft that carries the brake as true follower. Beside this functional inversion in service there is also a similar inversion in design or application, well exemplified by Fig. 397. There the engine mechanism or slider-crank is shown first as used in an engine with slide 4 driving, then as applied in a press (or pump) with crank 2 driving.

(*b*) **Inversion of Mechanism.**—The variations thus far described may be considered superficial, above what is kinematically an essential sameness. The deeper variation suggested

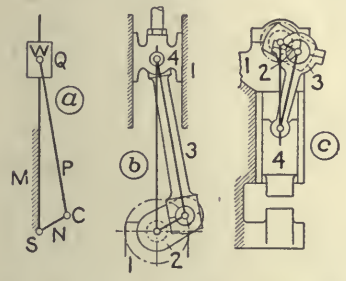

FIG. 397.—The engine mechanism, primary form.

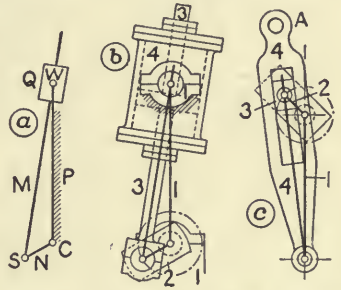

FIG. 398.—The connecting-rod member as frame.

in Sect. 2 (*g*) is a more important matter. There it is stated that a mechanism is derived from a kinematic chain by fixing proportions and choosing frame member: and it is with this last item of choice that we are now concerned.

All possible inversions of the engine mechanism are shown by Figs. 397 to 401. In order to keep track of changes, the skeleton at (*a*) of each figure carries the same lettering throughout, on members and at turning joints. Designation of the joints is appropriate to the engine arrangement, with S at shaft, C at crank and W at wrist pin; and the members are named according to this most prominent application. The drawings of machines have the members numbered in circular order, beginning with frame 1 and putting 2 on the piece that has uniform rotation; except that in Fig. 401, which is not a useful machine, member 2 is the one supposed to receive driving force.

Figure 398 (*b*) shows the oscillating engine, with slide member Q elaborated into a cylinder on trunnions and the common engine frame changed to a piston and rod: this striking example

of pair inversion is emphasized by sketches of the two forms of member M in Fig. 402. The crank-and-lever mechanism in Fig. 398 (*c*) is used for a quick-return drive, with suitable extension beyond A.

In Fig. 399 the short member with two turning elements, the crank of the engine, becomes frame. This Whitworth quick-

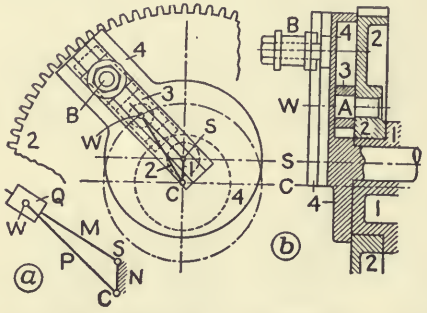

FIG. 399.—Whitworth quick-return motion.

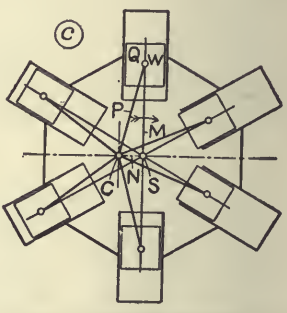

FIG. 400.—Rotating-cylinder airplane engine.

return motion is used in several machine tools. Gear wheel 2, turning on axis C and carrying pin A, corresponds with the connecting rod of the engine and acts as driver. Piece 4 on

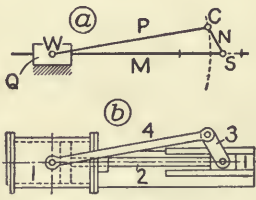

FIG. 401.—The slide block as frame.

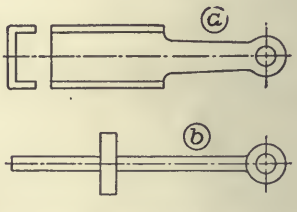

FIG. 402.—Illustrating pair inversion.

axis S is driven with variable angular velocity and carries crank pin B which can be adjusted to various radial distances from S, to change length of stroke of the slide which it drives.

The special engine outlined in Fig. 400 is sometimes used in airplane service. The cylinders are mounted upon what is structurally (if not in our usual kinematic sense) a wheel-like frame which turns on axis S with uniform rotation, while the connecting rods take hold of fixed crank pin C.

The fourth case, got by holding fast the member equivalent to the slide block of the common engine, does not have any practical

application—but compare it with Fig. 507. Figure 401 (*b*) shows an engine which might be made to work, but the lack of any fixed axis for the member 3 which has complete rotation prevents its use as a crank shaft.

As regards the inversion illustrated by Fig. 402, to which allusion has already been made, note that form (*a*) represents not only the frame in Fig. 397 (*b*) but also the slotted lever in Fig. 398 (*c*).

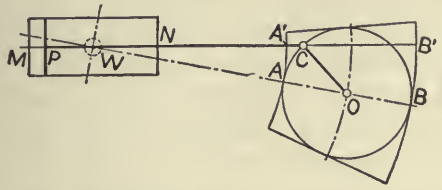

FIG. 403.—Piston diagram for the oscillating engine.

(*c*) **Piston Movement in Inverted Engines.**—At this point it is appropriate to see how the diagrams for piston position, as developed in Sect. 26, are affected by inversion of engine mechanisms. The outline of an oscillating engine in Fig. 403 has the piston reduced to a disc without thickness and distance CP is made equal to OW. But whether or not the piston thus has W for the middle of its stroke, the end positions M and N are determined by putting the crank on dead centers OA and OB and making AM = BN = CP. Then if tangent arcs AA′ and BB′ are

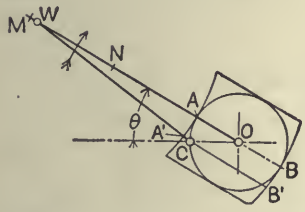

FIG. 404.—Figure of mechanism.

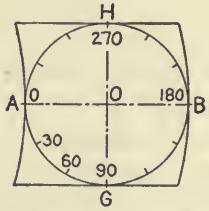

FIG. 405.—Service diagram.

Piston diagrams for the rotating-cylinder engine.

struck from W and a radial line WA′CB′ is drawn through any crank position C, the location of C on A′B′ will be the same as that of piston P on stroke line MN.

A single unit of the rotary-cylinder engine of Fig. 400 is represented by outline OWC in Fig. 404. In the common engine the crank is what may be called the **master member,** whose position on the frame is datum for the instantaneous form or

configuration of the mechanism. As regards relative motion the inversion in Fig. 404 is a simple case, in that we have merely interchanged frame and master member. Imagine a plane fast to rotating member OW, carrying the diagram of Fig. 355: then fixed crank OC turns on the rotating diagram just as it would if the latter were fixed and the same rate of relative turning preserved between OW and OC. Angle θ runs relatively backward, however, so that for consistency the angle scale must be reversed in the fixed diagram, Fig. 405.

(*d*) **Relative Motion.**—This one simple example of inversion, run through completely in Figs. 397 to 401, is an excellent illustration of a very important principle of kinematics. If (*a*) to a kinematic chain we give definite proportions, (*b*) make one member the reference member by attaching to it a plane of representation and (*c*) depict the other members on that plane; then a particular change in the relative position of one member will bring all the others into a definite new configuration, whether the reference plane is at rest or moving. In other words, with given proportions of members relative movement is independent of absolute movement. For determinative purposes it is, of course, far more convenient to have the reference member at rest; but this is for convenience, not because it gives a particular form of motion.

The idea and perception of relative motion, especially as between moving bodies or machine members, plays an important part in kinematic study and analysis. It is a concept to be realized, not a conclusion to be reasoned out.

(*e*) **A Notation for Simple Chains.**—We now turn toward a more general view of the subject of linkage mechanisms, building upon the foundation in Sect. 2 (*g*) to (*i*). Continuing at first the discussion of simple kinematic chains, a convenient and concise way of expressing their composition is desirable. This is secured by using something like a chemical formula, in terms of the kinematic pairs. Let T stand for turning, S for sliding, K for screw and H for higher pair, and use subscripts to indicate numbers greater than one. Then the slider crank is (T_3S), the crossed slider-crank is (T_2S_2) and the quadric with only turning joints is (T_4); while (S_3) stands for Fig. 21 and (TSK) for Fig. 22. The parentheses signify that a chain is represented, and without them the same notation can be used for links, the slider-crank chain containing two T_2 and two TS links.

A variation of the scheme is to use $(TS)_4$ to represent a general lower-pair quadric or $(TH)_3$ for gear and cam units such as Figs. 60 to 63 and 71 to 85, the external subscript showing number of links. The reason for not counting the roller on a cam follower will be given later, in Art. (*l*).

(*f*) **Various Four-joint Quadrics.**—The chain (T_4) with four turning pairs has many applications and various forms. Most symmetrical is the jointed parallelogram in Fig. 406 (*a*), which represents the coupled cranks on a pair of locomotive driving wheels. The "parallelogram" can also be crossed as at (*b*), where DC turns opposite to AB at a relatively variable rate. Figure 407 shows the short side of a quadrilateral held fast and the mechanism so proportioned that both of the long members jointed

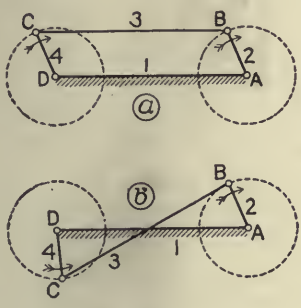

FIG. 406.

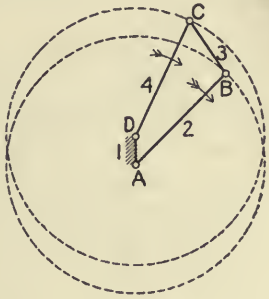

FIG. 407.

Jointed quadric chains for complete rotation.

to the frame can have complete rotation. If crank 2 turns uniformly crank 4 will have variable angular velocity; and the device is sometimes employed instead of the Whitworth motion as a quick-return drive for the "slotter" or vertical planing machine.

The Corliss valve gear outlined in Fig. 408 furnishes examples of four distinctive jointed quadrics. Unit I or ABCD has in eccentric AB a short crank arm which makes a complete rotation and through rod BC gives a symmetrical oscillation to the rocker arm CD. In Unit II or DEFG this oscillation is transmitted to wrist plate 6, which (so far as its function in this section of the mechanism is concerned) is sufficiently represented by radius GF: the motion of GF is similar to that of DE but is changed in angular magnitude. The valve arms, oscillating on axes K, P, N and Q, are driven by the wrist plate through connecting rods; but no attempt is here made to show the releasing gears whereby

arms 8 and 10 alternately take hold and let go of their valves. Quadric units III and IV differ in that quadrilateral GHJK is open while GLMN is crossed: both however are so proportioned that the oscillations of arms KJ and NM are decidedly unsymmetrical with respect to the mid-position in which the mechanism is drawn. This valve gear has the special feature that with the

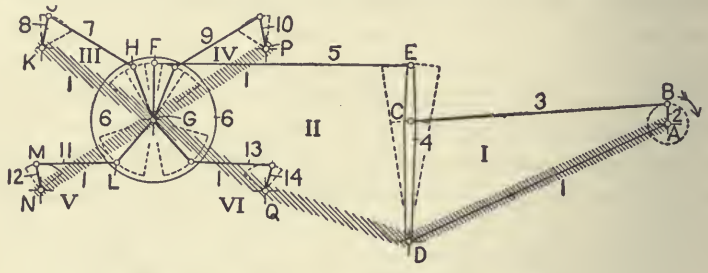

FIG. 408.—Mechanism of Corliss valve gear.

high number of 14 members it is made up wholly of quadric units, six in number. As to multiple-pair kinematic links, note that 1 enters into seven pairs, 4 into three and 6 into six.

(*g*) **Further Quadric Variations.**—Running over Figs. 397 to 401 we see that the four inversions there given are distinctive because they involve proportions as well as kinds of links. In chain (T_3S) there are only T_2 and TS links, in a symmetrical arrangement; then as regards kind of frame and sequence of other members, cases 1 and 4 of that group are alike in having a TS base while cases 2 and 3 have a T_2 base.

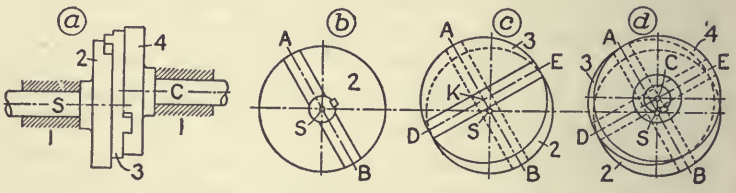

FIG. 409.—The Oldham coupling.

Chain (T_2S_2), in the form (TTSS) and based on one of the TS links, has been shown in Figs. 350 and 351. With link SS for frame it becomes the geometrical ellipsograph in Sect. 30, where a striking example of inversion will also be found. Based on its TT link this mechanism has the application in Fig. 409, forming a flexible coupling to transmit uniform angular velocity between

shaft sections which may be out of line. Side view (*a*) gives the whole mechanism while end views (*b*) to (*d*) show successive assembly. The TS links take the form of slotted discs 2 and 4 keyed to the shaft ends and cross 3 is an intermediate disc carrying two raised slide bars at right angles with each other. Center K of the cross travels in a circle of which line CS is a diameter. Since the two slides AB and DE have an invariable angle between them the two shafts must turn exactly together.

In Fig. 410 three mechanisms having the same purpose and very nearly the same effect show interesting kinematic variation.

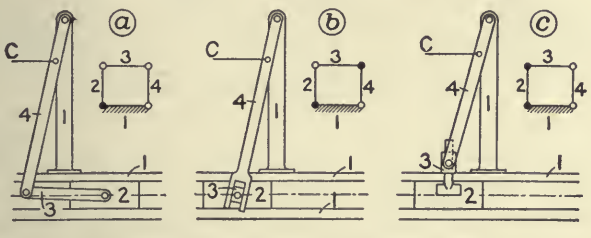

FIG. 410.—Reducing-lever indicator rigs.

The purpose is to give at cord C a miniature copy of the motion of engine crosshead 2, this cord to actuate the paper drum of a steam-engine indicator. We have here chains (T_3S) and (T_2S_2) arranged for oscillating motion only. Incidentally, the three possible locations of sliding pairs in a quadric chain are illustrated. Cases (*a*) and (*c*) correspond with the two slider-cranks while (*b*) shows the other possible arrangement (TSTS). Why cannot we have (TS_3) and (S_4)?

(*h*) **Variation as to Constraint of Motion.**—The three states of a kinematic linkage with respect to relative motion have been illustrated in Fig. 18, which shows (T_3) locked against motion, (T_4) constrained to definite motion and (T_5) free or loose for indefinite motion. Going to a six-link chain, in which we might hold three links after the manner of Fig. 18 (*d*) and yet leave the other three free to move, it may be said that such a combination has two degrees of looseness.

It is characteristic of these simple chains, whether used alone or in combination, that they are (or, having regard to actual mechanism skeleton, can be) represented by open polygons, enclosing clear areas. For a distinctive name call them *clear* chains.

Now the showing in the first paragraph may be summed up thus: *To introduce a link into a clear chain adds one degree of freedom or relaxes one degree of constraint; to take out a link adds one degree of constraint.*

In contrast with the preceding statement, the introduction of a new link across a clear chain, connecting new joints on non-adjacent links, adds a degree of constraint. Thus the quadric

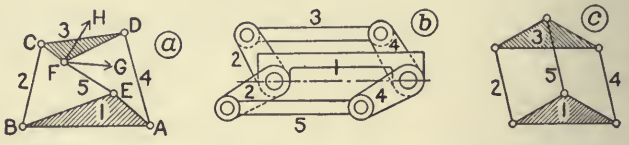

FIG. 411.—A special five-link mechanism.

ABCD in Fig. 411 (*a*) is locked by new link EF. Taking 1 as frame and considering joint center F as a point on link 3, this F tends to move in direction EG—got by means of the instant center of link 3, which will be explained in the next section. As a point on link 5 center F tends to go in direction EH. Since simultaneous different motions are impossible, it stands still. In further illustration of the principle stated take clear (T_5), add another link after the manner of Fig. 411 (*a*) and see that it changes from a loose chain to the constrained arrangement in Fig. 20 (*b*) or Fig. 415 (*b*).

The remainder of Fig. 411 shows a special exception. Sketch (*b*), representing a pair of coupled locomotive axles, corresponds with (*a*) as a linkage but is not locked. Evidently this is a case of special proportions, and its generalized skeleton at (*c*) shows that the repetition or addition of jointed parallelograms might be kept up indefinitely.

(*i*) **Considerations as to Pair Combinations.**—In the composition of $(TS)_n$ chains absolute freedom of choice as to pair combinations is not allowed. The two simple restrictions, illustrated by Figs. 412 and 413, are as follows:

1. A single turning pair, in a chain of the form (TS_n), is of no effect and such a combination may not be used.

2. If two sliding pairs are in line or parallel they reduce to the value of a single sliding pair.

In Fig. 412 (*a*) it is evident that no angular movement can take place between links 3 and 4, that joint 34 might as well be locked,

and that the mechanism is equivalent to the (S_3) form at (*b*).

In Fig. 413 (*a*) link 1 can slide but the rest of the chain is locked, so that practically we have arrangement (*b*). Parallelism reduces constrained (T_2S_2) to locked (T_2S).

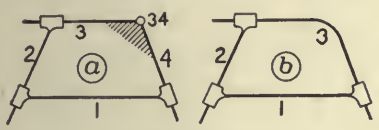

FIG. 412.—The (TS_n) mechanism.

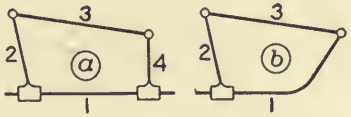

FIG. 413.—Two slides parallel.

The first principle above may be restated in the form that in any $(TS)_n$ chain there must be at least two T pairs. Knowing that the combination (S_3) is by itself free for constrained motion, the question may arise whether this property carries over into the quintic chain (T_2S_3), so that with two added links there

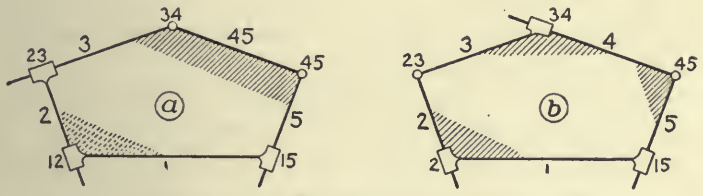

FIG. 414.—Discussion of (T_2S_3) chain.

may be two degrees of looseness. The answer to this question is illustrated by Fig. 414 as follows:

Holding any one of the S joints 12, 15 or 23 in Fig. 414 (*a*) we have left (T_2S_2) which is constrained, in normal agreement with Fig. 18 (*d*). It might seem necessary to lock links 3, 4 and 5 in order to just leave the remaining (S_3) constrained, which would be like the regular requirement for a six-link clear chain; but since Fig. 412 has shown (TS_3) to be equivalent to (S_3) it appears that to lock but one of the T joints is sufficient. In no respect, then, does the grouping together of three S pairs raise the freedom of this chain above what is normal to a general five-link unit. Figure 414 (*b*) is added in order to make clear that there can be no question regarding this matter when the three S pairs are not in direct sequence.

(*j*) **Chain Extension and Constraint.**—The scheme of chain extension introduced in Sect. 2 (*h*), with the forms reproduced at (*a*) and (*b*) of Fig. 415 as its first results, is carried forward

in (*c*) and (*d*). The first example shows again the possibility of extending either two adjacent or two nonadjacent links before adding two new links. At (*d*) we have three quintic units depending upon the primary constraint of one quadric.

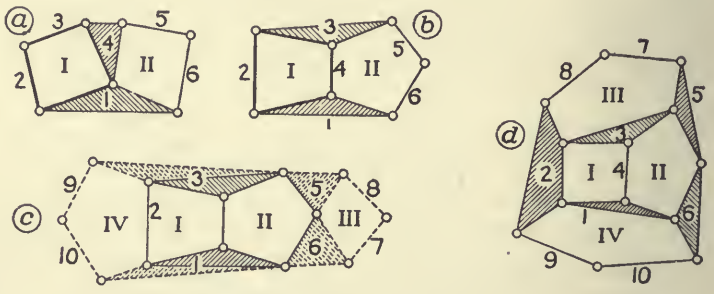

FIG. 415.—Normal chain extension.

A simple formula that will express relations in the extensions of (TS) chains, and will test the constraint of any such mechanism, can easily be derived as follows:

Let j = number of joints or pairs and n = number of links in a chain. Turning back to Sect. 2 (*h*) we see that after the initial kinematic pair the development or addition of a unit is accomplished by adding two links and three pairs, so that beyond that first pair the relation is $2j = 3n$. For the first pair, with $j = 1$ and $n = 2$, this relation is modified to $2j = 3n - 4$. Then for the whole chain

$$j = \frac{3n - 4}{2} = \frac{3}{2}n - 2. \tag{107}$$

One way to express the operation is to say that each new link carries with it $\frac{3}{2}$ new joints. Now in making or adding an (S_3) unit the addition is one link and two pairs, so that a new link brings two new pairs; also each new link means a new chain. The unit (S_3) is called a *prismatic* chain because all of its sliding members are of prismatic form. We have now that for every prismatic chain (number symbol p) there has been added one-half more joint than would be found for the same number of links with only (TS) units. For lower-pair chains in general the **criterion of constraint** is then

$$j = \frac{3n + p - 4}{2}. \tag{108}$$

To see the meaning of an inequality in the formula, try Eq. (107) on (T_3) and (T_5). For the first, $j = 3$, $n = 3$ and $\frac{3}{2}n - 2 = 2\frac{1}{2}$; j is in excess and the chain is locked. For the second, $j = 5$, $n = 5$ and $\frac{3}{2}n - 2 = 5\frac{1}{2}$; j is deficient and the chain is loose. Evidently j represents constraining influence.

Letting c stand for number of unit chains and leaving out of account the infrequent (S_3) unit, the following tabulation shows conditions for (TS) chains:

Units.....	c	1	2	3	4	5	6	etc.
Links.....	n	4	6	8	10	12	14	etc.
Pairs.....	j	4	7	10	13	16	19	etc.

When applying the formula care must be taken to count all the joints that exist. In Fig. 416 (*a*), for instance, only six joints appear at first sight, instead of the ten joints that an eight-link mechanism ought to have. But there are four joints

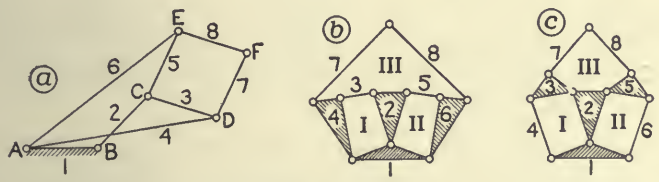

FIG. 416.—A mechanism with multiple joints, the Peaucellier straight-line motion.

A, C, D and E where three links come together; and since a joint or pair can involve but two links each of these points really represents two joints.

When sketching the chain diagram for any such mechanism having multiple joints, be sure to spread out the links so as to show all the pairs distinctly. Generally one link will carry a pin and the others will have eyes. Without exact knowledge of the detail of this construction (and even with it, since pin and eye may be interchanged at will) there is ambiguity or uncertainty. This leads to differences of interpretation such as are represented by diagrams (*b*) and (*c*). Between them change is made only at joints D and E, and as regards these the third possibility exists of assigning the double joints to links 7 and 8. Note that in (*b*) quintic unit II is 14786, at (*c*) it is the more obvious 23785.

(*k*) **Making Chain Diagrams.**—In drawing the chain skeleton of a mechanism that is at all complicated, care must be taken to get it into simple final shape. The two examples in Figs. 417 and 418 will show the kind of questions or difficulties likely to be encountered. In the six-link mechanism of Fig. 417 the quintic units 12345 and 12346 are rather more obvious than the necessary quadric 1546; but the chain diagram gives the latter its due prominence, even without being turned into the more open form of Fig. 415 (*b*). This example illustrates the fact, not quite so

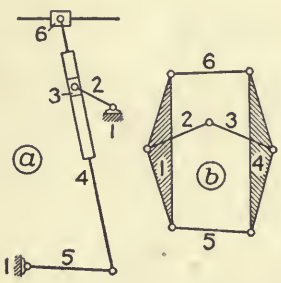

FIG. 417.—Crank-and-lever mechanism, special form.

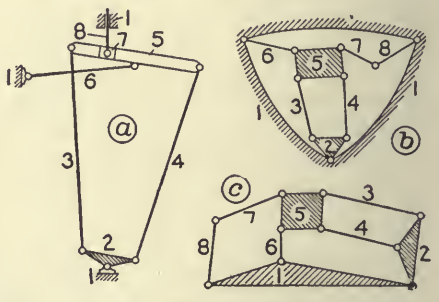

FIG. 418.—Modified Stephenson link motion.

outstanding in Fig. 415, that if two nonadjacent links of a quadric unit are extended either of the intermediate links can be counted in the quintic unit formed by adding two new links. In Fig. 415 (*b*) the second quintic form is 12356. What are the two extra ones in Fig. 415 (*c*)?

Sometimes it is best to make frame 1 surround the other links as in Fig. 418 (*b*), in order to keep a little nearer to the actual arrangement of the mechanism; but in such a case the apparently enclosed linkage 124578 must not be considered a six-link unit. In other words, when nonadjacent links 1 and 5 of quintic 12356 or 12456 are extended to make 16578, we may not count the longer way around the original quintic but must follow the shorter path. With the chain redrawn as at (*c*) the temptation to count a six-link unit disappears.

The chain formula devised in Art. (*e*) is not useful for extended chains because the number of possible combinations becomes too great. Even if we strung together a series of unit formulas there would be no way of showing which links the successive units had in common. It is worth while, however, to show kind of units by bracketing the simplest possible symbols and prefixing the

number of links. The common $(TS)_4$ and $(TS)_5$ units are well represented by the mere numbers 4 and 5. For the occasional three- and four-link prismatic chains P_3 and P_4 can be used. In the next article characteristic higher-pair units H_3 and H_4 are presented. Of this notation we have then examples as follows:

Fig. 416 = 8(4-4-5); Fig. 415 (*c*) = 10(4-5-4-5);
Fig. 418 = 8(4-5-5); Fig. 415 (*d*) = 10(4-5-5-5);
Fig. 420 = 6(4-H_3-H_3).

(*l*) **Chains with Higher Pairs.**—The common forms of chain with higher pairs are the three-link or ternic combinations (T_2H) and (TSH), both essentially represented by Fig. 419 (*a*). The first stands for the wheel pair like Fig. 55 (with frame bar, of course) and the cam with oscillating follower, Fig. 72. The use of a roller on follower gives the apparently quadric form in Fig.

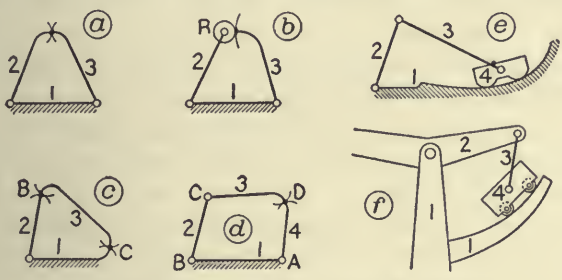

FIG. 419.—Simple chains with higher pairs.

419 (*b*), but the roller has no kinematic effect and may just as well be ignored. Chain (TSH) represents the gear and rack in Fig. 60 and the cam with sliding follower, Fig. 71 or 82. Even (S_2H) appears in Fig. 83.

A single higher pair imposes very little constraint, both gear teeth and cams depending upon lower-pair mechanism for the mere location of their contact. It is because of this small effect that the amount of such lower-pair mechanism is normally reduced from four to three links, and that such combinations as (*c*) and (*d*) of Fig. 419 are unconstrained. Commonly higher pairs are forced-closed; and when self-closure is secured, as by equidistant cam profiles and by gear teeth without backlash, the pair is not to be counted as doubled.

In order to insure the definite relative positioning of two bodies two independent higher pairs are needed, as shown in Fig. 419

(*e*); and with this condition the normal unit linkage is again the quadric. A possible realization of this infrequent chain is seen at (*f*), which shows an arrangement for making a crane counterweight move in and out as jib 2 is swung up and down to vary radius of lift; other mechanism would be needed, of course, to control position of jib.

(*m*) **The General Criterion of Constraint.**—It is a practical fact that unit chains containing higher pairs generally keep close

FIG. 420.—A chain with mixed units.

to the frame and are little subject to inversion, so that commonly the determination of constraint is a mere matter of inspection. But even though logical completeness rather than utility is the motive, it is of interest to extend the criterion of constraint so as to cover chains containing higher pairs.

First, as an illustrative example, consider in Fig. 420 the addition of chains (*a*) and (*b*) of Fig. 419 to an original quadric (T_3S), then the embodiment of this skeleton as an engine driving a cam shear like Fig. 73 through a pair of gears. Development of these ternic units involves the addition of one link, one lower-pair joint and one higher pair, while the quadric $4(T_3H_2)$ of Fig. 419 (*e*) adds two links, two joints and two higher pairs. One joint per link is less by one-half than the normal rate for lower-pair extension; and since in the present scheme links added and higher pairs added are numerically the same, we modify Eq. (108) to

$$j = \frac{3n + p - 4}{2} - \frac{1}{2}h = \frac{3n + p - h - 4}{2}. \qquad (109)$$

To count roller R in chain (*b*) of Fig. 419 as a link would disturb this relation, for then the addition would be two links, two joints and but one higher pair. Including the roller in Fig. 420 the numbers are $j = 7$, $n = 7$, $h = 2$, which give the false equation

$$7 = \frac{21 - 2 - 4}{2} = 7\frac{1}{2}.$$

Deficiency of j means looseness of chain, which is exactly what would exist if the roller were of any other form than a circular

cylinder. Again, to count two higher pairs at the contact of either tight gear teeth or a positive cam would make the criterion show locking of chain; and this would be true if the two higher pairs were not of interdependent, noninterfering form or relation. In the roller, in gear teeth and in equidistant cams we have cases of special proportions, somewhat analogous to the condition in Fig. 411.

SECTION 29. PRINCIPLES OF PLANAR MOTION

(*a*) **Conditions of Machine Motion.**—It is an outstanding fact that planar or coplanar motion is decidedly the prominent type in machines. Important general statements regarding such motion have been made in Sect. 2 (*e*) and in Sect. 26 (*a*). Preceding a fuller application of the facts and principles there stated, a deeper look into the mechanical and geometrical conditions underlying them is appropriate.

A complete turning pair, arranged so as to maintain an axis and to prevent any relative motion along that axis, constrains all points of either member to move in parallel planes with respect to the other member: so also does a prismatic sliding pair, whether self-closed or force-closed. In mechanisms of the linkage class, with members joined by lower pairs, parallelism of motion planes and of axes is almost unavoidable. Only by bringing the strong pair constraints into coincidence of direction can conflict and interference be avoided. Examples of the few possible nonplanar linkages will be found in Sect. 31.

The machine analysis into driver, connector and follower, proposed in Sect. 28 (*a*) and based on the fundamental description in Sect. 1 (*b*), is especially appropriate for simple mechanisms containing higher pairs. Driver and follower are jointed to the frame by lower pairs, which exert a strongly *directive* effect upon their motions, enforcing individual coplanarity with the frame. Using *transmission* by means of the more flexible but less definite higher pairs it is comparatively easy to pass from one direction of plane or axis to another, as by means of bevel gears, Fig. 64, helical gears, Fig. 67, turned belt-and-pulley pairs, Figs. 111, 112 and 114, guided rope drives and cylindric cams, Fig. 78.

In general the position of a rigid body in space is made definite by fixing the location of three points of the body, these points not lying in a straight line. In mechanism this is changed to the

fixing of a line or axis and the location of another contact line or a parallel axis. The latter condition underlies the general sufficiency of two-point determination of body motion in planar kinematics.

(*b*) **Kinds of Body Motion.**—For a body in a plane there are two simple, unmixed types of motion and a general type of which the first two are really special cases and into which they com-

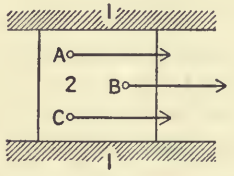

FIG. 421.—Straight-line translation.

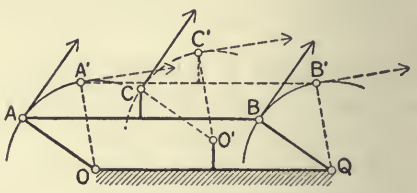

FIG. 422.—Circular translation.

monly enter as components. The first simple motion is **translation,** rectilinear for slide 2 in Fig. 421, curvilinear for bar AB in Fig. 422. In translation the body suffers no change of angular position, a line upon it remaining always parallel to a fixed reference line, as AB to OQ in Fig. 422. All points of the body move in like paths and at any instant all velocities are parallel

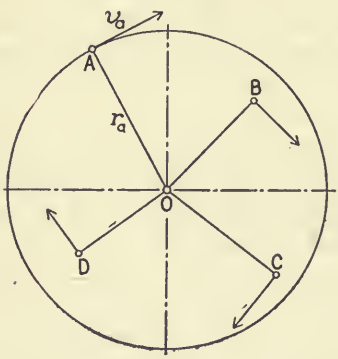

FIG. 423.

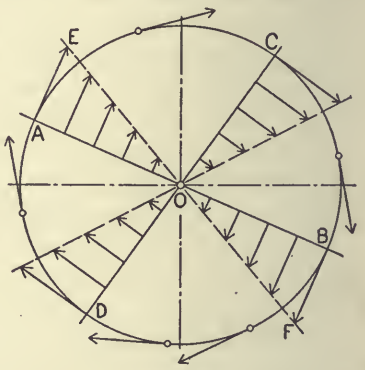

FIG. 424.

Rotation on fixed centers.

and equal: in Fig. 422 this is illustrated for a random point C on bar AB, by getting on frame OQ the center O′ of its circle path.

The second simple motion is pure rotation, or turning on a fixed center—understanding that in any machine diagram a

center is the end view of an axis. If the circle in Fig. 423 represents a disc turning on center O at angular velocity ω—compare Sect. 7 (*b*)—then the linear velocity of any point A, B, C or D is perpendicular and proportional to the radius from O to that point. The formula relation, from Sect. 7 (*b*), is

$$v = r\omega \text{ or } \frac{v}{r} = \omega; \tag{110}$$

but graphically, as between vector v and radius r the ratio is compounded of ω and the two scales of representation, for velocity and for dimensions. This ratio can be expressed and applied by drawing on the body plane an inclined line, such as EF with respect to diameter AB in Fig. 424.

Considering a rotating plane of undefined extent, three relations between positions and velocities of points are shown by Fig. 424. A diameter AB is the locus of all points which (at the instant) have velocities of the same angular direction. A radius OA carries all the points whose velocities are not only parallel but in the same linear direction. On a circle (from center O) lie all points whose velocities are alike in amount or in length of vector. With free choice as to inclination and length of radius it is possible to find on such a rotating plane a point that will have any assigned velocity.

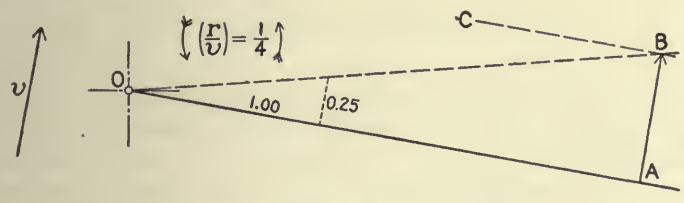

FIG. 425.—Location of assigned velocity.

Example 13. If the machine outline is drawn half-size, the velocity scale is 1 in. = 10 ft. per sec. and angular velocity ω is 20 radians per second, what will be the length ratio (v/r) on the drawing?

At 1 ft. actual radius v is numerically equal to ω. At the end of a 6-in. radius on the drawing, which represents the actual foot, the velocity vector should be $20 \div 10 = 2$ in.; then $(v/r) = 2/6 = 1/3$.

Prob. 221. On a quarter-size drawing and with the velocity scale 1 in. = 6 ft. per sec. the graphical ratio is $(v/r) = 0.3$; find ω.

Example 14. Given a velocity v, laid down at random on the fixed plane of the drawing, Fig. 425, and the center O on which a plane is rotating as indicated; find the one point on this plane that has velocity v.

Draw OA perpendicular to given v and express rotation effect by means or ratio line OB. A parallel CB at distance v from OA locates the outer end of vector AB and the particular point A which has v for its velocity.

Prob. 222. Given velocity v_a of a point A on a plane which is rotating at rate ω or (v/r), find the point which has some other assigned velocity v.
Suggestion: Work from v_a to center O, then from O out to required new point.

(*c*) **The General Type** of planar motion combines the two single forms, in a way that is given first illustration by Fig. 426. Here line AB represents a bar whose end points A and B are guided on paths MM and NN. Consider its change of position from A_1B_1 to A_2B_2: this may be conceived as the resultant of two component actions, first a translation with A or B to the parallel position A_2B' or $A'B_2$, then a rotation on A_2 or B_2 to the final position A_2B_2. In case (*a*) we have translation with and rotation about A as **master point,** in case (*b*) the master point is at B.
Shown thus for a large displacement and with the two components in distinct sequence, the analysis may seem crude and

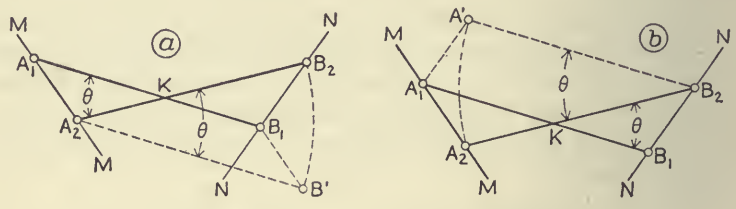

FIG. 426.—Resolution of plane displacement.

not very convincing. Its effectiveness appears more clearly when we pass from change accomplished to change in the making and think of the components as simultaneous rates of change or velocities. As a definite example consider the engine mechanism outlined in Fig. 427, where connecting rod CW has point C constrained to a circle path and point W to a straight-line path. If we know angular velocity ω for crank OC the linear velocity CB or v_c is readily found. If the rod could move in translation with C, point W would have the same velocity as C, shown by WD. Actually, pin center W is kept in its path by the fact that a turning on C accompanies the advance with C. Due to this turning point W has the second component velocity WE or u_{cw} (necessarily perpendicular to radius CW), which must have such a value that resultant WF will lie along path WO.

Detached triangle (*b*) of Fig. 427 shows the full velocity relation between guided points C and W, or the method of passing

from one point to the other: lay off primary or given velocity v_c as pc, draw the line of direction for unknown velocity v_w, then cut off its length by cw perpendicular to the direction pq of the line CW that joins the two points on the body. Cross velocity cw, divided by radius or length CW, will give angular velocity

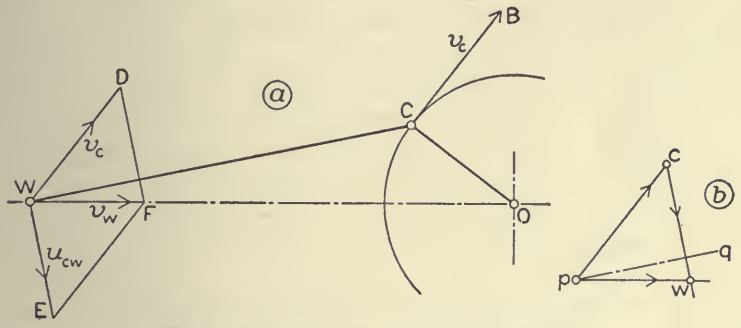

FIG. 427.—Velocity derivation or transfer.

ω for the rod or body. An alternate scheme of data is to have v_c and ω for the rod as known quantities, whereupon pc-cw will be determinate in Fig. 427 (*b*) and pw will be found in direction as well as length.

Three important principles have been introduced or suggested by the examples in Figs. 426 and 427. These are, (1) the resolution of planar motion into translation with and rotation on a master point, (2) the velocity-triangle relation between two points of a body and (3) the interchangeability of master points. As to the last, it appeared in Fig. 426 that either A or B might be master point; and it is evident in Fig. 427 that the relation expressed by triangle (*b*) will hold or can be worked in either direction, as between C and W.

These fundamental principles are basic concepts, to be realized with use and familiarity rather than reasoned out. Their fuller development and application belongs to the quantitative study of velocity, and our immediate application need go no farther than to establish the ideas of the instant center and the universality of rotation as the general type of motion.

(*d*) **The Instant Center.**—For a moving planar body let the data be, linear velocity v_a of master point A and graphical angular velocity (v/r). In Fig. 428 take first any two points B and C, draw radii AB and AC and lay down the cross velocities

u_b and u_c due to rotation on A; then adding v_a (vectorially) we get the resultant velocities v_b and v_c. This is an instance of the second method or scheme of procedure stated under Fig. 427 (*b*).

Next find the particular point where v is zero, or where u is just equal and opposite to v_a. This must be on a line AP perpendicular to v_a. In order to locate it, either draw ratio line AD with respect to AP, using parallel ED to find where the

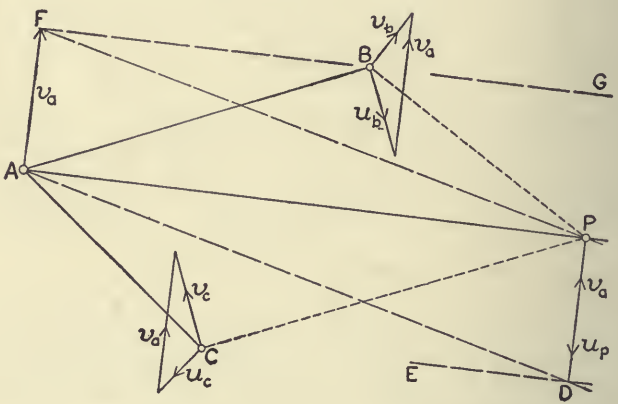

FIG. 428.—The master point of zero velocity.

u-vector equals v_a in length; or else draw FP with respect to FG, conceiving it as opposing a longer and longer u to v_a until neutralization is attained at P.

Now change to P as master point, drawing radii PB and PC. Since the body is rigid and angular velocity (v/r) prevails unchanged, the effect of rotation about P is to give each point a velocity perpendicular to its radius from P and proportional in the ratio (v/r); and because P has no primary velocity of its own the rotational effect becomes total velocity at each point. The zero point P, instantaneously at rest, is called the **instant center** of the body.

Now if a body has any planar motion other than pure translation—practically, if its two necessary guided points do not have parallel paths—a zero point can always be located as in Fig. 428; and the whole motion of the body then becomes an instantaneous rotation about that point as center. For this reason we say that rotation is the universal type of planar motion. Even straight-line translation may be considered the special case of rotation

about an infinitely distant center. The failing case is curvilinear translation of the kind in Fig. 422.

For a machine member the motion data are usually, not one master point and the angular velocity as in Fig. 428, but two master points definitely guided as in Fig. 426 or 427. Then since each velocity must be perpendicular to the radius from instant center, it follows that the latter must be at the intersection of lines drawn from the given points perpendicular to the paths or velocities. This relation furnishes the usual means of determination, which, however, is not illustrated until we come to Figs. 433 and 434, where it appears as part of a more general scheme.

Prob. 223. Make a construction like Fig. 428, with four well distributed secondary points instead of two. Find P, then satisfy yourself by trial that the originally determined v's are perpendicular and proportional to the radii from P.

(*e*) **The Relative Center.**—As stated and illustrated in Art. (*b*), it is possible to find on a rotating planar body a point with any assigned velocity. Having two such bodies, turning on centers P and Q in Figs. 429 and 430, let it be required to find the point of common velocity. This really means the common position of two points, one on each body, that have the same velocity.

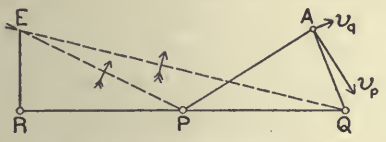

FIG. 429.—Like rotations.

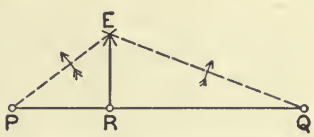

FIG. 430.—Unlike rotations.

The point of common velocity.

First take a random point A, Fig. 429: on plane P its velocity is $v_p = PA \times \omega_p$, on plane Q its velocity is $v_q = QA \times \omega_q$, and these agree in neither direction nor magnitude. To secure like direction of velocities there must first be like direction of radii, which can be secured only by making the radii lie together, along the line through centers P and Q. On this locus line the position of the point sought can be found if the ω's are known, by drawing ratio lines from the two centers to a determining intersection at E.

If the angular velocities are alike in direction point R will lie outside of PQ, beyond the center (here P) on which the rate of turning is greater. For the particular case of equal ω's, or in the

jointed parallelogram of Fig. 422, there is no intersection and no point of common velocity. If the turnings on P and Q are opposite point R will fall between those centers, as in Fig. 430.

In order to connect up the idea of the point of common velocity with that of the instant center as set forth in Art. (*d*), consider the example in Figs. 431 and 432. The gear-pair unit first shown is a simple and complete embodiment of Fig. 430, main-

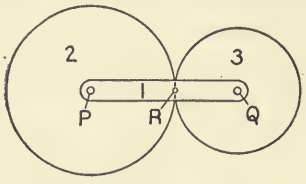

FIG. 431.—Frame-bar fixed.

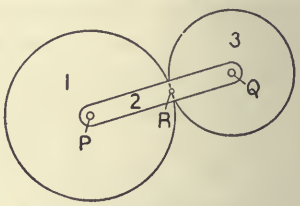

FIG. 432.—One wheel fixed.

Instant and relative centers.

taining a constant velocity ratio on fixed centers; common velocity exists at the point R of pitch-circle contact. Now invert the mechanism, holding the large gear fast in Fig. 432 and getting a simple cyclic train—see Sect. 22 (*b*). This makes R the instant center of wheel 3 on wheel 1, or its one point of zero velocity, while Q becomes the point of common velocity between 2 and 3.

This simple but representative example leads up to the general statement or definition that **the point of common velocity between two moving planar bodies is the instant center from their motion relative to each other,** or the point where either is turning upon the other. The instant center for absolute motion, derived as in Fig. 428 or like 13 in Figs. 433 and 434, is no more than a particular case under this definition, being the one point of the moving body that has the same velocity (zero) as the point beneath it on the fixed body or frame.

Instant center is the general title, properly covering all points of common velocity, whether one or both bodies are moving. It is often convenient, however, to make the distinction of taking *instant* centers between moving members and frame, *relative* centers between pairs of moving members. To cover the change in kind of centers that takes place during inversion, as instanced by Figs. 431 and 432, the general principle of relative motion from Sect. 28 (*d*) holds, namely, **that the relative motions of the members of a mechanism depend wholly upon dimensions and geometrical configuration, not at all upon what is held fast.**

(*f*) **The Theorem of Three Centers.**—In a grouping of three things three pairings can be made, so that in a group of three machine members there are three instant centers. Then the reasoning that puts point R on line PQ in Figs. 429 and 430 establishes the very useful theorem of three centers, namely, **that the three centers among any three members of a mechanism must lie on the same straight line.** This principle furnishes an effective means for locating instant centers, adequate for all mechanisms evolved by the simple extension described in Sect.

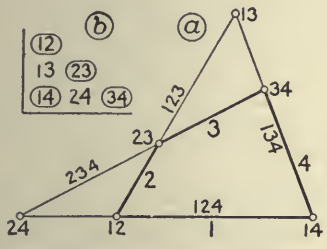

FIG. 433.—Four-pin quadric.

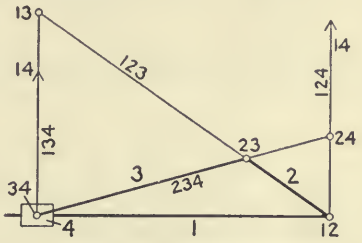

FIG. 434.—Slider-crank.

Finding instant centers in quadric chains.

2 (*g*) and Sect. 28 (*j*). A few elementary examples will be given, then the subject will be left for fuller development in connection with velocity determinations.

For either of the quadric mechanisms in Figs. 433 and 434, the first thing after numbering the links and joints is to write out the combination table like Fig. 433 (*b*), showing all the possible centers. In this table the joints or permanent centers are indicated by drawing circles around their designating numbers, leaving 13 and 24 (between nonadjacent links) to be found: be sure to read these doubled numbers "one-three," "two-four," not "thirteen" and "twenty-four." To get 13, take first 2, then 4, with 1 and 3: center 13 must lie on line 12-23 or 123 and on 14-34 or 134, hence at their intersection as indicated. Center 24 is found in similar fashion. In Fig. 433 it appears that the instant center between two opposite links of a (T_4) chain is at the interesction of the other two links. In Fig. 434 center 14 lies at infinity and all parallel verticals run toward it. Note how, for absolute center 13, this scheme includes the method of perpendiculars to velocities as described at the end of Art. (*d*).

Two examples of the failing case of the instant-center concept are shown by Figs. 435 and 436. In the parallelogram linkage

relative centers 13 and 24 really do not exist at all, because there are no points of common velocity for the pairs of links involved. If we conceive 13 as at the infinitely distant intersection of lines 123 and 134, the fact that it will have an infinite velocity due to the rotation of those lines gives it a far higher degree of indefiniteness than that of a slide-movement center like 14 in Fig. 434.

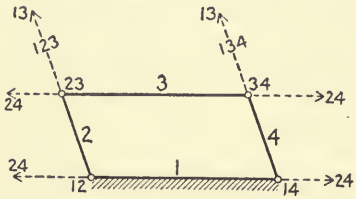

FIG. 435.

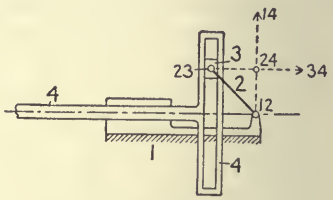

FIG. 436.

Failing cases of the instant center.

In the crossed slider-crank, Fig. 436, there is no center 13 because between members 1 and 3 the relative motion is again circular translation.

(*g*) **Centrodes.**—As the members of a mechanism move upon each other their instant centers travel along definite paths on the planes of the respective members. The simplest and most useful of these center paths or centrodes are found in the pitch circles or curves of gear wheels, as instanced in Figs. 431 and 432. The more intricate forms occurring in linkage mechanisms will be sufficiently represented by two examples from the slider-crank.

The curves for center 13, between connecting rod and frame, are determined in Fig. 437 by repeated plottings of point P as the intersection of vertical WP and crank line OCP. For the fixed centrode C1 this is a direct operation. For the moving centrode C3, shown with respect to the particular position CW of its member, further graphical work is required. In general, having found P′ as the apex of triangle C′W′P′ on any positon of the rod, it is necessary to transfer this triangle to CWP_1 on the particular exhibit position CW, getting point P_1 of centrode C3. Each curve has two branches, running up and down to infinity for crank at 90 deg. and at 270 deg. How the curves roll together is indicated by numbering successive points in conformity with crank positions.

The center 24 between crank and slide is at B in Fig. 438, where line of rod cuts vertical center line GH. With both

members moving there are three loci of the instant center, here GH on the frame and the two centrodes on the respective members; the latter are plotted by methods analogous to that for C3 in Fig. 437. Again coincident points and corresponding

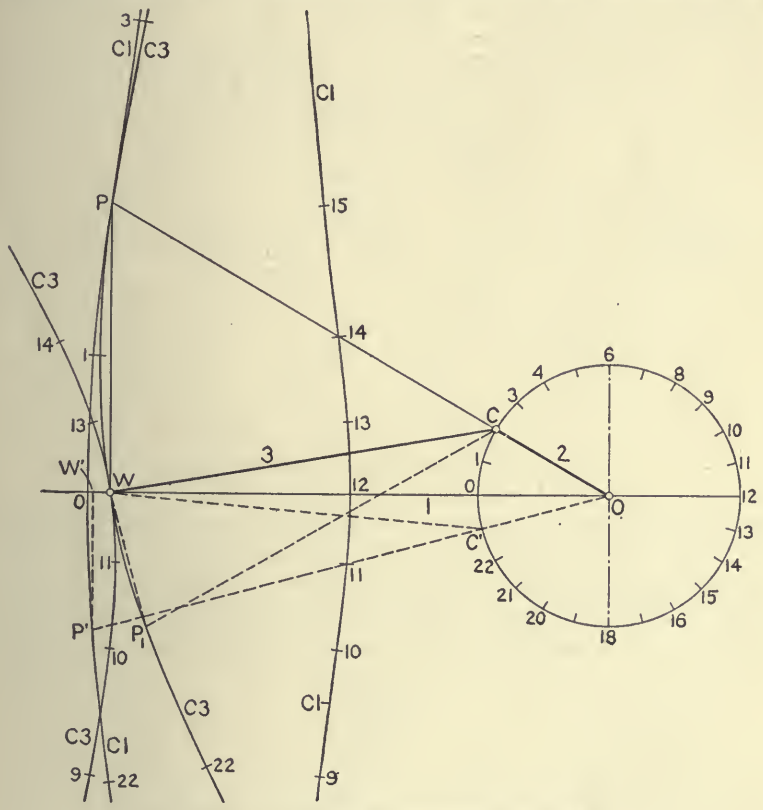

FIG. 437.—Centrodes, frame and connecting rod.

crank positions are indicated by like numbering. Understand that C4 is fast to slide 4 and moves to right and left with it.

(*h*) **Utility of Centrodes.**—The full idea of centrodes is that if these curves were given material backing and constrained to roll upon each other without slip they might replace the actual linkage and would continue to give the same relative motion of members. Such a scheme is, of course, wholly fanciful for curves of the kind drawn in Figs. 437 and 438. While a certain amount of interest attaches to the centrodes of linkage, they are of but little use for any quantitative purpose. The pitch-curve

centrodes of gear pairs are, on the other hand, of great importance and use; and in the more general class of noncircular gears their relations and proportions present some interesting problems.

As a supplement to the brief exhibition of centrodes in the last article we shall now consider two special cases in which a

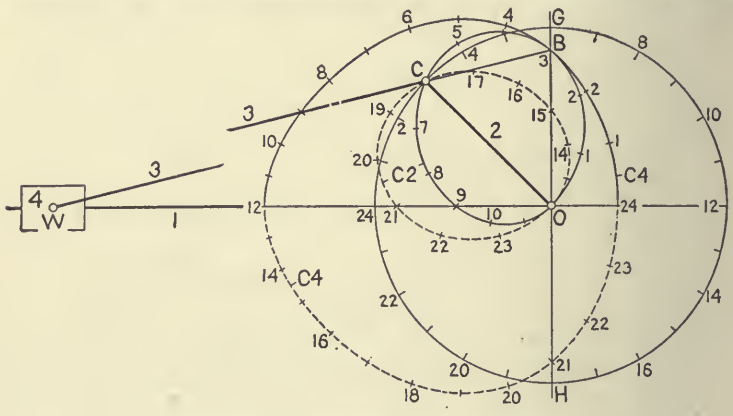

FIG. 438.—Centrodes, crank and slide.

linkage and a gear train are interchangeable, or in which linkage centrodes can practically be embodied and put to work as gear wheels. For the crossed slider-crank (T_2S_2), first with the cross fixed in Fig. 439, then with the cross sliding in Fig. 440, it is easily seen or shown that the centrodes are circles of 2 to 1

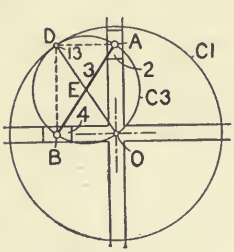

FIG. 439.

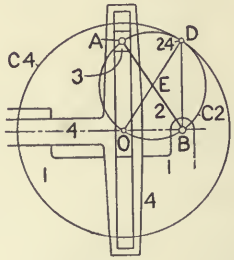

FIG. 440.

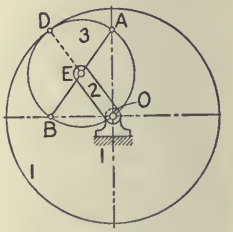

FIG. 441.

Crossed slider-crank and hypocyclic train.

ratio in diameters—this combination being a simplification of the curves in Fig. 438. The arrangement in Fig. 439 is readily replaced by the annular or hypocyclic train in Fig. 441; and a bar AB, fast to gear 3, will have the same motion as link AB of Fig. 439.

In the crossed parallelogram linkage, Fig. 442, center 24 is at intersection E. Drawing a line AC, the equality of triangles ABC and CDA is evident. It is then easy to prove that triangles AEB and CED are equal, whence AE + BE = AD = CE + DE. Since the sum of vectors AE and BE or of CE and DE is thus constant, centrodes C2 and C4 are equal ellipses, with A and B or C and D as foci. Very evidently these ellipses can be made the pitch curves of a pair of gears like those in Fig. 63,

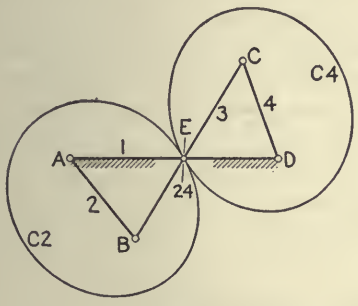

FIG. 442.

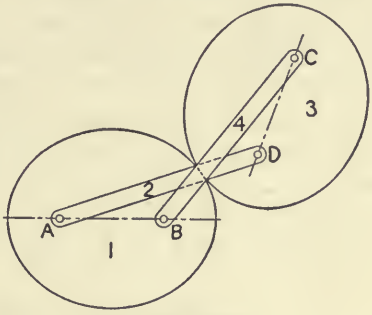

FIG. 443.

Crossed parallelogram and elliptical gear pair.

link BC becoming redundant as soon as rolling without slip is made positive by the formation of teeth. But it is made apparent in Fig. 443 that if one ellipse or short side is held fast as the base of an epicyclic train, then much better and more symmetrical constraint will be secured by retaining both arms.

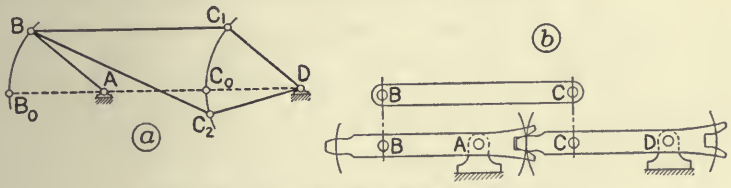

FIG. 444.—Passing dead center in Fig. 443.

(*i*) **Use of Centrodes at Dead Point.**—The parallelogram linkage has the unusual property of a double dead center, when the rod and both cranks are all in line, as at position B_0AC_0D, Fig. 444. Crank AB having been swung past B_0 up to B, the other crank may either have gone up to DC_1 or down to DC_2. For parallel motion positive action is secured by doubling the crank-and-rod arrangement, as shown in Fig. 411. To make the crossed

linkage positive it is necessary to use a portion of the elliptical centrodes after the manner shown in Fig. 444 (*b*), where cranks AB and CD are extended and carry at their ends the elements of a single tooth pair on the ellipses as pitch curves; note that link BC is lifted off so as to uncover the tooth pair now in action. At and near dead center the teeth relieve rod BC of the function of transmitting both motion and force, for which they are much more favorably placed.

This example of the localized use of centrodes in a linkage is probably unique in practical mechanism. Indeed the word "practical" may here be questioned, for while the mechanism of Fig. 444 might be used as a quick-return drive it is much inferior to the elliptical gear pair as regards force transmission.

Prob. 224. With 4-in. crank and 12-in. rod lay out the centrodes of Fig. 438.

Prob. 225. For a model of the mechanism in Fig. 443 sketch a scheme for supporting gear 1 that will permit complete rotation of 3 with the two links.

SECTION 30. VARIOUS MECHANISMS

(*a*) **Geometrical Mechanisms.**—In a broad sense every mechanism is a geometrical apparatus, because of its property of definite motion. Here, however, the term is used in a restricted sense, to designate appliances for producing some of the simpler and more useful geometrical forms, whether lines or surfaces. The word produce has degrees of meaning or covers degrees of process. Thus the graphical operation of drawing circles and lines, by means of compass and straight-edge, contains a mechanical element but is not completely a mechanism effect. In an engine the crank constrains crank-pin center to a circle path and the crosshead makes wrist-pin center travel on a straight line, without resistance to form of motion but under the action of transmitted forces. In the lathe and planer similar geometrical results are secured, but against the heavy resistance of the material that is being cut away, localized as the useful load resistance. Geometrical mechanisms have all the purposes suggested by these examples, ranging from the drawing of lines to the shaping of bodies.

The two machine-tool operations just named are applications of the first two lower kinematic pairs, and they offer good examples of inversion. In drilling and boring machines work piece stands

still and radius rotates, after the manner of crank arm and hand compass; in lathe and boring mill radius stands and work turns. Between shaper and planer there is a similar inversion.

When citing above the two simplest geometrical forms there was good reason for naming the circle first, because it is *generated* by a radius from a mere fixed point, whereas the straight line is *copied* from a form already in existence. This distinction brings out the reason why the lathe, as an accurate machine tool, is a good many years older than the planer. It does not, however, preclude us from saying that a geometrical form is generated even if straight-line guiding happens to be an element of the producing mechanism.

(*b*) **The Ellipsograph.**—As a mechanism, if not in the form of its product, the development of the crossed slider-crank (T_2S_2) into the ellipsograph is one of the simplest geometrical devices.

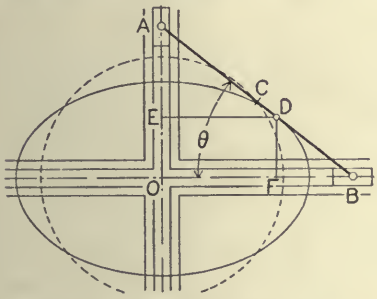

FIG. 445.—Inside tracing point.

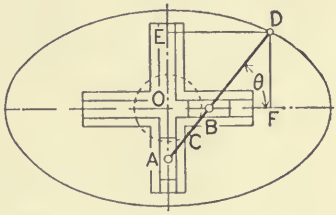

FIG. 446.—Outside tracing point.

The plain or rectangular ellipsograph.

If a bar AB in Figs. 445 and 446 has its two points A and B guided on perpendicular straight lines, any other point D will trace an ellipse. For the unique point C, midway between A and B, the ellipse becomes a circle; and as an outside tracing point gets farther away (or as AB approaches zero with D outside) the large ellipse approaches a circle

In either figure let DA = a and DB = b; then DE = x = $a \cos \theta$ and DF = y = $b \sin \theta$, whence

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 \theta + \sin^2 \theta = 1, \qquad (111)$$

the equation of an ellipse.

With the help of the equivalent one-to-two hypocyclic train from Fig. 441 it can clearly be shown that tracing point D does

not have to lie on line AB and that the two guide lines OA and OB may be oblique to each other. First we need a complete proof of the geometrical correctness of the action of this gear train, or of the fact that any point on the pitch circle of the smaller wheel traverses a diameter of the larger wheel. In Fig. 447 suppose that the position shown was reached by rolling from original contact of B with G, bar OC then lying along radius OG. Arcs BE and GE being therefore equal and the radii as 1 to 2, angle ECB is twice angle EOG, or horizontal CK (parallel to OG) bisects angle ECB. Then EB = 2 × CD and since OE = 2 × OC point B must lie on line OG.

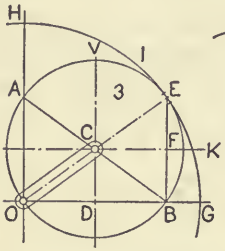

Fig. 447.—Hypocyclic mechanism.

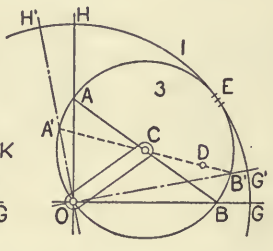

Fig. 448. Tracing point off bar.

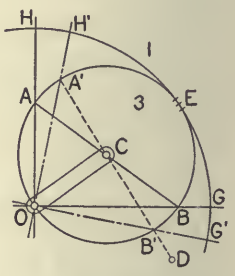

Fig. 449.

Now in Fig. 448 or 449 suppose that we really have points A and B guided as in Fig. 445 or 446, but that D is off the center line of bar AB. Drawing in the gear wheels or centrodes as equivalents, we pass through D and C a line which crosses the circumference of circle 3 at A′ and B′. Like all other points of that circle, these trace diameters of circle 1, so that the effect in each case is as if the dotted mechanisms were in action after the manner of Figs. 445 and 446. It can easily be shown that the angle GOG′ of axis deflection is half of the angle DCB on small wheel or tracer bar.

(*c*) **The Universal Ellipsograph.**—Figures 448 and 449 make clear the possibility of the oblique ellipsograph, since points A and B′ or A′ and B can simultaneously follow paths not at right angles. In Fig. 450 we start with oblique axes MM and NN, calling departure MOM_0 the obliquity of the combination. Take straight bar AB and pass the smaller centrode or gear circle through its ends and center O. The center C of this circle is an important point on bar AB. It must lie on a diameter BA′ that

runs to vertical axis OM_0: then since arc $AA' =$ arc MM_0 it follows that angle $ABA' = MOM_0$, or that the base angles of isosceles triangle CAB are each equal to the obliquity of axes.

The most general statement for the ellipsograph is that if any triangle moves with two of its corners guided on intersecting straight lines the third corner will trace an ellipse. First in

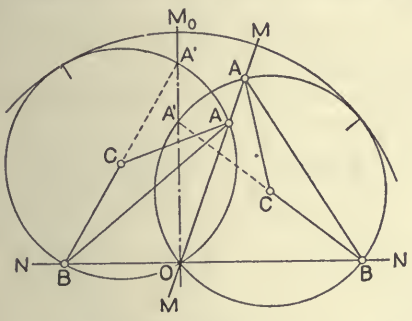

FIG 450.—Oblique guide lines.

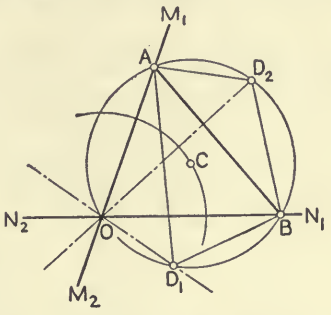

FIG. 451.—Generating straight lines.

Fig. 451 we show the particular condition that apex angle AD_1B or AD_2B is equal to the axis angle M_1ON_1 or M_1ON_2: this necessarily puts points D_1 and D_2 on the small centrode circle, so that each traces a straight line, equal in length to the diameter of the larger centrode. Examples of the general condition are given

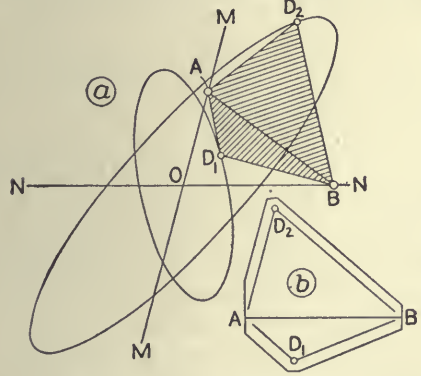

FIG. 452.—Sample ellipses.

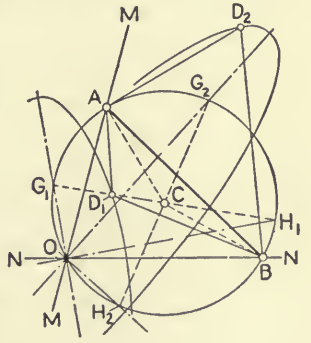

FIG. 453.—Determination of axes.

in Fig. 452, which shows also at (*b*) the form of a paper template to be used in plotting the ellipses.

Determination of ellipse axes for these double examples is shown in Fig. 453. Take any position of AB, locate C and draw

the circle. Through D and C pass a diameter having the ends G and H. Then the principal axes of the ellipse pass through and are loci of these G and H points.

Application of the ellipsograph to its ostensible purpose of drawing or cutting ellipses will be discussed later, in Art. (*o*). Here we next take up a large group of straight-line "motions" derived from this mechanism.

Prob. 226. In Fig. 451 establish the relation between base angles DAB, DBA, and the parts into which OD divides MOM.

Prob. 227. With assumed proportions plot two ellipses after the manner of Fig. 452, determining their axes as in Fig. 453.

(*d*) **True-circle Straight-line Motions.**—In Fig. 454 (*a*) is shown the plain rectangular ellipsograph, with the fact that

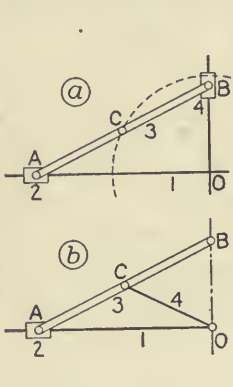

FIG. 454.

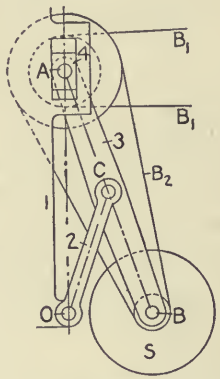

FIG. 455.

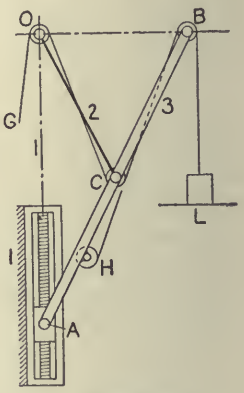

FIG. 456.

True-circle mechanism and applications.

middle point C travels on a circle. In sketch (*b*) the slide block at B is replaced by a radius rod or crank OC, and center B now generates a straight line. We have passed from (T_2S_2), cross S_2 fixed, to the common slider-crank with the special proportion that rod AC equals crank OC and that the crank has only oscillatory motion. It is because radius OC enters into all mechanisms evolved from the form in Fig. 454 that we call them the true-circle class, in contrast with the approximate devices considered in Art. (*f*).

Figure 455 shows a grooving saw, for taking cuts of constant depth in the top face of a board on its table. Driving belt B_1 comes from a countershaft pulley so located that the small up-and-down movement of shaft A does not affect the length and

tension of belt. In Fig. 456 is outlined a simple constant-level luffing crane. By luffing is meant radial or in-and-out movement of load, produced by swing of jib 3. Evidently pin-center distance BCO is constant, but the necessary use of fairly large pulleys introduces a small error. If the hoist drum is on frame 1 there must be three pulleys, if the drum is mounted on the jib at H only end pulley B is necessary. With pulleys, what happens to load level as the crane is luffed out?

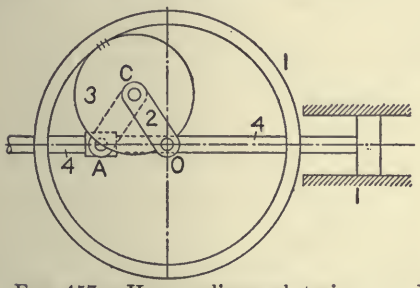

FIG. 457.—Hypocyclic crank train.

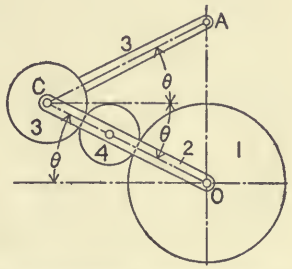

FIG. 458.—Special epicyclic train.

An interesting combination of gear train and linkage is seen in Fig. 457, where crank OC rolls gear 3 within 1, crank CA is fast on 3 and the straight-line movement of pin A is applied to driving a piston rod: the sketch suggests use of opposed cylinders, which would be a good pump arrangement.

Figure 458 shows how the same straight-line motion can be produced by means of an epicyclic train. The essential thing is that with arm 2 held fixed gear 3 shall turn twice as fast as gear 1 and in the same direction; this is secured in Fig. 458 by making 3 half the size of 1 and putting in idler 4 (size immaterial) for double reversal.

Prob. 228. Using pulleys of 18-in. diameter, what will be the variation in height of load due to 30 deg. of jib swing in Fig. 456, with the one-pulley and the three-pulley arrangement?

Prob. 229. Apply to Fig. 457 the criterion of constraint, Eq. (109) and interpret result. How about Fig. 458?

Prob. 230. Make sketches (as many views as needed) to show clearly how the drive in Fig. 457 will be arranged so as to permit full rotation without interference, providing ample bearings where needed. Then elaborate the scheme so as to have a second pair of cylinders with stroke line vertical.

(*e*) **Constant-level Luffing Cranes.**—For a long-reach crane the elementary arrangement in Fig. 456 has control mechanism OCA too big with respect to jib length AB; also the center of

gravity of jib moves up and down, so that there is not equal ease of luffing in either direction. In Figs. 459 and 460 are shown two designs with the same true-circle mechanism OCA, with extended but balanced jib, and with compensation of rope length so that

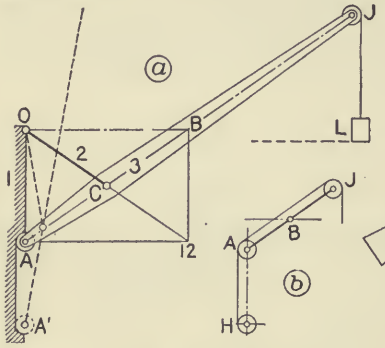

FIG. 459.—Crane with floating jib.

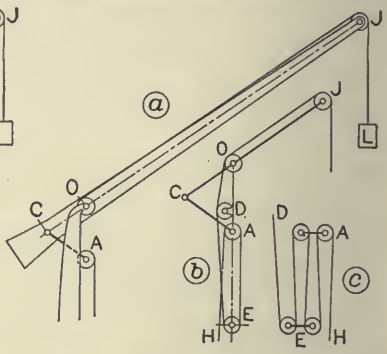

FIG. 460.—Use of compensating pulleys.

load remains at same level during luffing. Of course, if no weights are lifted or lowered the only work performance is that against friction.

For correct relations in Fig. 459 length AJ is four times AC or CO; then center B moves on a horizontal line, the jib turning at

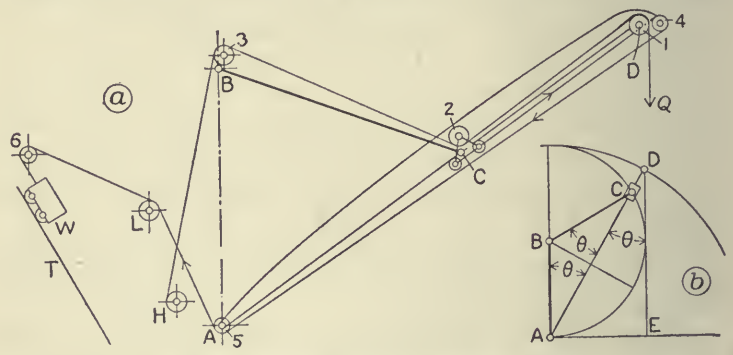

FIG. 461.—Special Mitchell-Williams design.

the instant on center 12. Pulley arrangement is shown by diagram sketch (*b*), where it appears that even pulley effects are balanced at J and A. Would it be better to bring the center of gravity of jib out to B or to leave it (as naturally due to construction for bending strength) a little inside of B? Note use of roller wheels at A, instead of sliding guide. Crosshead A can be moved

by a screw as in Fig. 456, by a crank and rod, or by a hydraulic plunger.

In Fig. 460 the jib is pivoted at O and arm OJ is eight times arm OC; then the vertical movement of J is just four times that of crosshead A. The rope arrangement is shown by sketch (*b*), with spread-out diagram (*c*) to make compensating action clear. By having two pulleys on a fixed axis at E and two on crosshead A, change in height of A is quadrupled for the rope, just compensating the change of length JL. Again the net effect of guide pulleys J and O is zero.

A different geometrical scheme is used in Fig. 461. Consider diagram (*b*): with AB = BC and AD = AB + BC, we have AC = (AB + BC) cos θ = AD cos θ = DE. Then length CD-DE is constant, as is also BC-CD-DE. On the crane slide block C becomes a little carriage, jointed to radius rod BC and running on the under side of jib. The lifting rope runs over pulleys 1, 2 and 3 to hoist drum H: remember the convention that crossed center lines indicate a fixed axis. By minor adjustments, in the location of pulleys 2 and 3 and in a slight slant of carriage track from line AD, the effects of wrap and unwrap during luffing are very nearly neutralized.

The common tendency of jib weight and of load is to swing the jib down or to pull C toward A. Position is controlled and luffing effected by a rope from carriage C out to pulley 4, then back over pulley 5 at A to drum L. To compensate for weight of jib another rope on drum L runs to counterweight W, which goes down as the jib rises. Since CD + DE = AD in diagram (*b*) we have that movement of C along jib is equal to vertical movement of D and therefore in constant ratio to rise and fall of center of gravity of jib. Because of this constant velocity ratio a uniform balance can be maintained as the crane luffs. By the use of an inclined track T partial balancing of load Q is secured, as its overturning moment varies on the crane body.

Prob. 231. Sketch suitable compensating arrangements for ratio OJ/OC equal to 6 and to 4 in a crane like Fig. 460.

(*f*) **Approximate Straight-line Motions.**—In the exact form of ellipsograph straight-line motion, Fig. 454, pivot O lies right on the path of point B. If this condition is undesirable it can be avoided by choosing some other point D (that is, other than middle point C), plotting its ellipse with B truly on the straight path, then fitting a circle to as much as possible of this ellipse

and getting the mechanism in Fig. 462. Form (*a*) has the path of B well beyond all supporting parts. Form (*b*) is likely to be a little more accurate, but link QD crosses the path of B. At the end of longer ellipse axis *a*, case (*a*), the radius of curvature is b^2/a and QD will be a little longer; at the end of shorter axis

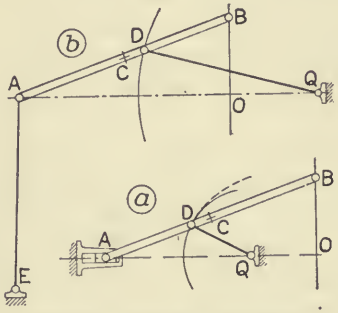

FIG. 462.—Grasshopper motion.

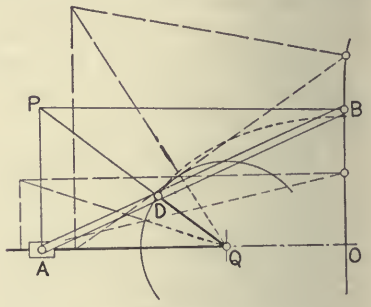

FIG. 463.—Testing point path.

b, case (*b*), the radius is a^2/b and QD will be a little shorter. These devices give good approximation up to an inclination of about 25 deg. (of AB with AO), so that the effective total "stroke" of B is about 0.8 AB. For this range A has so short a movement that it can be almost exactly guided by means of a long radius bar, as in Fig. 462 (*b*).

The most sensitive test of correctness of action of such approximate straight-line mechanisms is furnished by the instant center.

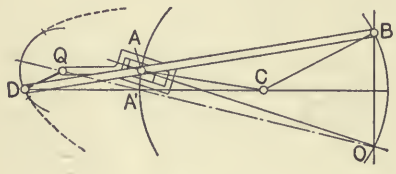

FIG. 464.

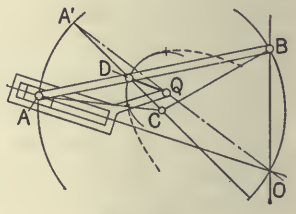

FIG. 465.

Oblique grasshopper motions.

Thus in Fig. 463 we find center P, draw radius PB, and by its slight departure from the horizontal find how much the generated line is inclined from the vertical at B.

Instead of actually plotting the ellipse to get DQ, a good scheme is to put B on the true path at O and at points about two-thirds the way out to ends of range (up and down), then pass a circle through the three D-points thus found. This will fairly

well balance up the departures of circle from ellipse: at first it goes a little outside the ellipse, then swings more strongly inward, as indicated by comparison with dotted curve in Fig. 463.

Just to show the possibilities of the ellipsograph scheme two oblique arrangements are drawn in Figs. 464 and 465. Start with inclined axes OA and OB and straight bar AB. Find center C and draw the smaller centrode circle. Take D on bar AB, extended in the first case, draw a diameter through D and C and locate axis OA′ of the ellipse traced by D. Plot this ellipse, fit circle to it and put in radius bar QD.

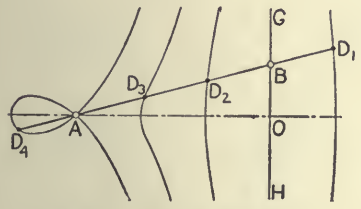

FIG. 466.—Plot of conchoids.

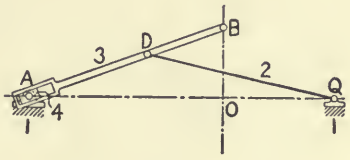

FIG. 467.—Conchoidal mechanism.

Let a rigid line pass always through a fixed point A, Fig. 466, and have one of its points constrained to straight-line path GH. Then other points will trace variations of the conchoid curve, all symmetrical with respect to AO, which is perpendicular to GH. To a portion of this curve a circle arc can be fitted, with an effect similar to that in Fig. 462. Embodied in Fig. 467 this is seen to give the slider-crank inversion that is used in the oscillating engine.

Prob. 232. Grasshopper motion, Fig. 462; take AB = 8″ and by trial find proportions that will put Q at 3″ from BO (toward A) and give best approximation over a range of 6″ of movement of B. Having the mechanism, test it by instant centers at four points, namely, for B at 1½ and 3 in. up and at ¾ and 2¼ in. down. Also plot the whole path of B, traced during a complete rotation of QD.

Use the circle scheme suggested above, which does not require the plotting of ellipses.

Prob. 233. With OA = 8″ design a conchoidal mechanism, for the conditions of the last problem and with similar testing of result.

Prob. 234. With obliquity of 20 deg. and AB = 5 in. lay out oblique ellipsograph straight-line motions as follows: Instead of taking D on AB put it on AC or on BC extended backward, Fig. 465, getting QD relatively longer than in that example but keeping Q to the left of OB. Determine range of good action in each case and test several points by means of instant center.

(*g*) **Symmetrical Guiding.**—This title covers devices in which the tracer-bar member is an isosceles triangle, with its base corners guided in symmetrical paths The special exact mechanism in Fig. 468 is obviously the simplest case of Fig. 451. For a more general type, start in Fig. 469 with the same pair of inclined axes OM and ON, draw a centrode circle on bisecting line OH, and choose two tracing points K_0 and L_0 symmetrical with respect to OH. Here these points are taken on radii CA and CB, so that OA and OB are the axes of the ellipses which they

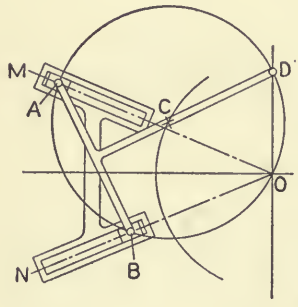

FIG. 468.—Reuleaux straight-line motion.

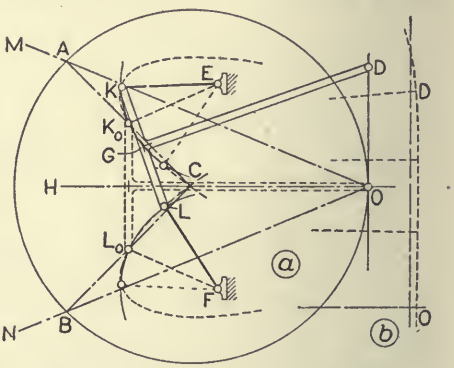

FIG. 469.—Derivative from ellipsograph.

trace. Over a moderate range circle arcs can be fairly well fitted to the true curves and embodied in guide links EK and FL. Dotted diagram (*b*) shows the result of an about double-size plot of the mechanism here derived, with a vertical straight line drawn close beside the path of D for comparison. The nearly horizontal lines shown at intervals up OD are instantaneous radii or normals to path.

Beside showing a derivation or relationship, Fig. 469 suggests a scheme of design, useful when triangle DKL is the primary given quantity. Linkage EKLF can be laid out by the trial method of choosing angle MOH and length OA, then plotting path of K and fitting an arc to it:* successive trials would be made until good proportions were found. With the linkage given the question to be determined is the best location of tracer point

* Understand that for this purpose OA is a tracer bar, to be guided like DA or DB in Fig. 468: it is a line definitely located in the moving plane of body DKL.

on center line GD. For this we use the instant center after a manner suggested in Fig. 470. The linkage having position ABCD, find center 13 at P, draw a vertical PF (perpendicular to the desired straight-line path) and find where it crosses center line GE, at F. If a series of F positions can be even moderately well represented by a line parallel to AD the location of tracing point is indicated.

The completely isosceles form of this mechanism, shown in Fig. 470, is found to give good results: by making AB = BE = EC = CD we insure that the path of E shall pass through

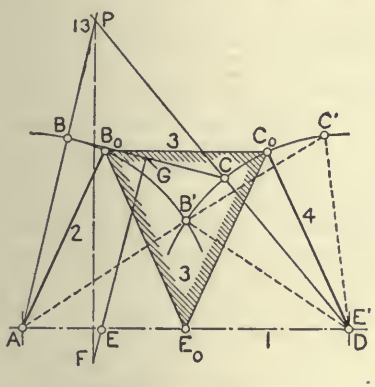

FIG. 470.—Roberts motion.

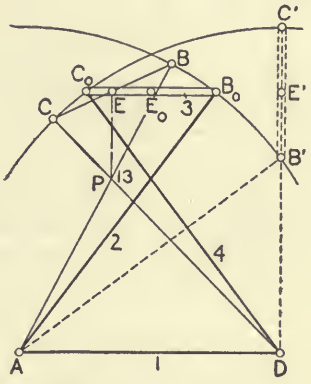

FIG. 471.—Chebicheff motion.

pin centers A and D. As drawn the linkage is very close to the limiting proportion that makes AB′C′ a straight line when E is at D: for best working it ought to be a good deal taller with respect to a given base AD.

(*h*) **Compensatory Guiding.**—Into the working of the linkage mechanism derived in Fig. 469 enters a considerable element of *compensation*. That is, the degree of correctness attained depends largely upon whether the departures of circles from ellipses neutralize each other as errors or whether they combine accumulatively. In Fig. 471, while yet having an arrangement symmetrical with respect to the vertical center line, we introduce the principle of full **compensatory guiding.** This means that as a straight tracer bar moves and one guide point is deflected from the path of tracer point E, the other point receives a compensating deflection with respect to that path. Proportioned as in Fig. 471, with coupler BC half of base AD and rods AB and DC of such length that when DC′ is vertical B′C′ will

lie upon it and have E′ at same height as E_0, the mechanism traces almost exactly a straight line out to E′.

The outstanding example of this type of tracer-bar guiding is found in the Watt parallel motion, Fig. 472, which has had some important applications. Tracer point E is so located as to

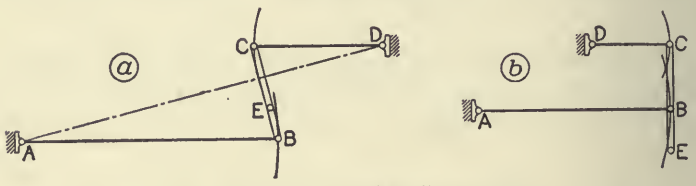

FIG. 472.—Watt straight-line motion.

divide bar BC into segments inversely as the nearer radius bars, or so that EB: EC = DC: AB. The argument for this location is that the sidewise deflections of points B and C are almost inversely as their radii of path (less for the long radius, more for the short one), hence they balance or compensate at a point located according to the proportion stated. Middle or reference

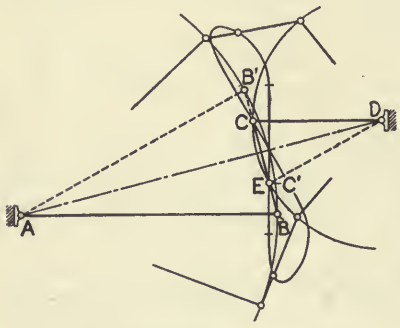

FIG. 473.

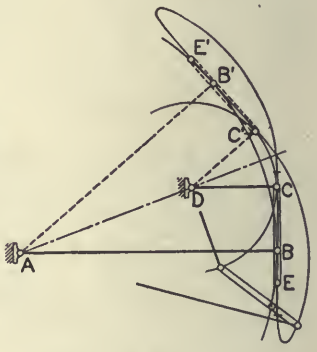

FIG. 474.

Full plot of Watt motions.

position is determined by parallelism of bars AB and CD, and the path of E is perpendicular to this bar direction (or nearly so): obliquity of tracer bar BC, measured in this position by its departure from a perpendicular to the bars, is an important dimension. Form (*a*), with opposite guide links, is the useful one; the other is a possible rather than a practical arrangement.

In Figs. 473 and 474 are shown complete paths of point E, for the two forms of linkage. Dotted outlines, with primed letters,

show the reversed mid-positions, the whole curve being in every case symmetrical with respect to line AD.

Prob. 235. Given DG = 4″ and KL = 2.5″ in Fig. 469, find proportions to make D trace a straight line 4″ long: treat the mechanism as a derivative of the ellipsograph. Plot path of D, running beyond its straight range; and within that range test direction at several points by means of instant center.

Prob. 236. In a linkage like ABCD of Fig. 470 let AD = 6″, BC = 3.5″, AB = DC = 4″: by the instant-center method find best location of tracer point E, then plot and test its path as directed in the last problem.

Prob. 237. For Fig. 470, with BC = 0.5 AD, find length of AB and BE that will just make AB′C′ a straight line when E is at D.

Prob. 238. Plot and test path of E in Fig. 470, for the limiting proportions just found, also for AB = 0.8AD and for AB = AD. In the first (shortest) case plot the whole path of E. Use AD = 5″.

Prob. 239. Find length of rods AB and CD in Fig. 471 (in terms of base AD) for the condition drawn and described in Art. (*g*). Plot the whole path of E, then try effect of making the rods either longer or shorter than the particular value computed.

Prob. 240. In a Watt motion like Fig. 472 (*a*) let AB = CD and BC = ⅔AB. Try it first with BC perpendicular to AD at midposition, plotting the whole curve, determining length of its useful part, and applying the instant-center test to several points of that part. Then try the effect of increased and of diminished obliquity, drawing a separate figure for each case.

Prob. 241. Similarly investigate a Watt motion, type (*a*), with AB = 2CD and BC = CD.

The graphical problems in this group are of the variation type, in which the effect of a variable can advantageously be tried out by assigning different data (over a well chosen range) to different members of a class or group. In the plotting of movements and point paths it is best to use an accurate template, as already suggested by Fig. 452.

(*i*) **Copying Devices.**—This name describes a class of devices which divide in constant ratio a straight line of varying length. They embody the simple geometrical proposition that a line parallel to the base of a triangle divides the sides in the same ratio. The class is composed mostly of variations of the pantograph, which makes solid bars serve as one side, base and parallel, leaving the variable side of the triangle purely geometrical. As shown by Fig. 475, it is derived from the jointed paralellogram, with sides extended as necessary. In general a straight line drawn across the figure will intersect all four of the sides, as at E, F, G and H. Three of these points are chosen for use, as E, F, H, or E, G, H.

The pantograph is a standard instrument for enlarging or reducing maps and drawings, being made adjustable so as to

provide different ratios of change: any full catalog of drawing supplies will show examples. The essential thing in making adjustment is to be sure that the three points are in line; otherwise some peculiar results will be obtained. If in line for one

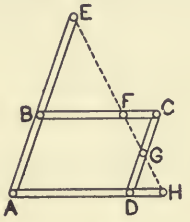

FIG. 475.

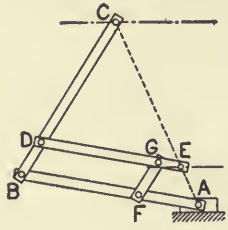

FIG. 476.

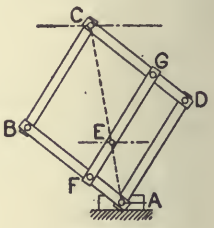

FIG. 477.

The pantograph and its applications.

position the points must remain in line as configuration changes.

Two applications of the pantograph as reducing motion for the engine indicator are given in Figs. 476 and 477, the first suitable for long-stroke, the second for short-stroke engines. Point A is the fixed joint and pin C is on the engine crosshead. The cord is attached at E, and must be led off parallel to the stroke-line of C.

Figure 478 shows the multiple-unit pantograph, called the "lazytongs," here likewise applied as an indicator rig. For the

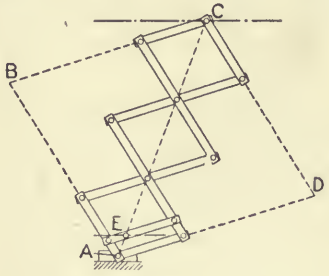

FIG. 478

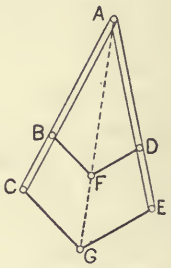

FIG. 479.

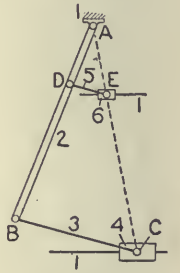

FIG 480.

Various constant-ratio mechanisms.

big simple parallelogram ABCD is substituted a series of smaller ones with successive sides in common. This mechanism is compact, especially when folded up close, but the multiplicity of joints is not favorable to maintained accuracy.

The six-link arrangement in Fig. 479 is a possible ratio mechanism, with A, F and G as active points, but it has nothing to

recommend it as a substitute for the pantograph. The reducing motion in Fig. 480 typifies the scheme of similar unit mechanisms, small (T_3S) form ADE being proportioned exactly like larger (T_3S) form ABC. For the use shown it is less flexible than the pantograph and is more complex of construction than the simpler pendulum devices of Fig. 410.

(*j*) **Applications of Straight-line Motions.**—In the early days of steam engineering the straight-line motion had a place in

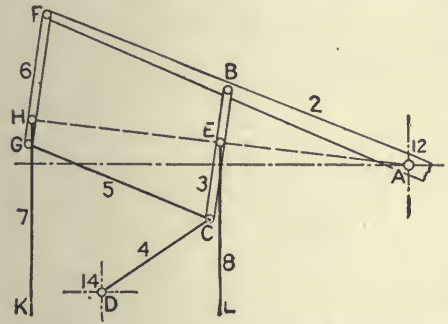

FIG. 481.—Outline of beam engine with Watt motion.

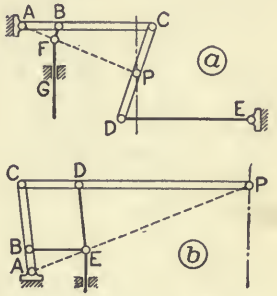

FIG. 482.—Watt and pantograph indicator mechanisms.

heavy machinery which it no longer holds. Developed by Watt from the earlier Newcomen mine pumps, the first power engines naturally had a "walking beam" between cylinder and shaft. Partly because of undeveloped shop facilities for producing planed guides, partly to simplify frame construction and save space, it was desirable to couple the upper end of piston rod to the beam as directly as possible. The outline in Fig. 481 shows how this was accomplished. Piece 2 is half of the oscillating beam or lever; from the end of its right-hand half a connecting rod ran to the crank pin. Linkage ABCD is a Watt motion, giving to point E an essentially straight-line path. Pantograph ABFGC produces an enlarged copy of this movement at H. The main piston rod took hold at H, while E drove the rod to condenser pump.

An important present use of the straight-line motion is in the pencil mechanism of the engine indicator. The function of this mechanism is to give a magnified copy of the movement of the indicator piston, as variable pressure (against resistance of spring) causes that piston to move up and down. To the kinematic requirements of straight-line path and constant travel

ratio for the pencil is added the necessity of minimum mass, in order that the instrument may be quick-acting without serious disturbance by inertia. The first indicator to use such a mechanism (Richards, 1863) had the Watt motion ACDE in Fig. 482 (*a*), with pantograph effect ACP-BF for proportionality. This magnified the motion of piston, but yet carried too much mass

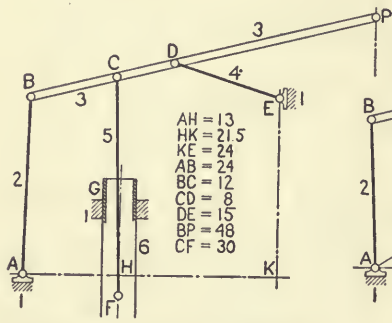

FIG. 483.—Thompson indicator.

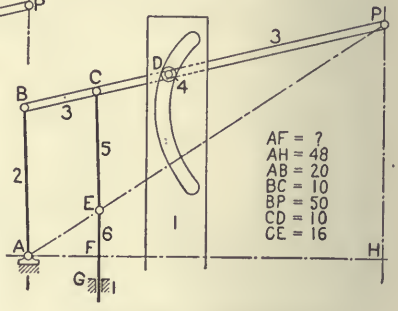

FIG. 484.—Tabor indicator.

for high speed, in coupler CD and extra arm ED. Figure 482 (*b*) shows a much later application of the complete pantograph, with the single disadvantage that the guiding of P in path depends upon that of E, and a close fit of piston rod in guide is not desirable because friction must be minimized.

The next two indicator mechanisms, Figs. 483 and 484, use the grasshopper motion of Fig. 462, either directly or with the replacement of guide link by cam slot—the latter being laid out

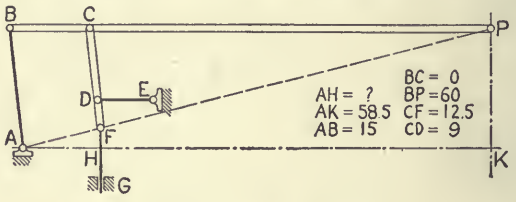

FIG. 485.—Crosby indicator.

for exact travel of pencil P on a vertical straight line. Note that Fig. 483 does not conform to the pantograph scheme, but the error due to such departure is exceedingly small.

The Crosby mechanism, Fig. 485, is essentially a six-link chain, with guiding all concentrated on member CF and dependent upon radius rod DE: in Figs. 482 (*a*), 483 and 484 there is a complete

and constrained quadric unit outside of the piston rod and its connection.

Prob. 242. Design a pantograph rig like Fig. 476 to get an indicator card 4 in. long from an engine of 48-in. stroke, with *vertical* height from A to C about 30 in.; also one like Fig. 477 for a 3-in. card from a 12-in. stroke, distance AC about 12 in.

Prob. 243. Design a flexible pantograph rig like Fig. 476 to give indicator cards between 3 and 4 in. long from engines of 24, 30, 36, 42 and 48 in. stroke. Take AB equal to BC and choose this length; make DE and FG longer than drawn and put in a series of holes so that DE can be moved up and down on bars BC and FG, pin E being at the same time moved in and out on DE. Decide upon the spacing for these holes that will give best service.

Prob. 244. Apply the criterion of constraint, Eq. (109), to Fig. 481 and explain result.

Prob. 245. For pantograph proportions in Figs. 484 and 485, and in the unit of measurements given, what are the correct lengths of the dimensions marked AH and AF? What are the ratios of pencil to piston movement in the three dimensioned examples?

Prob. 246. Plot Fig. 485 to a large scale, say 5 units to the inch, first with BP horizontal as here drawn, then with P at 16 units up and down. Find arc of circle that will pass through the three positions of D, thus determining link DE and pin center E. For the two extreme positions find instant center of pencil bar BP, thus testing direction of path of P. Methods for the last requirement will be found in Part II, Sect. 48.

Prob. 247. (*a*) Reasoning in general terms, does the use of long coupler CF in Fig. 483 (instead of pantograph proportions) introduce much error in travel ratio?

(*b*) Analyze the guiding of pencil in Fig. 485, considering effect of looseness in pin joints D, E, F and slide G: compare this mechanism with Figs. 483 and 484 as regards positiveness of pencil guiding.

The question of close analysis for travel ratio (or velocity ratio) in a mechanism like Fig. 483, as also in the approximate indicator rigs of Fig. 410, will be considered in Part II, as a velocity-ratio problem.

(*k*) **Parallel Motions.**—The most important parallel movements are those produced in machine tools, fundamental to all accurate machine construction: the prismatic slide predominates, forms are simple, precision and rigidity are the essential elements. Here, however, we are interested in a group of various devices from other fields, selected with the purpose of showing different ways of making a body move without rotation.

1. *Parallel Rolling.*—The form of parallel rule in Fig. 486 (*a*), intended for accurate work on the drawing board, has a heavy metal body and two short milled rollers R, R, fast on an axle. Exactly the same effect is secured in the ordinary travelling crane, with the (carrying and propelling) wheels at the two ends

driven by a common shaft, from a motor on the side of the bridge. It was early found that friction was sufficient to insure parallel movement, provided that the wheels were of truly equal diameters and that the crane was set squarely in position upon its tracks. The one exception to positive gearing together of the traversing wheels is found in the very long outdoor gantry bridges, used for handling ore, etc.: a gantry crane is a bridge with legs, that can run on tracks at ground level. Here separate motors on the two carrying trucks are tied together electrically; but the structure is not quite rigid, turntable support being provided between

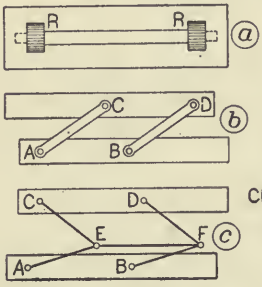

FIG. 486.—Parallel rules.

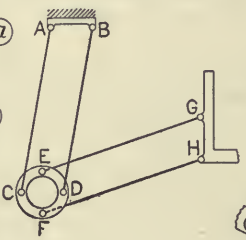

FIG. 487.—Universal drawing machine.

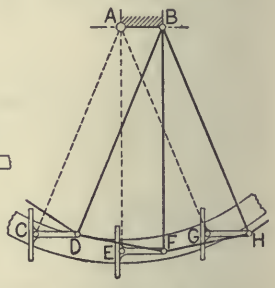

FIG. 488.—Feathering paddle wheel.

truck and crane foot. Only a few angular degrees of freedom are desired, and the electrical control must permit adjustment of relative motor positions.

2. *Parallel Linkages.*—These are commonly applications of the jointed parallelogram, as seen in Figs. 486 to 488. The parallel rule in Fig. 486 (*b*) is an old-fashioned piece of draftsman's equipment: the duplicate linkage at (*c*), necessarily having an intermediate parallel rod EF, is of greater range, with more freedom of position for the moving bar. The "universal" drafting machine in Fig. 487 is an interesting substitute for T-square and triangle: it depends on the rigid angle between diameters CD and EF of the coupling ring; and instead of the rigid arrangement here sketched at GH there is another circle, so that the straightedges can be set and clamped at any slant.

The simplest form of feathering paddle wheel, with strictly parallel movement of the paddles, is represented by Fig. 488. Connecting links DF, FH, are not needed at bottom and top of wheel, but are put on for the sake of positive passage of the dead points on the horizontal diameter; and the polygon -DFH- might

just as well be a light extra wheel rim. Generally this linkage was modified so as to give the paddle a small angular movement during its passage through the water; and with the abandonment of parallelism the dead or ambiguous points could be avoided.

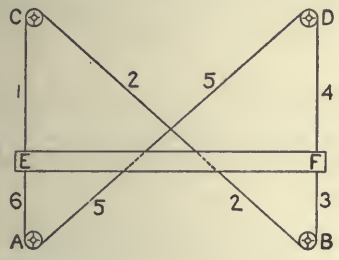

FIG. 489.

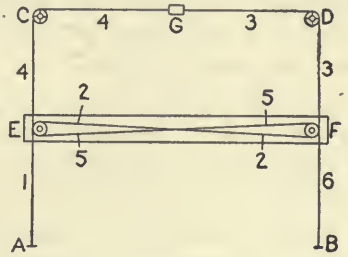

FIG. 490.

Cord devices for parallel movement.

3. *Cord Devices.*—Two effective schemes, both used on drawing boards (and occasionally in light machinery) are shown by Figs. 489 and 490. Following the cord segments in sequence of numbering, it is easy to see how parallelism is maintained as straightedge EF moves up and down. At G in Fig. 490 is a

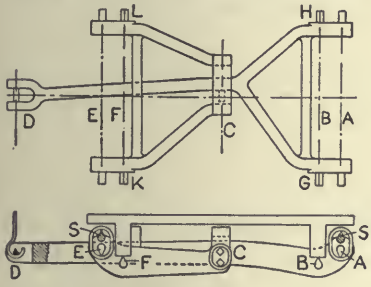

FIG. 491.

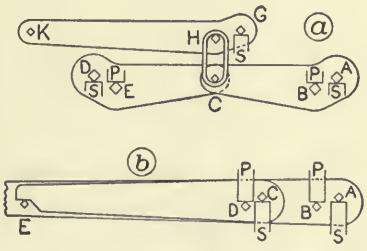

FIG. 492.

Lever arrangements for platform scales.

clamp on cord 3-4, for adjusting angular position of bar EF: which way will cord 3-4 move when EF is given a small clockwise rotation?

4. *Lever Mechanisms for Platform Scales.*—Any appropriate lever arrangement for a platform scale must provide truly parallel movement of the platform: then no matter where the load is placed there will be a constant velocity ratio between load and poise, hence also a constant force ratio. The top view in Fig. 491 shows how the flexibility of three-point support really under-

lies the four-point support of the platform, being secured by the use of two triangular levers GHC and KLC. For light service the outer lever is prolonged beneath the inner, to receive the support from weighing lever. For heavy service the two primary levers act on a third, GK in Fig. 492 (*a*). Sketch (*b*) shows the first levers of a testing machine, with different lengths but having the same ratio of arms.

Prob. 248. Modify the scheme of Fig. 489 so as to avoid running cord segments 2 and 5 diagonally across back of drawing board. Make clear diagram of new arrangement. How provide for easy change of bar slant? *Suggestion,* Use double pulleys at C and D.

(*l*) **The Peaucellier Mechanism.**—The next device to be taken up is predominantly geometrical, constituting an advanced example of the scheme of building a mechanism about a proposition of

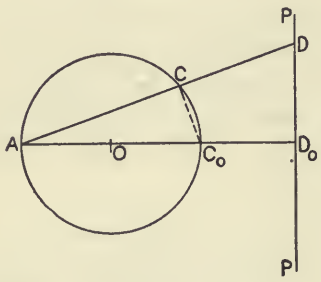

FIG. 493.—Geometry for straightline generation.

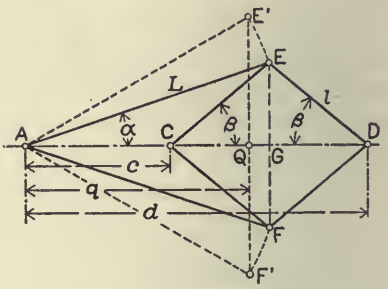

FIG. 494.—The Peaucellier linkage.

geometry. In Fig. 493 straight line PP is perpendicular to extended diameter AD_0 and ACD is any secant line from A. By means of similar triangles AC_0C and ADD_0 it is shown that $AC \times AD = AC_0 \times AD_0 = \text{constant}$. Consider now the linkage in Fig. 494, consisting of a jointed rhombus CEDF and equal radius bars AE and AF. Using symbols as indicated,

$$AC = c = L \cos \alpha - l \cos \beta,$$
$$AD = d = L \cos \alpha + l \cos \beta.$$

So that $cd = L^2 \cos^2 \alpha - l^2 \cos^2 \beta = L^2 \cos^2 \alpha - l^2 + l^2 \sin^2 \beta$. But vertical $EG = L \sin \alpha = l \sin \beta$, therefore

$$cd = L^2 - l^2 = q^2. \qquad (112)$$

Graphically the constant q is got as the base of right triangle AE'Q, having L and l as hypotenuse and one side—or by folding

the rhombus vertically flat so that its four sides lie together on line E′F′.

Figure 495 shows joint A fixed and a radius rod BC added to constrain point C and represent the radius of the circle in Fig. 493. Within the limits of possible swing (what fixes them?)

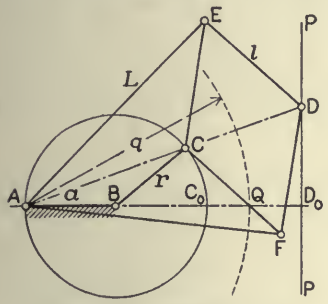

FIG. 495.—Direct form.

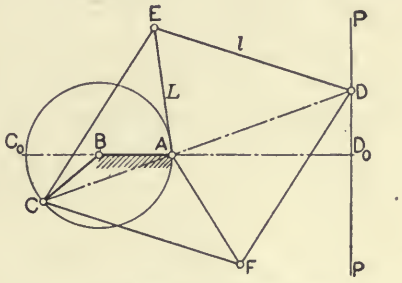

FIG. 496.—Inverted form.

Complete Peaucellier straight-line motions.

point D traces a perfect straight line. An alternate form of the mechanism, with L shorter than l, is drawn in Fig. 496: it has the effect of interchanging points A and C_0 of Fig. 493. How is limit of swing now determined? Prove Eq. (112) from and for Fig. 496.

To have pole A right at the end of a diameter in Fig. 493, or base a equal to radius r in Fig. 495, is a special case of the law

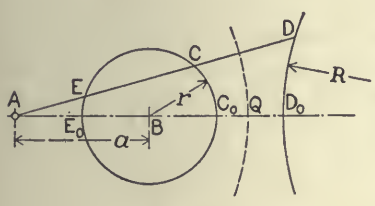

FIG. 497.

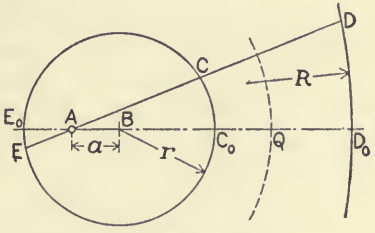

FIG. 498.

Geometry for generation of circular arcs.

of constant segment-product on secant ACD. If a is greater or less than r, as in Figs. 497 and 498, point D describes a true arc of circle. Consider any secant ACD in relation to basal secant AC_0D_0. Constructing so as to make $AC \times AD = AC_0 \times AD_0$, we have in either figure

$$\frac{AD}{AD_0} = \frac{AC_0}{AC}.$$

But for the circle itself, whether with secants in Fig. 497 or with chords in Fig. 498, $AC \times AE = AC_0 \times AE_0$, whence

$$\frac{AC_0}{AC} = \frac{AE}{AE_0}.$$

This gives the generally prevailing proportion $AD : AD_0 = AE : AE_0$, so that curve DD_0 must be similar to EE_0 and therefore a circle.

The ratio of size is evidently $R : r = AD_0 : AE_0$. Now AE_0 is $a - r$ or $r' - a$ and $AD_0 = q^2 \div AC_0 = q^2 \div (a + r)$: then the radius of the generated circle is

$$R = r \times \frac{q^2}{(a + r)(a - r)} = r\frac{q^2}{a^2 - r^2}. \qquad (113)$$

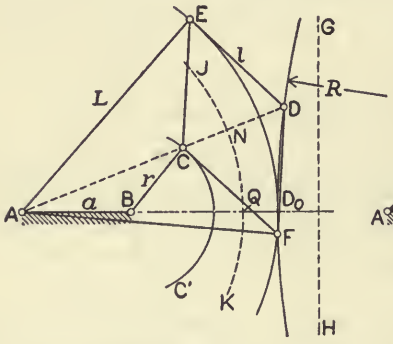

FIG. 499.

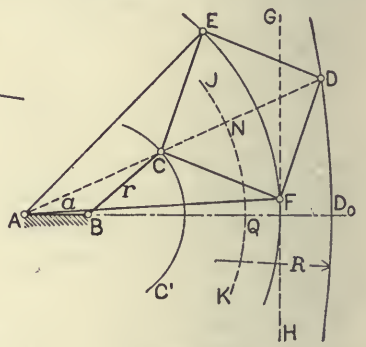

FIG. 500.

Peaucellier "Universal Compass."

Embodiment of Figs. 497 and 498 appears in Figs. 499 and 500. The proportions of both are represented by $l = 2$ in. and $L = 4$ in., making $q^2 = 12$. For Fig. 499, $r = 1.3$ in. and $a = 1.7$ in., wherefore

$$R = 1.3 \times \frac{12}{2.89 - 1.69} = 1.3 \times \frac{12}{1.20} = 13 \text{ in.}$$

For Fig. 500, $r = 1.5$ and $a = 1.0$, so that

$$R = 1.5 \times \frac{12}{2.25 - 1.00} = 1.5 \times \frac{12}{1.25} = 14.4 \text{ in.}$$

(*m*) **Automobile Steering Rigs.**—Under this title will first be shown a design which exactly meets the ideal requirements of wheel configuration for true running on a curve. Geometrically its simple result is reached through a rather intricate relationship. Mechanically it is too complex and contains elements of a kind

not well adapted to the conditions of service. Comparison of the simpler form universally used, approximate but good enough, will furnish an excellent example of the balancing of influences in a compromise solution.

Approaching the problem from the geometrical side, start with isosceles triangle GAB in Fig. 501. Let its external angles E_0AC_0 and F_0BD_0 be established as rigid figures, representing the short

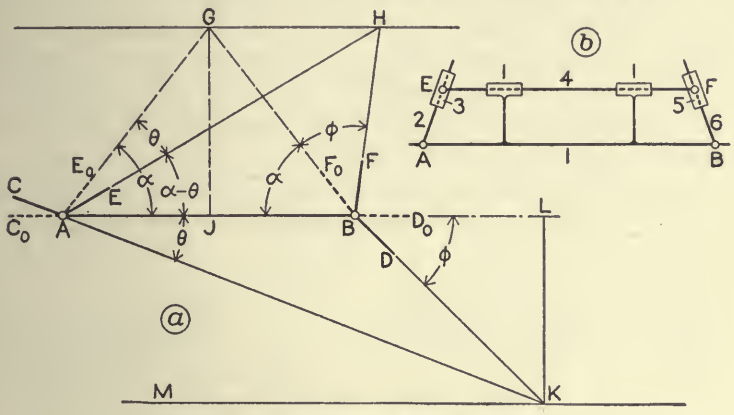

FIG. 501.—Davis steering-gear mechanism.

swivelling axles with their actuating arms. On line GH, parallel to base AB, choose any point H as apex of a new triangle HAB, turning the rigid angles into conformity with sides HA and HB, as EAC and FBD. The new directions of CA and BD intersect at K, and we are interested in the size relationship between LK and JG.

Triangle AJG or BJG gives, very simply, $JG = \frac{1}{2}AB \tan \alpha$. From ALK we have $AL = LK \cot \theta$ and from BLK, $BL = LK \cot \varphi$, whence $AB = LK (\cot \theta - \cot \varphi)$. From these expressions comes the ratio

$$\frac{JG}{LK} = \frac{1}{2} \tan \alpha (\cot \theta - \cot \varphi). \qquad (114)$$

Now in triangle HAB, $\dfrac{\sin(\alpha - \theta)}{\sin(\alpha + \varphi)} = \dfrac{BH}{AH}$. Again, triangles AGH and BGH are equal in area, whence

$$\tfrac{1}{2}\, AG \cdot AH \sin \theta = \tfrac{1}{2}\, BG \cdot BH \sin \varphi;$$

but $AG = BG$, so that

$$\frac{\sin \theta}{\sin \alpha} = \frac{BH}{AH} = \frac{\sin (\alpha - \theta)}{\sin (\alpha + \varphi)}.$$

Expanding and combining like terms,

$$2 \cos \alpha \sin \theta \sin \varphi = \sin \alpha(\cos \theta \sin \varphi - \sin \theta \cos \varphi);$$

and division gives

$$1 = \tfrac{1}{2} \tan \alpha(\cot \theta - \cot \varphi),$$

so that in Eq. (114) LK = JG.

Now the effect of triangle HAB can be produced by the mechanism at (*b*), where rod EF establishes a parallel to base AB, at constant length and at constant height, so that altitude to apex H will also be constant. Then if AC and BD are the front axle journals and MK is the line of the rear axle, point K will be an exact instant center, about which the wheels will roll without any individual component of cross slip.

Obviously, the sliding joints in this mechanism would be hard to keep properly clean and lubricated and the whole contrivance

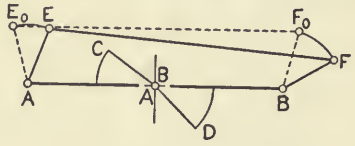

FIG. 502.—Front arms.

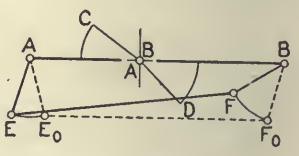

FIG. 503.—Rear arms.

Common steering-gear linkage.

would be likely to wear loose and rattle. For these reasons the simple quadric linkage outlined in Figs. 502 and 503 is regularly used, preferably in the second form, or with the coupling rig back of the axle.

The general requirement, as seen from Fig. 501, is that the inner wheel (there right-hand) shall turn farther than the outer wheel. To secure this effect the jointed linkage must be so proportioned that an outer arm (as AE) swings past a position perpendicular to axis AB while the inner arm swings farther to one side—fuller discussion of this question of mechanism geometry will be found in Part II, Sect. 44 (*k*). Then if the arms are in front of the axle they must flare outward—just the reverse of Fig. 501 (*b*)—while if behind the axle they incline inward. To save diagram space and for better comparison the two axle positions are drawn together, at the middle of each figure. A close analysis of this mechanism will be found in Part II, Sect. 44 (*n*), taken up as a case of plotted movement.

(*n*) **Cutting Arcs of Circles.**—Several devices for machining arcs of large radius are shown in Figs. 504 to 506, all of them

applications of the quadric chain (T_2S_2). In the first arrangement piece 2 is a planer table with stroke line CD. To it is bolted a base plate carrying a cross guide for block 3 and surfaces that support the work table 4, which swings on a fixed pivot at B. It is not necessary that tool T be on a cross line through B, but this particular condition gives a cutting direction parallel to CD.

The second rig, Fig. 505, is arranged as for a slotting machine, tool T having a vertical cutting motion, perpendicular to the plane of the drawing. Piece 1 is the base of the machine, saddle

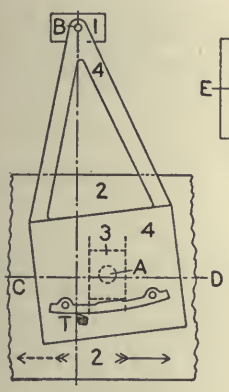

FIG. 504.

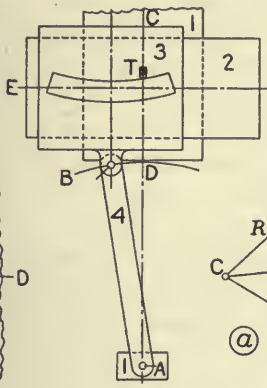

FIG. 505.

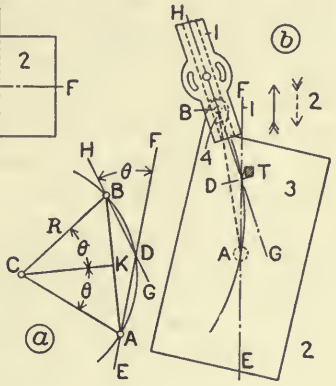

FIG. 506.

Devices for cutting flat arcs of circles.

2 moves in direction CD and 3 slides crosswise on 2. No matter where the work is bolted on Table 3 the arc cut has AB as radius, because this is a case of circular translation; consequently pivot A must be adjusted for any desired change of radius.

The third device, Fig. 506, has not the obvious radius in action that appears in the other two. Its geometrical principle is shown at (*a*). If we have a circle ADB on center C and a fixed chord AB, then as point D moves along its arc the angle ADB is constant. Conversely, a rigid angle EDH, moving on guide points A and B, traces with its apex D an arc of circle. This is really an inversion of the ellipsograph, for now it is a question of the path traced by a point of the cross, upon a plane fast to the bar; and the circle here shown agrees with the smaller centrode circle in that mechanism. Angle θ or FDH, supplement to EDH, is measured by half of arc AB, hence is equal to half the apex angle of isosceles triangle ACB. Radius R is then equal to $\frac{1}{2}$ AB/sin θ.

In the embodied mechanism (*b*) work Table 3 is pivoted at A to a base plate bolted on planer Table 2, so that A must move along line EF. Another pin B on piece 3 is guided along line GH, by a slotted bar bolted beneath the planer crossrail. If tool T is placed at intersection D it will trace an exact arc of circle on Table 3. Now a planer tool is positioned (very nearly) at what would be a fixed horizontal line across Fig. 506. For this reason it cannot generally be brought exactly to intersection D, so that the device is mostly used in an approximate fashion, giving very fair results for short, flat arcs.

(*o*) **Cutting Ellipses.**—Beyond its moderate practical utility, a full discussion of this problem is of great kinematic value and interest, because it offers so broad an example of mechanism inversion or of various ways of combining elements to a common end. The problem is somewhat more intricate than that of merely generating the curve, for two reasons: it involves the presentation of a proper cutting edge and such arrangement of the mechanism that, with needed strength, there will be no interference of parts. Virtually without exception, workable devices are derived from the basal form in Fig. 446, where the tracing point is far out from a short bar AB. As with the subject of Art. (*k*), there are a number of distinct schemes, which can be presented in logical order under subtitles.

1. *Cross Held, Cutter Bar Turns.*—An almost exact application of Fig. 446 is shown by Fig. 507. Spindle 2, carrying tool arm BD (keyed fast), has bearing in a head or saddle that can slide sidewise (along line EBP), as on the arm of a radial drill. Upon this spindle is also keyed an eccentric of center A, connected by rod 3 to a fixed pin. The error due to arc instead of straight line as guide for point A is insignificant. Practical objections are that the eccentric will be much in the way on a drill spindle and that it is not adjustable in throw or radius.

What is really required for production of the ellipse is a rotation of arm BD (or AD) plus a harmonic reciprocation. The latter element may be separated mechanically and applied either to spindle or to work table. Thus in Fig. 507 a crank might be mounted on the machine arm, geared to turn exactly with the spindle, and coupled to the spindle head by a rod or a cross slot. In Fig. 508 crank OA is supposed to be thus positively geared with spindle 1, but now its motion is imparted to work Table 3 and the spindle axis is immovable. It is not even necessary that

BD and OA turn in the same direction, provided only that they are parallel when in the direction (here horizontal) of table sliding.

2. *Work Turns, Cutter Arm Held.*—This is the general scheme of the long used elliptic chuck, shown in Fig. 509, which contains the elements of the Oldham coupling, Fig. 409. In its design the principal problem is to avoid interference of parts. Here 1

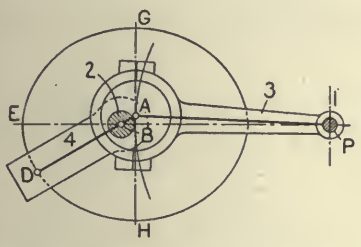

FIG. 507.—Fixed cross, on radial drill.

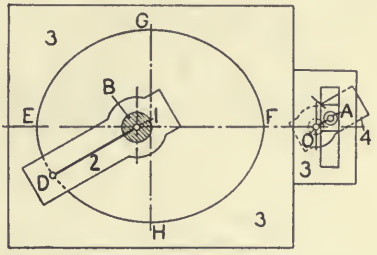

FIG. 508.—Separate harmonic reciprocation of work.

is a driving head screwed on the lathe spindle. In special frame 2 is carried ring support 3, which can be adjusted so as to vary offset BA. On 3 turns ring 4, having projections to fit the T-slot on the back of cross member or face plate 5. View (*b*) shows assembly up to the face plate, view (*c*) gives the latter alone, with its right-angled slots dotted and an ellipse outlined.

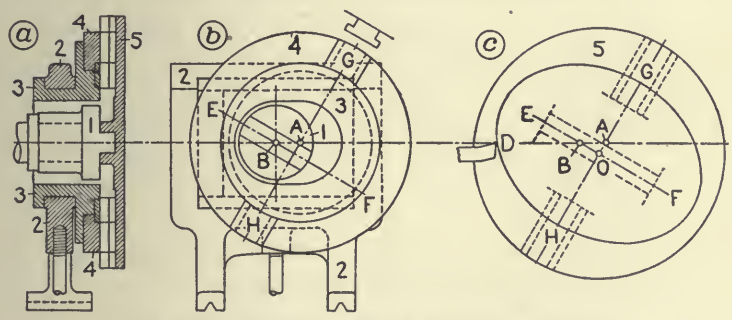

FIG. 509.—The elliptic chuck, slotted-cross style.

Now in Fig. 509 we may say that plate 5 turns with spindle 1 and is given a harmonic reciprocation along (rotating) diameter EF, by the action of ring 4 through slide GH. A highly ingenious variation, partly outlined in Fig. 510, substitutes a gear-train effect analogous to that in Fig. 508. Point O represents the axis of lathe spindle and circle 1 is a gear fast to the headstock: on it roll the equal gears 2 and 3, held in the chuck body at axes

A and C. These gears carry crank pins B and D, with provision for varying arms AB and CD. One crank, as AB, operates the cross-sliding face plate to which work is fastened; the other drives in opposed reciprocation a second slide back of the face plate. By the combined motion of the two slides, of approximately

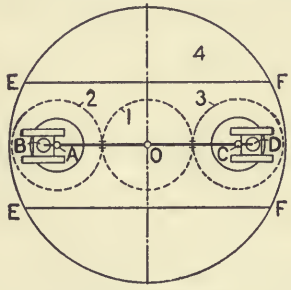

FIG. 510.—The Pryibil elliptic chuck.

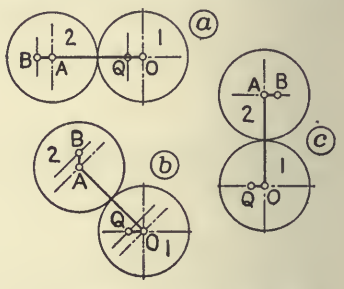

FIG. 511.—Action of gearing.

equal weight, the center of gravity of the system is kept practically at axis O, giving a good dynamic balance at high speed.

Action of the gear-train mechanism is explained by Fig. 511. In one revolution of the chuck gear 2 makes two turns in space or one turn with respect to arm OA. Equivalence to the common form, Fig. 509, is shown by putting in its second fixed axis Q and

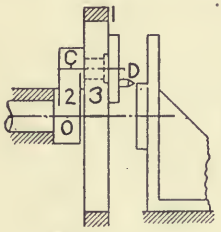

FIG. 512.

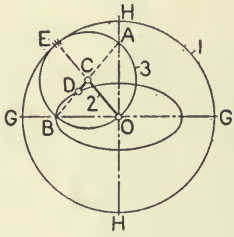

FIG. 513.

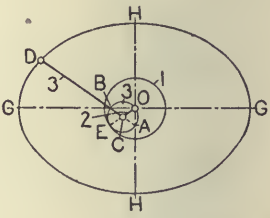

FIG. 514.

Direct use of hypocyclic ellipsograph.

drawing through Q the center line of the second slide, GH in Fig. 509. With mechanism as shown, which axis of the ellipse will lie along AC, major or minor?

3. *Use of the Hypocyclic Train.*—For this type of ellipsograph, as outlined in Fig. 513 with inside tracing point, a facile embodiment is shown in Fig. 512. Annular gear 1 is fast to frame, crank 2 rolls gear 3 within it, tracer D is on a plate attached to this gear, and the work piece is on a fixed angle plate. Striking

objections are, the large size and clumsiness of the mechanism, its lack of flexibility and the fact that the tracer bar makes all kinds of angles with the curve, so that only a scribing (not a cutting) tool could be used. The last fault disappears with outside tracing point, Fig. 514, but here the gear pair becomes too small for strength and even for practical construction.

Now in this special cyclic train proportions are such that with arm 2 held fixed gear 3 turns in the same direction as 1 and

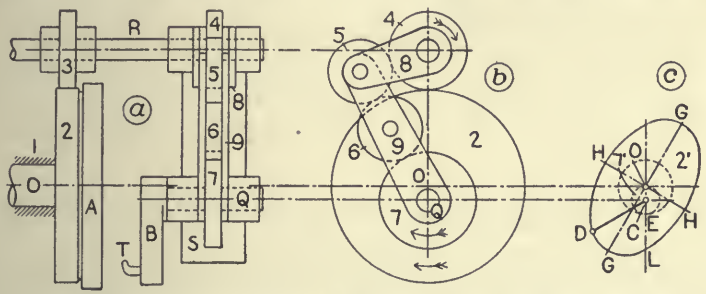

FIG. 515.—Use of extended gear train: both work and tool turn.

twice as fast. Keeping the two gear axes fixed, suppose that we produce this relative turning by means of an external gear train, as suggested in Fig. 515. Here gear 7 on axis Q is half the size of gear 2 on axis O, transmission gears 3 and 4 are equal and idler system 5-6 gives flexibility for adjustment of offset OQ. Face plate A being fast to gear 2 and tool arm B keyed with gear 7, an ellipse will be cut under the condition that *both work and tool arm are turning.* Snapshot view (*c*) shows angle DCL twice GOL (from initial coincidence of DC and OG on line OL), indicating that the tool travels clear around the curve during the revolution of face plate A. The centrodes 2′ and 7′, resultant from the transmission train and identical with the actual small pitch circles of Fig. 514, are dotted in on this same diagram (*c*).

For combined ruggedness, flexibility and readiness of adaptation to an existing machine this scheme is the best of all that have been discussed.

Prob. 249. Look up the Pryibil chuck, Fig. 510, in catalog or in journals of November or December, 1909; study its complete construction and make a clear sketch of the device for varying crank arms.

(*p*) **Toggle Mechanisms.**—As the slider-crank mechanism approaches dead center there is a rapid rise in the velocity ratio

of crank to slide, or in the ratio of possible resistance Q to driving force P, Fig. 516: just where the advance of slide stops, or where crank and rod are in line, that ratio becomes infinity (ideally, or without friction). This effect of getting a tremendous "purchase" by straightening out a flat angle between two mem-

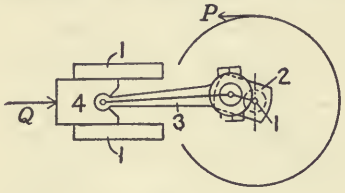

FIG. 516.—Toggle effect in crank press.

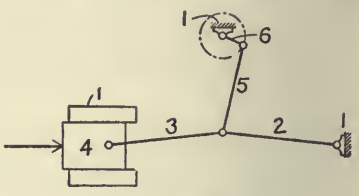

FIG. 517.—Standard toggle mechanism, press or rock crusher.

bers is called a toggle effect. The name toggle comes from and strictly belongs to the combination 1234 in Fig. 517. With crank-and-rod drive thus applied to joint 23, this is a standard mechanism for rock crushers and for heavy presses that must exert a big force during a large part of their "stroke." The direct crank-and-rod drive in Fig. 516 belongs to the lighter type of

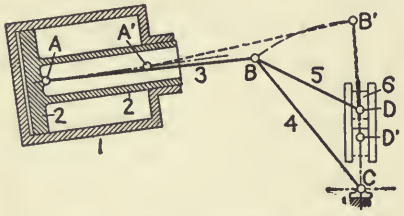

FIG. 518.—Allen pneumatic riveter.

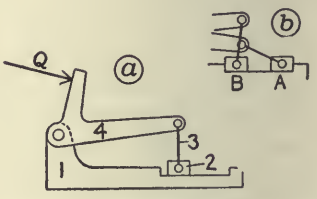

FIG. 519.—Gripping mechanism of friction clutch.

press, except when only the last little bit of the stroke is used for heavy work, as in coin impression and in the extrusion of a collapsible tube from a flat planchet.

The pneumatic riveter outlined in Fig. 518 contains the alternate toggle form 1456 in which member 4 is in tension and 5 in compression: by this mechanism a moderate air pressure on piston 2 is enabled to produce a big "closing-up" pressure on the rivet.

In effect a toggle mechanism is a (T_3S) unit operating near its dead point. Figure 519, which is typical of the gripping

mechanisms used in friction clutches, shows an inversion to slide as driver and also a distinct utilization of the principle of self locking. As a converse to the rapid rise of the resistance Q surmountable by a given driving force P, we have that with a given Q the required P tends to vanish at dead point. In Fig. 519 this occurs in position (*a*), where link 3 is perpendicular to the path of slide 2. For security the final position B, sketch (*b*), is a little past dead point, so that the clutch holds itself definitely locked.

Other linkage mechanisms will be given and discussed in the opening section of Part II, where they will furnish problems first in the plotting of movements, later in the determination of velocities and accelerations.

SECTION 31. CHAMBER TRAINS AND NONPLANAR LINKAGES

From the viewpoint of kinematics the title Chamber Trains covers the class of rotary pumps and motors that function by means of actual volume displacement—this in contrast with the more important class of pumps, blowers, turbines, etc., that operate by means of the kinetic action of streams of fluid. Planar chamber trains fall into two divisions, (*a*) those having a single rotor or impeller placed eccentrically within the cylindrical casing, which are really built around some form of the slider-crank linkage; and (*b*) those having two rotors, each concentric with its part of the casing, the two working together like a pair of gears.

(*a*) **Eccentric Trains with Turning Crank.**—In Fig. 520 the simple rotor R is a rigid eccentric on shaft O, while the other two examples have an inner eccentric E and an outer body R which need not rotate but only oscillates. Always, the variable displacement volume is contained between contact Q (of rotor with casing) and a sliding partition S. In what is essentially the primary slider-crank form of the first two examples this partition has the motion of an engine piston. The geometrical function of connecting-rod link CP is variously taken: in Fig. 520 the pair at P is force-closed, as by connecting the top of cylinder S to the high-pressure side of the machine, while in Fig. 521 it is mechanically positive. With slide S a part of the rotor, in Fig. 522, there is inversion to the linkage of the oscillating engine, Fig. 398.

(*b*) **Eccentric Trains with Fixed Crank.**—Figures 523 to 525 show a one-piece rotor R turning on a fixed axis, so that contact Q holds an unchanging position and partitions S, S must rotate. When they are radial to the rotor the chief mechanical problem

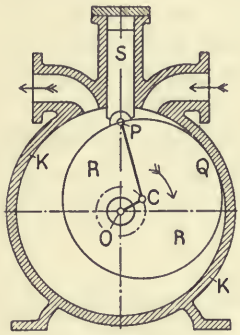

FIG. 520.

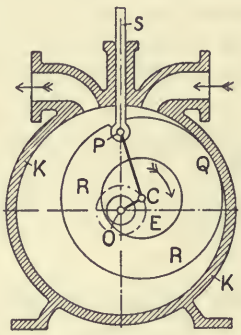

FIG. 521.

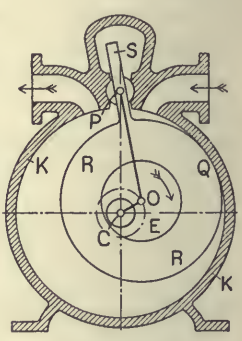

FIG. 522.

Turning-crank chamber trains.

is to keep them in contact with the casing, or, again, to realize connecting-rod member CP. In Fig. 523 ring segment AB, integral with block D, fills a groove turned in the end of casing: the two segments are of such length that their ends are close

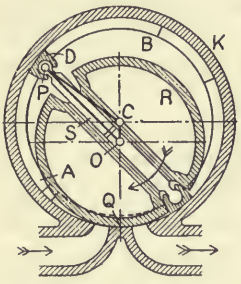

FIG. 523.

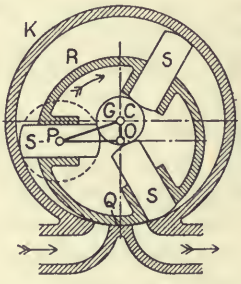

FIG. 524.

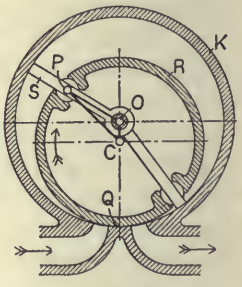

FIG. 525.

Fixed-crank chamber trains.

together when passing Q, so as not to open a channel between inlet and outlet; and for symmetry of force action there can be a ring and groove at each end of D, or in both heads of the casing.

Another scheme is shown by Fig. 524, where each slide S is a segment of a cylinder, in double contact with the casing and with a central guide cylinder G. Although this guide might be a loose

piece, held in position by the three plungers, it may better be fast to one head. With the latter arrangement, which is definitely necessary in Fig. 525, there can be but one shaft section, carrying the rotor on its end. Note the inversion between connecting-rod and frame members (using names from the engine form of the slider-crank) in Fig. 525 as compared with the other two examples.

(*c*) **The Rotary-plunger Mechanism.**—Consideration of the rotary-cylinder engine in Fig. 400 will bring out its essential kinematic resemblance to Figs. 523 to 525, the cylinder frame

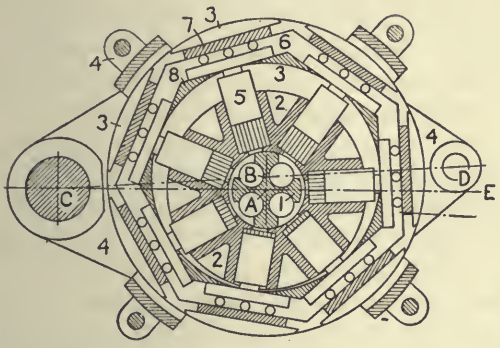

FIG. 526.—Rotary oil pump, "Oilgear" hydraulic transmission.

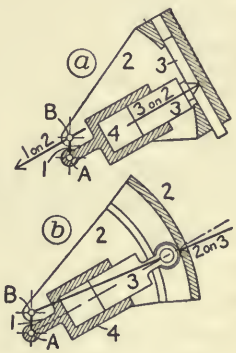

FIG. 527.—Analysis of torque action.

corresponding with rotor R. This similarity is even stronger in Fig. 526, where the cylinders are turned inward and casing K, although modified in form, has real existence. Here cylinder block 2 turns on fixed hollow shaft 1, axis A, the necessary valve action being secured by means of this relative motion. Casing 3, given the form of a polygon in order to eliminate side pressure between plungers and cylinders, turns with 2 but on axis B. Plunger 5 and flat head 6 are fastened together as one rigid piece, the head having a roller bearing on reaction plate 7 (fast in 3) for the heavy pressure of instroke and a plain bearing on surface 8 for the light pressure of outstroke. Casing 3 has its bearing in swing-frame 4, which is pivoted at C and is moved and held by a double hydraulic plunger that takes hold at D. By altering offset AB the delivery of pump can be varied from full capacity to zero, and by swing past AE reversal of effect is secured.

This pump is half of a variable-speed hydraulic "transmission," shaft 1 carrying a motor of like form but with fixed eccentricity AB. Since the pump has variable delivery with constant speed while the motor has constant capacity per revolution, the speed of motor can be varied at will with an accuracy dependent upon the sensitiveness of the control device that takes hold at D.

The outline of one unit in Fig. 527 (*a*) makes clear the fact that it is really a crossed slider-crank, rotating on the crank member as frame. The layout of principal forces, due to oil pressure, shows that with driving torque applied to member 2 (which represents the casing or member 3 of Fig. 526) there will be no torque on cylinder 4: hence in Fig. 526 the only side pressure between plungers and cylinders is that needed to overcome machine friction. Sketch (*b*) shows that with a cylindrical casing, whether it stands or turns, there must be side pressures because constraining force 2-on-3 is now not along the cylinder axis. In this case the torque must be applied to (or taken from) cylinder 4.

(*d*) **Chamber Wheel Trains.**—These are well represented by the group of examples in Figs. 528 to 531. The two rotors turn on axes A and B, with their pitch circles touching at C; and on these same pitch circles, usually outside of the casing, there is a pair of common multitooth gears to produce and insure correct relative turning. The six-tooth rotors in Fig. 528 will almost drive each other; and with even as few as six teeth it is possible to form profiles that will insure pitch-circle rolling—see Sect. 33 (*a*). Sometimes ordinary gears of twelve or more teeth are used, but with lowering of capacity because the radial depth of the tooth zone is less. The two-tooth rotor in Fig. 529 is regularly used in large "positive" blowers of the Root type. Figure 530 represents a type of design that has the serious fault of only a sharp-edge contact during parts of the cycle, together with the pocketing of some fluid in the space shown around C and its return from outlet to inlet. In Fig. 531 we have an example of unequal rotors, here 2 to 1 in ratio of pitch-circle diameters. Note in Fig. 531 that the tooth is all outside the pitch circle on rotor 1, the space all inside the pitch circle on rotor 2.

Chamber trains in general have been invented or designed for use as both engines and pumps. With steam they run most seriously into the two great difficulties of leakage on the one hand or excessive friction on the other, beside that of uneconom-

ical handling of the steam; and no rotary steam engine has ever been a commercial success. As small air or water motors they can be made to work fairly well. They are most successful as blowers, delivering air against moderate pressures (of a few

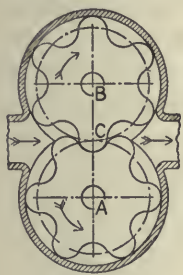

FIG. 528.

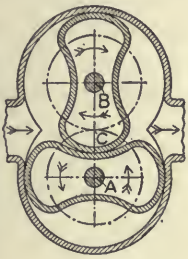

FIG. 529.

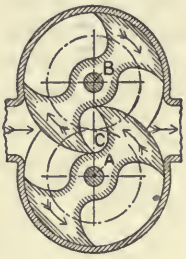

FIG. 530.

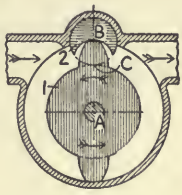

FIG. 531.

Chamber wheel trains.

pounds per square inch), and as pumps also for comparatively low pressures, especially with thick or sticky fluids. The absence of valves is an excellent feature in connection with the last-named kind of service.

Prob. 250. As pumps, would Figs. 520 to 525 deliver a uniform or a pulsing flow? How about Fig. 526 in this respect? Would the 120-deg. partition spacing of Fig. 524 be good with a liquid? Give a reasoned explanation for each answer.

Prob. 251. Assuming no loss by leakage, what is the volume handled per revolution by the single-rotor machines of Figs. 520 to 525 and by the gear-pair machines of Figs. 528 to 531, in terms of casing and rotor dimensions or areas?

Prob. 252. For Figs. 523, 524 and 525 sketch lengthwise sections (along axis), showing a workable construction in each case. Include arrangement of shaft and bearings and construction of casing and of rotor for assembly and for access.

(*e*) **Use of Ball-and-socket Joints.**—Coming now to linkages "in space," the simplest departure from coplanarity is to have cranks or arms in two planes, commonly at right angles, and connect them by a rod with ball-and-socket joints. The example in Fig. 532 has the special feature of a large driving eccentric, formed as the middle segment of a sphere. The purpose of this two-plane quadric linkage is to actuate a piston valve that lies across the top of a pair of vertical cylinders in a single-acting steam engine. The eccentric is not really fast to the shaft as sketched, but is carried by a shaft governor.

The use of members combining spherical and sliding pairs is seen in Fig. 533, where the arms of piece 4 slide (and turn) in balls 3 and 5 while the latter turn in cranks 2 and 6. The device is equivalent in overall effect to a pair of miter gears, but could be adapted to other shaft angles by changing the arm angle on piece 4. Since the two quadric units 1234 and 1456 are alike and symmetrical the transmission is uniform, follower 6 and driver 2 having always the same angular velocity. Each arm of oscillat-

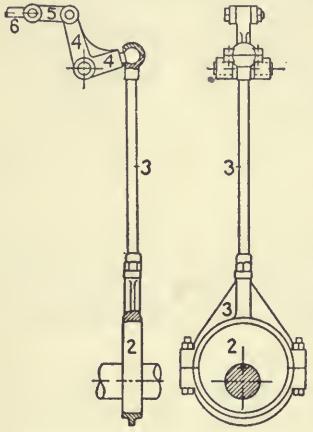

FIG. 532.—Valve drive of Westinghouse compound engine.

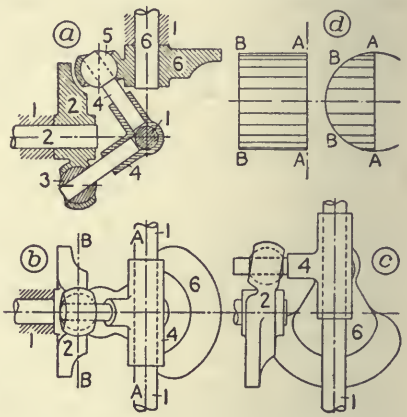

FIG. 533.—The Almond right-angle transmission.

ing slide 4 traverses or "describes" the conoidal surface sketched at (*d*), which has a circle for base and a line equal and parallel to a diameter of that circle for apex. Conditions as to force or power transmission are least favorable in the middle position shown by views (*a*) and (*b*), where the instantaneous motion of 4 is wholly sliding, and most favorable at an end position like (*c*) where 4 has only turning. The whole mechanism runs in an oil bath within a tight casing, and its power transmission is of the light-force, high-speed type.

(*f*) **Spheric Linkages.**—In Art. (*a*) of Sect. 29 emphatic statement has been made of the fact that the strong individual coplanarities of turning pairs tend to interfere unless the axes of such pairs are parallel. A simple but geometrically universal way of avoiding interference is to have all the axes of a linkage converge to a common point, making them radii of a sphere. This substitutes triangles for rectangles (not lines) as the essential geometrical forms of links, arcs for straight lines in a surface

representation of the linkage, and cones for cylinders as the paths of axes. The linkage outline can be shown upon a single sphere, as in Figs. 534 and 535, but in construction to avoid bodily interference the parts must lie and move in concentric spheres of differing radius. In general, links of the mechanism reduce to arcs of "great" circles of the sphere, while point or joint paths are "small" circles.

The true and complete representation of a spheric linkage by a single plane drawing is a rather troublesome problem in projec-

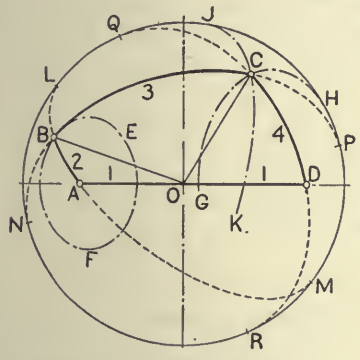

FIG. 534.—A quadric linkage on a sphere.

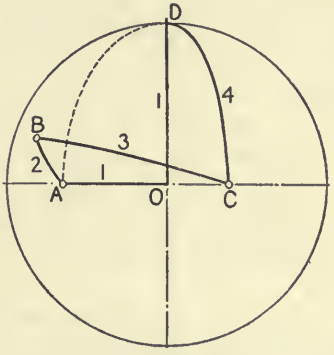

FIG. 535.—Spheric equivalent of the slider-crank.

tion, requiring that ellipses be plotted from all kinds of oblique positions of the circles which they represent. In Fig. 534, where only the final "front" view is shown, we make the simple start of taking frame member 1 in the horizontal plane, so that it appears as a straight line AD. Crank AB, at a particular angle, is an arc of semiellipse LBAM and the path of point B projects as BEF. To close the linkage we must swing CD on D and BC on B: the small-circle paths GCH and JCK are more difficult to draw than the great-circle arcs NBCP and QCDR. If spheric linkages were really used it would become necessary to devise a practical scheme for drawing on spherical sheets of paper.

On a sphere the straight-line path of planar sliding becomes the arc of a great circle. Consequently, as shown in Fig. 535, a link of quadrant length is equivalent to the common straight-line guide.

(*g*) **The Universal Joint.**—Also known in the older literature of mechanism as Hooke's joint, this is the one spheric linkage that is practically useful. No matter how constructed and pro-

portioned it has essentially the form in Fig. 536, with two semicircular forks 2 and 4 pin-jointed to right-angle cross 3. The important question is, What kind of motion does follower 4 receive from uniformly rotating driver 2?

For geometrical purposes we need use but one arm AB or CD of each fork and can reduce the cross to a quadrant link BC connecting them, in Figs. 537 and 538. Emphasis now passes strongly to the circle paths PBRS and PQCS in which pins A and C respectively travel. Corresponding with Fig. 536 link BC would be at PQ. Taking this as initial or zero position, we have angle POB $= \theta_1$ for the displacement of shaft 2 and QOC $= \theta_2$ for the coincident displacement of shaft 4. In the spherical triangle PBC let

side BC or plane angle BOC $= a = 90$ deg.;
side PB or plane angle POB $= b = \theta_1$;
side PC or plane angle POC $= c = 90 + \theta_2$;
angle BPC or diedral angle QOR $= \alpha =$ angle between shaft axes.

For this case of sides a and b with angle α known, the formula from spherical trigonometry is

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha. \tag{115}$$

Since $a = 90$ deg. and $\cos a = 0$, this reduces to

$$\cos \alpha = -\cot b \cot c \text{ or } \cot c = -\frac{\cos \alpha}{\cot b};$$

but $\dfrac{1}{\cot b} = \tan \theta_1$ and $\cot c = -\tan \theta_2$, therefore

$$\tan \theta_2 = \cos \alpha \tan \theta_1. \tag{116}$$

A complete set of results by Eq. (116), at each 15 deg. of θ_1 around the circle for $\alpha = 40$ deg., is given in Form 32. It appears that at each quadrant point angles θ_2 and θ_1 are together; but within the quadrants the follower alternately lags and leads, in like successive cycles of departure from the movement of the driver.

For a graphical illustration of the action which is made definitely quantitative by Eq. (116), first turn the mechanism into the position in Fig. 538, symmetrical with respect to a plane bisecting the angle α between pin-path planes PRS and PQS; then turn this PS plane horizontal (rotating the lower view in

Fig. 538 to the left through 90 deg.) and make in Fig. 539 a sort of development of the middle zone of the sphere—similar in idea to Mercator's projection as used in mapping the surface of

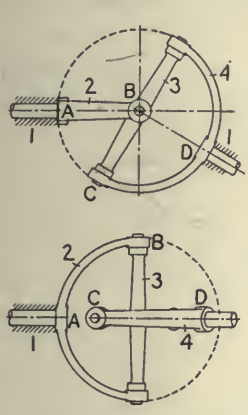

FIG. 536.—The universal joint.

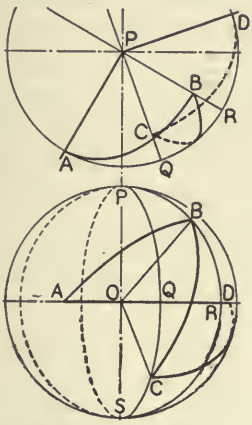

FIG. 537.—Layout for relations.

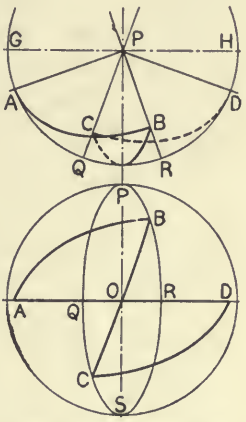

FIG. 538.—Layout for FIG. 539.

the earth. On this pseudodevelopment of the paths of B and C coupler BC is now drawn at half-quadrant intervals. In positions like 2 and 6 the effective length of BC suffers a shortening

FORM 32

θ_1	θ_2	$\theta_2 - \theta_1$	θ_1	θ_2	$\theta_2 - \theta_1$	θ_1	θ_2	$\theta_2 - \theta_1$
0	0		120	127° 0′	+7° 0′	255	250° 43′	−4° 17′
15	11° 36′	−3° 24′	135	142° 33′	+7° 33′	270	270	0
30	23° 51′	−6° 9′	150	156° 9′	+6° 9′	285	289° 17′	+4° 17′
45	37° 27′	−7° 33′	165	168° 24′	+3° 24′	300	307° 0′	+7° 0′
60	53° 0′	−7° 0′	180	180	0	315	322° 33′	+7° 33′
75	70° 43′	−4° 17′	195	191° 36′	−3° 24′	330	336° 9′	+6° 9′
90	90	0	210	203° 51′	−6° 9′	345	348° 24′	+3° 24′
105	109° 17′	+4° 17′	225	217° 27′	−7° 33′	360	360	0
			240	233° 0′	−7° 0′			

much like that of the engine connecting rod when it swings away from the stroke line. In positions like 4 and 8 great-circle quadrant BC is extended along what is really a small-circle distance, hence its effective length is increased.

The departure angle ($\theta_2 - \theta_1$) from Form 32 is plotted in Fig. 540, on θ_1 as base. While the waves of this curve are of like form

vertically they are not symmetrical horizontally, being alternately reversed right and left. If the angle base, instead of showing the movement of fork or shaft 2 on axis AD, were made to show the rotation of cross 3 on axis GH in Fig. 538, the distortion from harmonic waves would disappear.

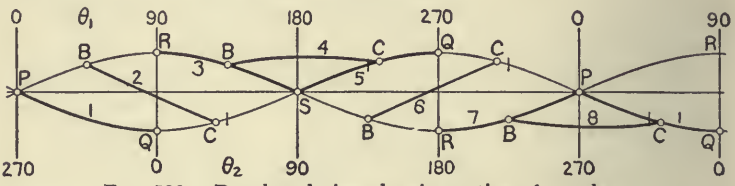

FIG. 539.—Developed view showing action of coupler.

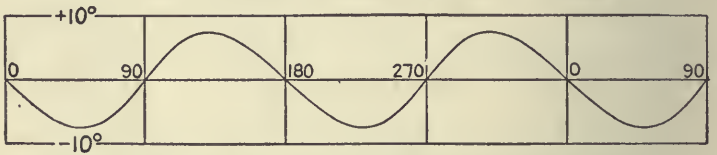

FIG. 540.—Angular departure of follower from uniform motion.

Positions, 2, 4, 6 and 8 of coupler BC in Fig. 539 are approximately positions of equal velocity for shafts 2 and 4. Exact equality exists where BR and QC, BS and SC, etc., are equal, or where BC makes the same angle with pin paths at both ends and departure angle ($\theta_2 - \theta_1$) is maximum. Thus for $\alpha = 40$ deg. as in Form 32 we have $\theta_1 = 45°$ and $\theta_2 = 37°27'$, with the difference 7°33′. After a trial or two a practically exact balance is found at $\theta_1 = 48°48'$ or 3°48′ ahead of 45° and $\theta_2 = 41°11.2'$ or 3°48.8′ behind 45°, with a corrected difference of 7°37′.

Variation of this maximum difference, as related to shaft deflection α, is shown in Table 6, up to the value $\alpha = 50$ deg., which is considerably beyond the range of practice. Remember that the total angular swing of the follower is twice the tabular value.

TABLE 6

α	1	2	3	4	5	6	7	8	9	10
0	0° 0′	0° 1′	0° 2′	0° 4′	0° 6′	0° 9′	0° 13′	0° 17′	0° 21′	0° 26′
10	0° 32′	0° 38′	0° 45′	0° 52′	1° 0′	1° 8′	1° 17′	1° 26′	1° 36′	1° 47′
20	1° 58′	2° 10′	2° 22′	2° 35′	2° 49′	3° 3′	3° 18′	3° 34′	3° 50′	4° 7′
30	4° 25′	4° 43′	5° 2′	5° 22′	5° 43′	6° 4′	6° 26′	6° 48′	7° 12′	7° 37′
40	8° 2′	8° 28′	8° 56′	9° 24′	9° 53′	10° 23′	10° 54′	11° 26′	11° 59′	12° 33′

(*h*) **Compensation by Pairing.**—For small shaft angles, and in rough machinery where a slight irregularity of motion is not seriously troublesome, a single universal joint may be used; but for smooth transmission it is necessary to employ two like "joints" in symmetrical arrangement, with a definitely directioned intermediate shaft between the two. If the three axes are coplanar, as in Fig. 541, intermediate OO′ must make the same angles α and α' with main shafts AO and A′O′: the latter may be either

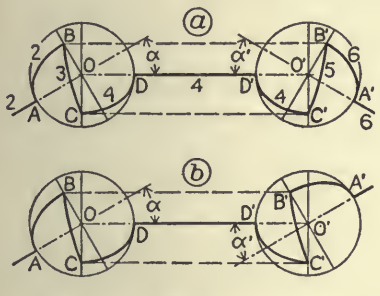

FIG. 541.—Side projections.

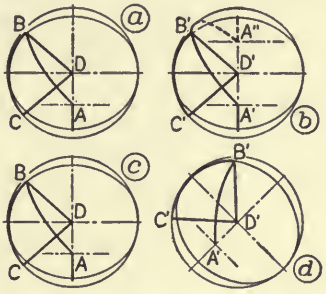

FIG. 542.—End projections.

Mutual compensation of two universals.

at angle $(\alpha + \alpha')$ or at $(\alpha - \alpha')$, the second condition making them parallel. Arms or forks CD and D′C′ being in the same plane through axis OO′, the two mechanism outlines show that any disturbing effect of BC is balanced by an equal and opposite effect of C′B′.

The end or side views in (*a*) and (*b*) of Fig. 542 help to make clear this compensation of irregularities. Drawing (*a*) is a section by a plane cutting intermediate OO′ and shows the left-end joint as viewed from the right, for either case of Fig. 541. Drawing (*b*) is a right-side view of joint A′B′C′D′, showing case (*a*) of Fig. 541 by A′B′ and case (*b*) by A″B′. It is evident that if arms AB and A′B′ are alike in angular position links BC and B′C′ will give equal displacements at arms CD and C′D′.

Suppose that the two shaft-axis planes AOD and A′O′D′, capable of relative rotation on axis OO′, are not in conformity. Thus in the second half of Fig. 542 view (*c*) is like (*a*) but (*d*) is turned 45 deg. clockwise as compared with (*b*), the two axis planes appearing in lines AD and A′D′. With this same angle between arms DC and D′C′ on the intermediate compensation is evidently maintained.

Parallelism of driver and follower as in Fig. 541 (*b*) is the usual condition in machine tools and in the main axle drive of an automobile—see Fig. 133. The type of arrangement at (*a*) has long been used for connecting the dial mechanisms of tower clocks; and it has an important modern application in the front-axle drives of four-wheel-drive trucks. When driven axles must swivel as shown in Figs. 502 and 503 the best way to transmit power is to use two universal joints at each wheel, with a short ntermediate between them like OO′ in Fig. 541.

Prob. 253. For a chosen or assigned value of α compute a table like Form 32, then plot a curve like the one in Fig. 540. Determine the maximum angular departure of follower from driver and its exact location.

Prob. 254. Sketch and discuss the drive for swivelling truck wheels as just described, arranging it so that the short intermediate shaft will go to correct angle in any position. Can you get perfect compensation? Is there any other available scheme of drive than by means of universal joints?

(*i*) **The Conic-crank Chamber Train.**—The simple spheric or conic crank, not extensive enough to be called a conic linkage, can readily be developed into a chamber train. Views (*a*) and (*b*) in Fig. 543 show the measuring element of the common small water meter. Hard rubber disc A, integral with ball B, has a conic rolling constrained by pin C against cone D. Inlet and outlet are on opposite sides of fixed diaphragm or partition E. The water current drives the disc, in a way that can best be understood by means of the cylindric section—by cylinder SS in view (*a*)—that is unrolled at (*c*). The horizontal arrows show direction of flow, those across portions of the serpentine section of the disc show the movement of the latter in the sense of volume displacement. Because the great-circle disc is rolling upon conical constraining surfaces that are of shorter circular length (with some friction) it has a continuous tendency to advance, hence presses against the left side of partition E as seen in top view (*b*). Pin C actuates the recording mechanism, through little crank F on shaft G. This whole unit is mounted within the outer, pressure-containing shell of the meter.

This chamber train has been used by a number of inventors of rotary pumps and engines, taking the form in Fig. 543 (*d*): note the device for securing surface contact with partition E while retaining freedom of motion. Troubles from leakage and friction —probably worse than in the cylindric forms—have naturally prevented success when it was a question of big pressure differ-

ence. The related piston or plunger engine is shown in Fig. 543 (*e*), where six cylinders are set in a circle, like the chambers of a revolver, and act upon rolling disc A. A good mechanical feature of the particular design is that heavy thrust is taken at

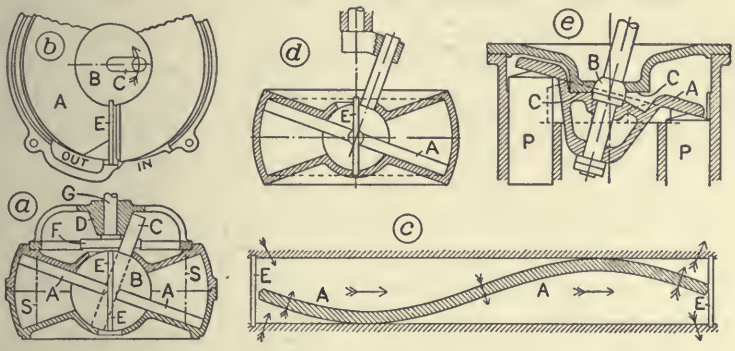

FIG. 543.—Spheric mechanisms with conic rolling of disc.

surfaces C, C, leaving to small sphere B only a minor guiding function. Lack of commercial success of this West steam engine was due not to inherent faults of the chamber-train engine, but rather to imperfections of steam action because of small cylinders and poor valve functioning.

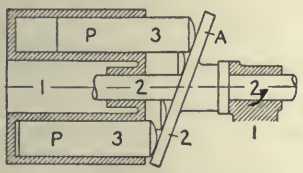

FIG. 544.—Disc turning.

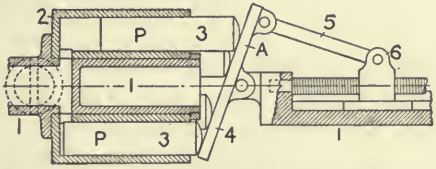

FIG. 545.—Disc standing.

Conic-rolling effect with inclined flat disc.

Exactly the same cycle of piston movement is secured by the mechanism in Fig. 544, where plate A is fastened obliquely upon shaft which rotates in front of the cylinder barrel. A useful modification or inversion, giving a variation of pump delivery like that from Fig. 526 and used in the same kind of service, is shown in Fig. 545. Here barrel 2 rotates and plate 4 is held fast, but at an obliquity that can be varied and reversed beyond its neutral at the vertical.

Note that in all these arrangements side pressure between plunger and cylinder cannot be avoided, also that the plunger-

and-plate pair is force closed. This means that during the low-pressure stroke, intake of a pump or exhaust of an engine, there must be enough excess above atmospheric pressure to maintain contact, as against friction and inertia of the plunger.

(*j*) **The Skew Parallelogram.**—Consider the planar parallelogram linkage ABC′D in Fig. 546 (*a*), having the random configuration determined by instantaneous angle θ_1. Drawing diagonal DB and keeping triangle DAB in the plane of the page, turn triangle BC′D on hinge line DB to position BCD, at

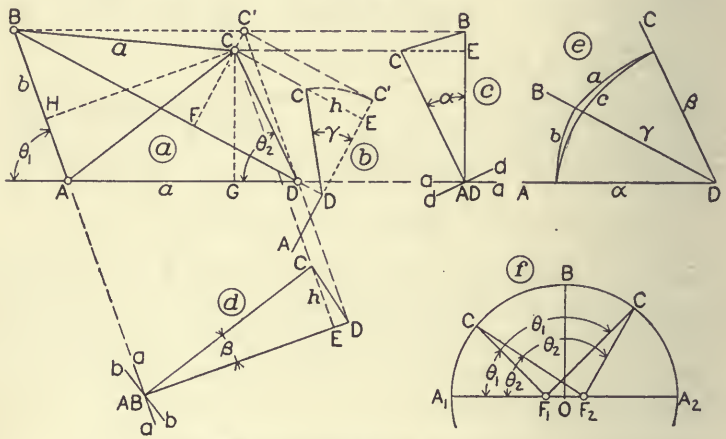

FIG. 546.—Bennett's skew parallelogram.

inclination γ as seen in view (*b*) and with point C at height h from plane DAB. Now with the help of diagonal AC define two more planes CDA and ABC; then if the axes at D and B are made perpendicular to these last planes there can be free turning at all four joints. The two axes at the ends of each side a or b remain perpendicular to the side, lying in parallel planes that are perpendicular to it; but with respect to the side as a common perpendicular they are now twisted, those connected by a at angle α, those connected by b at angle β. If we can show a relation between α and β that does not involve a variable such as γ or h it will be evident that the linkage is capable of continuous movement.

Draw projection (*c*) so as to get edge-on views or "traces" of planes ABD and ACD, with axes aa and dd perpendicular to them and at relative angle α; also projection (*d*) to show planes ADB and ACB and the similar relation for axes at ends of sides

b. In (*a*) show CG perpendicular to DA and CH to AB, their true lengths appearing as C-AD in (*c*) and C-AB in (*d*). Since triangles ACD and CAB have their sides respectively equal, angles CDA and ABC are also equal. Then in these triangles $CG = b \sin \theta_2$ and $CH = a \sin \theta_2$; but from views (*c*) and (*d*) we have

$$h = CG \sin \alpha = CH \sin \beta,$$

whence $$b \sin \alpha = a \sin \beta \quad \text{or} \quad \frac{\sin \alpha}{a} = \frac{\sin \beta}{b}, \tag{117}$$

meeting the stated requirement for continuity of motion without binding of joints.

It is obvious that angle θ_2, which measures the rotation of arm DC, is not in general equal to θ_1 of arm AB. Striking a sphere from D as center we get the spherical triangle shown at (*e*), with angle CDB as side *a*, ADB as *b*, ADC as *c* and the (diedral) angles α, β, γ, as indicated. The appropriate relation from spherical trigonometry is

$$\frac{\tan \tfrac{1}{2}(a + b)}{\tan \tfrac{1}{2}c} = \frac{\cos \tfrac{1}{2}(\alpha - \beta)}{\cos \tfrac{1}{2}(\alpha + \beta)}. \tag{118}$$

But $a + b = \theta_1$ and $c = \theta_2$, whence

$$\frac{\tan \tfrac{1}{2}\theta_2}{\tan \tfrac{1}{2}\theta_1} = \frac{\cos \tfrac{1}{2}(\alpha + \beta)}{\cos \tfrac{1}{2}(\alpha - \beta)}. \tag{119}$$

It can be shown, by a deduction too long to be reproduced here, that if an ellipse is drawn with its eccentricity—which is OF/OA in Fig. 546 (*f*)—having the value

$$e = \tan \tfrac{1}{2}\alpha + \tan \tfrac{1}{2}\beta, \tag{120}$$

then its radius vectors F_1C and F_2C will have the same relative motion as do arms AB and DC in mechanism (*a*).

The open and crossed parallelograms of Fig. 406, further shown in Figs. 435 and 442, are now to be considered as particular, limiting cases of this more general form. Discussion with respect to varying proportions will be found in the inventor's description, *Engineering*, vol. 76 or 1903II, page 777, which is reprinted in Dunkerley's "*Mechanism*."

Prob. 255. Assuming values of *a*, *b*, α and θ_1, draw the skew parallelogram as in views (*a*) to (*d*) of Fig. 546, computing β by Eq. (117) against its determination in (*d*) and θ_2 by Eq. (119). Then get eccentricity by Eq. (120), draw the ellipse like (*f*) and see whether the θ_2 thus determined agrees with that computed. Sample values are, $a = 5''$, $b = 3.5''$, $\alpha = 20°$, $\theta_1 = 70°$.

CHAPTER VII

FORM, ACTION AND PRODUCTION OF GEAR TEETH

This chapter presents the whole subject of gear teeth on circular gears in planar motion, as viewed from a predominantly geometrical standpoint. It stops short of the more general case of noncircular pitch curves and does not go into the question of tooth strength and force transmission. Along with toothed gears proper are included the two kinds of chain gearing, in Sect. 39.

The first three sections, numbers 32 to 34, cover the geometry of tooth profiles, developing the two common forms of curve, illustrating tooth action, working out the limitations of action in quantitative fashion and studying certain special conditions that arise in the annular pair. The next three, numbers 35 to 37, deal with tooth production, first discussing the older profile methods, then going into the processes of tooth generation.

The flexible involute system, with variable addendum and (on the same wheel) variable pitch circle and obliquity, forms the subject of Sect. 38. This is discussed especially with respect to generation by a single rack cutter of "standard" obliquity, developing the geometry of the Maag and any similar system.

As already noted, the chapter ends with a description and some analysis of pin and chain gearing.

SECTION 32. TOOTH PROFILES

(*a*) **The following ideas and facts** are fundamental or preliminary to a study of the form and action of tooth profiles:

1. The concept of toothed wheels as evolved from rolling cylinders, already stated in Sect. 4 (*e*). The teeth must be so formed as to permit and produce smooth rolling of pitch circles, without slip or irregularity.

2. The idea of the instant center, which has been fully developed in Sect. 29. For present purposes the condensed statement of Sect. 22 (*f*) covers enough ground. The two parts of Fig. 547, reproducing Figs. 431 and 432 but in reversed order, well

illustrate the ideas that are immediately useful. First, at (*a*), we have wheel 2 rolling on 1 in cyclic fashion and turning on instant center C. Then at (*b*) bar AB is held and the two wheels turn, with relative center or point of common velocity at C. Necessarily, not only because the centrodes are circles but under the general principle of the theorem of three centers, point C must lie on the line of centers AB.

3. The parts and dimensions of teeth, given in Sect. 4 (*f*). In the discussion to follow free use will be made of the module as a unit of measurement. This is the reciprocal of diametral pitch, or the quotient of diameter by number of teeth, hence may be defined as the length of diameter per tooth and be used as a linear unit.

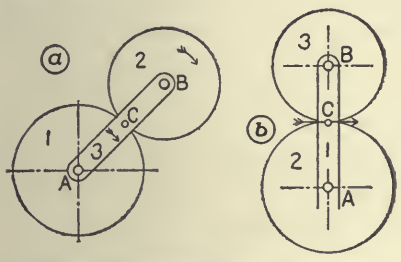

FIG. 547.—Instant center between gears.

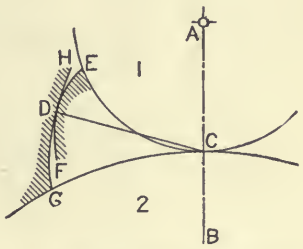

FIG. 548.—General kinematic requirement.

(*b*) **The primary kinematic requirement** for correct tooth action is illustrated by Fig. 548, where profiles EF and GH, attached to wheels 1 and 2 respectively, are in contact at D. Obviously this contact must be a tangency (not a crossing) of the curves; and in order that they may move on each other, maintaining contact yet not interfering with true pitch-circle rolling at C, it is further evident that the tangent curve elements at D must be perpendicular to the common rotation radius CD. Thus if we imagine 2 fixed and 1 rolling upon it, point D on 1 will be tracing a portion of curve GH necessarily perpendicular to CD; with 1 fixed and 2 rolling, D on 2 will likewise trace a portion of EF perpendicular to CD; and this relative motion will remain unchanged if the two wheels rotate together. The usual statement of the fundamental, really axiomatic proposition here illustrated is, *The common normal to a pair of tooth curves at their point of tangency must pass through the pitch point.* To help visualize it, sketch curves oblique to radius CD and see that they will have to separate as the pitch circles roll.

There are two general ways of producing tooth curves. The first is to combine with the two primary rolling wheel discs a secondary geometrical moving element which will trace on the two wheels, simultaneously, curves that fulfil the kinematic requirement just stated. The other method is to choose a suitable profile for one tooth, roll the pitch curves together, and let the given profile generate a mating curve which will be an **envelope** to its successive positions. To these processes the

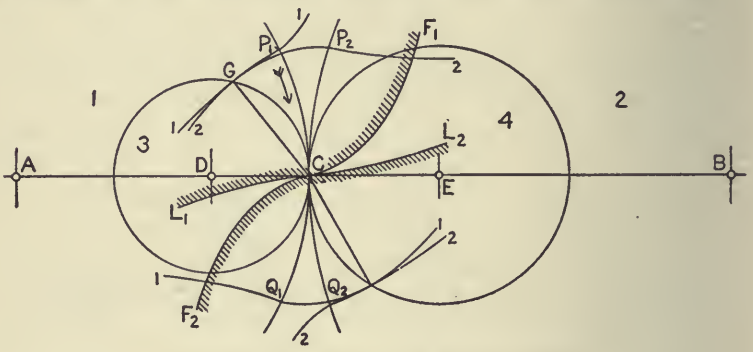

FIG. 549.—Generation of cycloidal profiles.

names "describing-generating" and "molding-generating" have been given. We shall call them simply **describing** (action by point) and **generating** (action by line).

The describing method is decidedly the one to be studied first, since such study leads most readily to an understanding of principles: indeed it underlies the other (which is of greater practical importance) because its principles aid the choice of *suitable* generating profile. Just as pitch circles are the simplest selection out of a range of possible curves, so for the describing function the geometrical forms actually used are the simple circle and straight line.

(*c*) **Cycloidal Tooth Profiles.**—Let the two wheels be conceived as simple plane discs 1 and 2, Fig. 549, bigger than the pitch circles but not yet limited to any particular diameter. These turn on centers A and B in a motion which is made uniform (relatively) by the definite and unchanging location of pitch point C—with noncircular gears point C would move back and forth along line AB. Now add the two smaller circles 3 and 4, having their centers D and E on line AB and each tangent to both the pitch circles at C. Letting all four circles turn together, on the respec-

tive fixed centers A, B, D, E, and in pure rolling at C, consider 3 with 1 and 2. A chosen tracing point G on circle 3 will describe a curve on the (moving) plane of wheel 1 and another curve on wheel 2: these two curves must have a common point (of tangency) at any instantaneous location of G; and since 3 is turning on both 1 and 2 at C the curves are normal to CG at G.

Understand that the curves shown in Fig. 549 are on moving bodies and are merely caught in particular, instantanous positions. Viewing the shaded profiles passed through C, we note that circle 3 describes a flank curve CL_1 on wheel 1 and a face

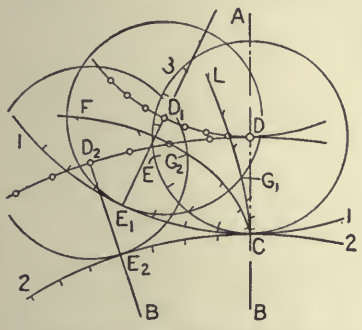

FIG. 550.—Drawing the epi- and hypocycloid.

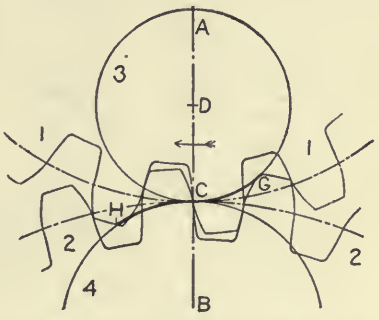

FIG. 551.—Cycloidal teeth in mesh.

curve CF_2 on wheel 2; and circle 4 completes the process by generating CF_1 and CL_2. The face curves, described by a point of the circumference of a circle which rolls on the *outside* of another, are called **epicycloids**; while the flank curves, produced by *inside* rolling, are **hypocycloids.** It is when the describing circle rolls on a *straight line,* for the rack, that the plain name **cycloid** applies.

In order to draw these cycloidal curves we hold the pitch circles fixed and roll the describing circle upon them, as shown in Fig. 550. Using a division so small that the difference between arc and chord may be disregarded, step off equal scales on the three arcs C1, C2 and C3. Roll circle 3 to contact at E_1 and at E_2, getting the locations of its center at D_1 and at D_2 by means of the radii E_1D_1A and BE_2D_2. Then measure off E_1G_1 and E_2G_2 each equal to CE, locating points originally at C and finding a point on each of the curves CL and CF. For real accuracy the lengths got by stepping off should be checked by computation: find the angle of pitch circle subtended by an arc equal to one

or two quadrants of the describing circle, then from *the chord equals twice the sine of half the angle* lay off this arc.

Curves from Figs. 549 and 550 are used to draw the teeth in Fig. 551, where the wheels have 18 and 26 teeth of the size $p_d = 6$. This figure is intended to give a general idea of tooth form and action, introducing several questions that will be fully discussed presently. One thing now to be noted is that the heavily lined compound arc GCH, comprising portions of the two describing

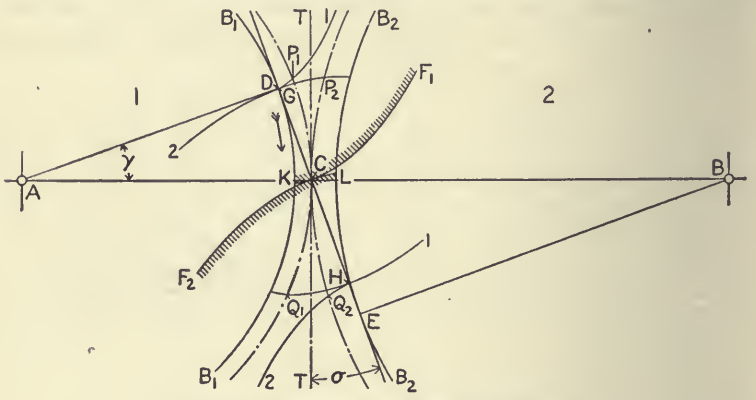

FIG. 552.—Generating involute profiles.

circles, is the **locus of contact** (that is, of the point of contact) for these teeth—this with gear 1 driving 2 in the direction of the arrow, or 2 driving 1 in the opposite direction.

(*d*) **Involute Tooth Profiles.**—In order to get practicable tooth curves by means of a straight line as describing agent, this line must be rolled, not on the pitch circles, but on two **base circles** established as seen in Fig. 552. Through C is drawn a line DE making an angle σ (sigma, here 20 deg.) with the common tangent TT. The base circles B_1B_1 and B_2B_2 are tangent to this line at D and E. If the line rolls on these circles without slip their circumferences will have the same velocity $\omega_1 AD = \omega_2 BE$. Since triangles ACD and BCE are similar, AC:BC::AD:BE; therefore $\omega_1 AC = \omega_2 BC$ and (if this proof is needed) perfect rolling of the pitch circles is maintained. There is, however, a complete replacement of the pitch-circle action by that of the combined line and base circles.

Now a chosen point of line DE will describe two curves on the moving planes of wheels 1 and 2, and since the line always passes through C the requirement as to common normal is perfectly met.

Each of these curves is an **involute** to its own base circle, to be drawn as shown in Fig. 553—the distinctive property of this curve being that BC is the radius of curvature at C. In Fig. 552 there is suggestion of the fact, much more fully developed in Art. (*e*) and in Sect. 33, that tooth action must be confined within the limits set by tangent points D and E, so that only a short portion of profiles KF_1 and LF_2 can possibly be used.

Below (or inside of) the base circle the flank of a described involute tooth is completed by a radial line, as in Figs. 554, 556, etc.: but in the proper working of such teeth this part of the flank does not come into action. With a high number of teeth,

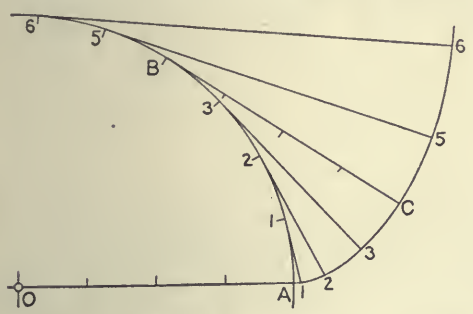

FIG. 553 — Drawing the involute.

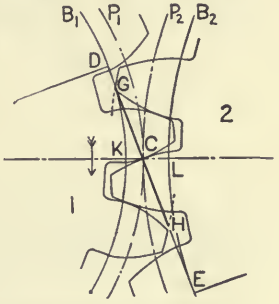

FIG. 554.—Involute teeth in mesh.

or with the diameter large relative to root depth, the base circle drops so far down that the whole tooth profile is involute. When we get to the rack the describing line like CB in Fig. 553 is unwinding from an infinite circle at infinite distance, or it has no angular movement relative to the plane of the rack; then since this line simply moves parallel to itself the profile described becomes a straight line perpendicular to it.

The wheels of Fig. 554 have the same dimensions as those in Fig. 551, with 20 deg. for obliquity angle σ: this makes them a little broader at the root than teeth of the old "standard" 15-deg. or 14.5-deg. obliquity. The locus of contact is now the portion GCH of the describing line.

(*e*) **The Action of Gear Teeth** is illustrated in detail by Figs. 555 and 556, using a set of good average proportions, as follows: tooth numbers 25 and 50; approximate size 2 diametral pitch, diameters 12.5 and 25 in., addendum 0.5 in.; for the cycloidal profiles diameter of both describing circles is 3 in.; for the involute profiles obliquity $\sigma = 15$ deg. Gear 1 is driving 2 from right

to left, as in Figs. 551 and 554, and the pairs of profiles, belonging to the active sides of one pair of teeth, are drawn at intervals of one quarter of the circular pitch p, on pitch-circle arcs P_1Q_1 and P_2Q_2. The reader should study these drawings carefully, running over the positions in the order of movement and of

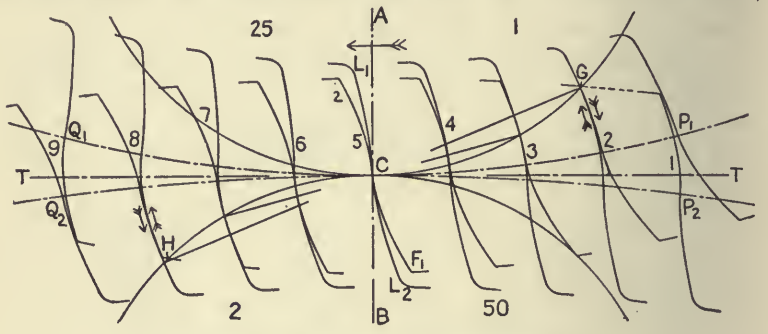

FIG. 555.—Action of cycloidal teeth.

numbering, seeing how the profiles come into and go out of contact and noting how the contact travels along locus GCH.

Contact and tooth action are limited by the addendum circle, of the follower at beginning point G, of the driver at ending point

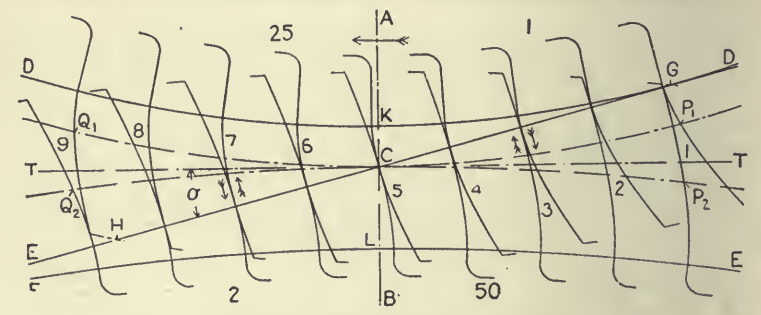

FIG. 556.—Action of involute teeth.

H. These limits are more clearly seen in Figs. 557 and 558, where full-line profiles are drawn at G and H. Tooth action from G to C is called approach, from C to H recess, measured (on the pitch circles) by the arc of approach PC and the arc of recess CQ: the whole distance PQ is called the arc of action. Driving pressure is exerted, during approach by flank of driver on face of follower, during recess by face of driver on flank of follower. There is sliding over the whole of each tooth face, but the active flank is

only the length GP_1 on driver and Q_2H on follower, so that the effective working depth, on the tooth profiles, is less than the total depth, between the addendum circles where they cross the line of centers.

Several points of comparison are to be noted, as follows:

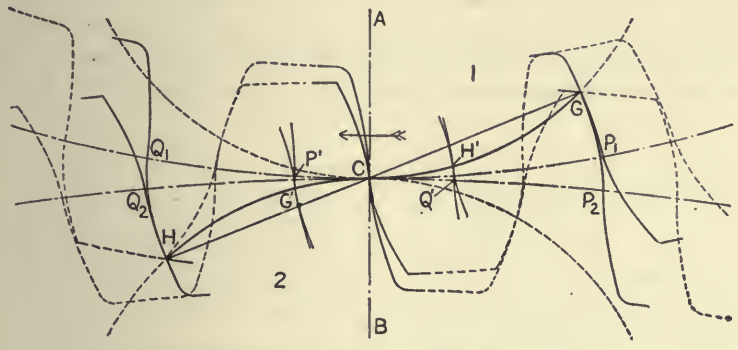

FIG. 557.—Approach and recess, cycloidal.

1. With curvature always in the same direction where they touch, cycloidal teeth have a broader contact than do involute teeth with their curvature always opposite.

2. The common normal like CG or CH is also the line of pressure of tooth on tooth (disregarding friction). With cycloidal

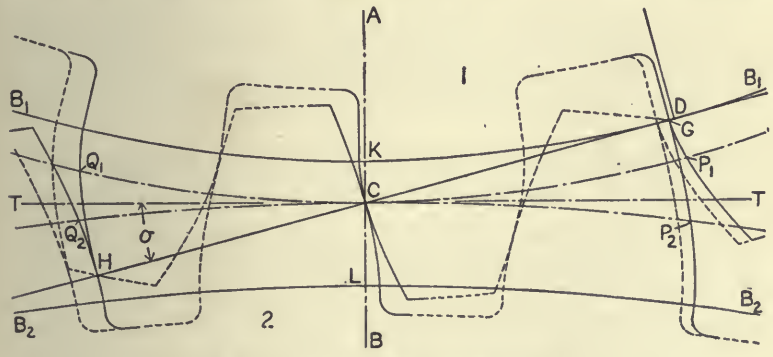

FIG. 558.—Approach and recess, involute.

profiles the obliquity of this line varies from zero at C to a maximum at G or H; with involute profiles the slant is constant.

3. Curvature of the contact locus in Figs. 555 and 557 makes the active flanks longer but the arcs of action shorter than in Figs. 556 and 558.

A question of much interest and some importance is whether there is contact of one or of two pairs of teeth during the whole or a portion of the cycle of tooth action. This is answered in Fig. 557 by drawing parts of the next pair of teeth, ahead of G at G′, behind H at H′, arcs GCG′, PCP′ and H′CH, Q′CQ being all equal to pitch p: note that here P and Q are used as general symbols, covering sub 1 and sub 2. From G to H′ and again from G′ to H there is double contact, but from H′ to G′ only one pair of tooth surfaces is in action. In Fig. 558 both PC and CQ

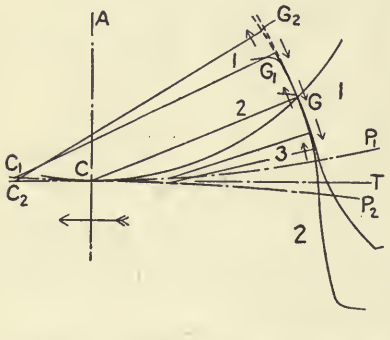

FIG. 559.—Tooth sliding: corresponding normals.

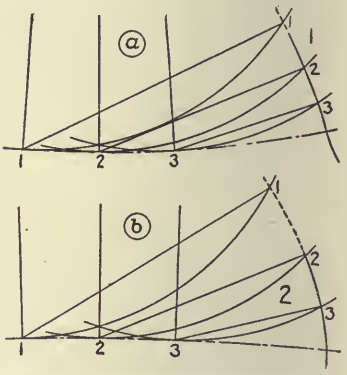

FIG. 560.—Separated profiles from Fig. 559.

are so nearly equal to the pitch that such extra profiles would fall very close to C and confuse the drawing: even without them it is easy to see that over all but a very short arc near C there is double contact.

(*f*) **Rubbing Action of Teeth.**—In contrast with the pure rolling of pitch circles, tooth curves both roll and slip upon each other. As can be clearly seen in Figs. 555 and 556, the teeth slide together during approach, apart during recess. Primarily this relative movement is due to the approach and separation of the pitch-circle arcs before and after contact; secondarily it is influenced by the manner in which the point of contact changes along each profile. Because of the movement of the teeth a new point on the profile of each is continually being exposed to the rubbing action of the other. A slow change of this sort tends to concentrate and intensify wear, a rapid change to diffuse and diminish it.

In illustration consider Fig. 559, which shows the pair of cycloidal profiles from Fig. 555 in position No. 2, with contact

at G and pressure line or common normal CG. To this drawing are transferred the CG lines from positions 1 and 3, each properly located on the plane of its own wheel. At position 1 lines C_1G_1 and C_2G_2 were in coincidence; during the motion from 1 to 2 they have separated by the amount shown; and the two lines marked 3, now separated, will come together when position 3 is reached.

The movement of contact point along profile can be seen more distinctly if the two teeth are separated, as in Fig. 560.

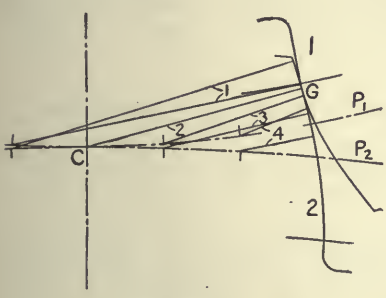

Fig. 561.

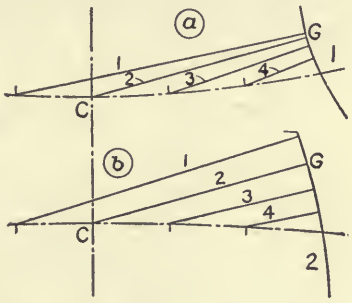

Fig. 562.

Analysis of involute action.

There is rapid change on both curves, greater on the face of tooth 2 than on the flank of 1, but similar in kind and approximately similar in amount to the velocity of relative slip.

Diagrams in exactly the same terms for the involute teeth of Fig. 556 are shown by Figs. 561 and 562, with lines drawn for position 4 also. There is now a marked difference between contact changes on face and flank, the movement slowing down toward zero as contact approaches the tangency limit at D, Fig. 558.

The exhibit of conditions in Figs. 559 to 562 opens up a line of investigation or analysis in which some interesting results can be obtained: but these results are of only second-rate importance because other considerations have so much greater weight in determining the form of profile to be adopted. However, the subject of roll and slip of teeth will be taken up as a part of the closer study of gear action in Part II.

Prob. 256. On radii of 9 and 12 in., with both describing circles of 3 in. radius, construct a set of cycloidal curves as in Fig. 549, all starting at pitch point C. Extend them to 120 deg. of describing-circle movement, using 8

divisions to cover this range; check the stepping off by means of angular distances on the pitch circles.

Prob. 257. On the same radii of 9 and 12 in., with obliquity angle equal to 20 deg., construct a pair of involutes through the pitch point as in Fig. 552.

Prob. 258. Cycloidal teeth as in Fig. 551.

Prob. 259. Involute teeth as in Fig. 554.

Using thick, hard drawing paper or a good grade of cardboard, cut accurate templates of the profiles laid out in Probs. 256 and 257, then use them for drawing the teeth in Probs. 258 and 259, which will be of the diametral pitch $p_d = 1$. Cut with a small, keen knife blade to points pricked through from the drawing, guiding the knife by a "variable" or "French" curve, and use a sharp chisel-edge pencil point. The template should carry a good length of pitch-circle arc, so that it can be accurately placed. For standard radial dimensions see Eqs. (6) and (130).

SECTION 33. LIMITATIONS OF THE PROFILE SYSTEMS

With any possible system of tooth curves some limitation of application will be encountered on the side of smallest gear or minimum number of teeth. In the two common systems of described or point-generated profiles there are very distinct limitations of form and of action, which must now be definitely understood.

(*a*) **Limits with Cycloidal Profiles.**—Without limitation the whole of the epi- and hypocycloids can be used in a tooth action which does not interfere with pure pitch-circle rolling. This is shown by Fig. 563, which gives an outline of the rotors or impellers of the well-known type of positive blower already shown in Fig. 529. With these complete curves there can, however, be real tooth action (in the sense of effective driving) over only certain portions of the revolution, each central about a point where profiles cross pitch circles: when obliquity σ rises to 90 deg., as in the position drawn, driving effect is zero. But these impellers are not required to drive each other, the maintenance of proper rolling being insured by an external pair of spur gears having the same pitch circles P, P.

A limit to form is imposed by the fact that when the diameter of the describing circle is half that of the pitch circle the hypocycloid becomes a straight line. This is shown by Fig. 564, using a relation as to angular motion that has already been stated in Sect. 22 (*c*) under Fig. 289: if arm OB turns through an angle θ from OA, radius CB on wheel 2 turns the same distance to the other side of vertical BD, and point C lies on the diameter with which OB and BC initially coincided: see also Sect. 30 (*b*).

If the rolling circle is bigger than half the pitch circle the hypocycloid bends to the other side of the diameter in a way that would make the flanks of a tooth draw together, curving inward from the radial lines in Fig. 565.

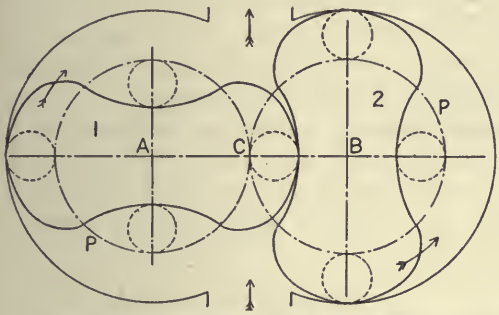

FIG. 563.—Impellers for Root blower.

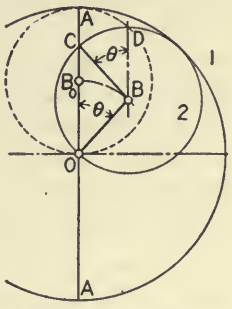

FIG. 564.—The straight-line hypocycloid.

In order to have interchangeable wheels there must be a uniform diameter of describing circle for all those of a given pitch, the same for both face and flank; and the straight-line hypocycloid is what fixes the size of this circle. The generally

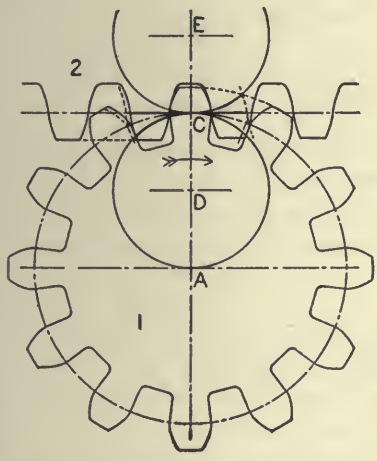

FIG. 565.—Twelve-tooth pinion with rack.

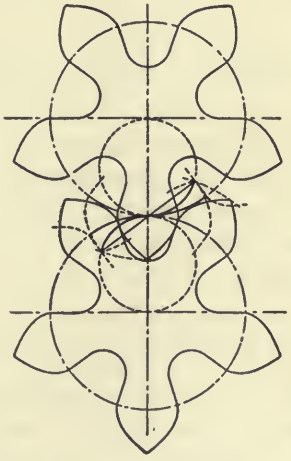

FIG. 566.—Five-tooth pinions.

accepted system has twelve teeth on its smallest pinion, on which the flanks are radial; then the rolling circle agrees in diameter with the pitch circle of a 6-tooth pinion and its radius is

$$r = 3 \text{ mod.} = \frac{3}{\pi} p = 0.9550p. \qquad (121)$$

This pinion, in mesh with its rack, is shown by Fig. 565.

The smallest possible pair of cycloidal pinions having radial flanks is drawn in Fig. 566. With pointed tooth and maximum addendum, the whole arc of action is just a little longer than one pitch. If there were but four teeth this arc would be shorter than the pitch and continuity of action would be lost.

(*b*) **Determining Range of Action.**—Turning back to Fig. 557 and remembering that both describing circles belong to each wheel, we see that one of the action limits G or H is wholly determined by either wheel: No. 1, by its addendum and face gen-

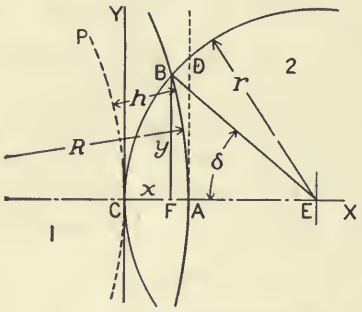

FIG. 567.—Geometrical relations.

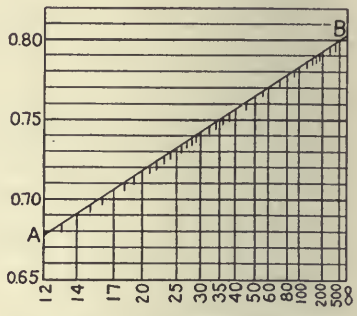

FIG. 568.—Tooth number and arc. Arc of action for standard cycloidal teeth.

erator, limits at H the action of its own face or what is recess if it drives, No. 2 similarly fixes the other limit at G. Enlarging the notation of the subject, let a_a and a_r stand for arcs of approach and recess, then with a second subscript to designate the wheel get the following:

If 1 drives, $a_{a1} (= a_{r2})$ = approach, $a_{r1} (= a_{a2})$ = recess;
If 2 drives, $a_{a2} (= a_{r1})$ = approach, $a_{r2} (= a_{a1})$ = recess.

Each wheel determines its own a_r or limits the portion of the *locus of contact* that lies *outside of its own pitch circle.*

For the tooth system represented by Fig. 565, with the describing circle given by Eq. (121), we now derive a definite relation between the number n of teeth on any wheel and its own face-action or recess arc a_r, using Fig. 567. Taking the module $1/p_d$ as unit, the describing circle CB is drawn with radius 3; CP is the pitch circle of any gear (n teeth), with radius $n/2$;

and its addendum circle AB, at distance $h = 1$, has the radius $R = 1 + n/2$. If the drawing is large and carefully made, arc CB (equal to a_r) may be measured with a good degree of accuracy; and since a 60-deg. arc of circle 2 equals the pitch, the ratio of CB or δ (in degrees) to 60 gives the value of a_r in terms of pitch p.

To get an analytical relation, refer both circles AB and CB to the coordinate axes CX and CY, letting CF = x and FB = y: write equations for the two circles and make them simultaneous, thus:

$$(R - h + x)^2 + y^2 = R^2; \qquad (r - x)^2 + y^2 = r^2;$$
$$h^2 + x^2 - 2hR + 2(R - h)x = -y^2; \qquad x^2 - 2rx = -y^2;$$
$$2(R + r - h)x = h(2R - h),$$
$$x = \frac{h}{2}\frac{2R - h}{R + r - h} \tag{122}$$

Substituting the particular values above,

$$x = \frac{1}{2}\frac{n + 2 - 1}{\frac{1}{2}n + 1 + 3 - 1} = \frac{n + 1}{n + 6};$$
$$\cos\delta = 1 - \frac{x}{3} = \frac{2n + 17}{3n + 18}. \tag{123}$$

The range a_r is from $0.6767p$ for a 12-tooth pinion to $0.8030p$ for a rack, these two limits being shown by dotted profiles on Fig. 565 and also appearing in the arcs CB and CD of Fig. 567, where circle CP is drawn for the smallest wheel. Variation of a_r with n can be shown either by a table or a diagram, and the latter is most compact when laid out as in Fig. 568 with a variable scale for n: the scheme is to adopt a straight variation line AB, then fit computed values of a_r to this line and thus locate division points on the base.

(*c*) **Limits for Small Equal Pinions.**—For the special small pinions of Fig. 566 the only primary specification is that the describing circles shall be half the diameter of the pitch circles. In Fig. 569 it is shown that the three major dimensions for action, namely, arc of action, addendum and maximum obliquity, can be found by graphical construction without the plotting of any profiles—although the profiles are drawn in for the sake of clearer understanding.

In each case CA is the pitch circle and CB the describing circle, just half as large. For six teeth, at (*a*), the half-pitch is 30 deg.;

this is laid out as CA, with CB of the same length covering 60 deg. of the smaller circle. A face profile originating at A has its instantaneously active contact point at B, on CB as locus. Draw radius ODB and see whether AD is less than half of AC, or tooth ABE that just reaches to B is smaller than actual tooth AGC on half-pitch AC; if this is true, then the actual tooth is

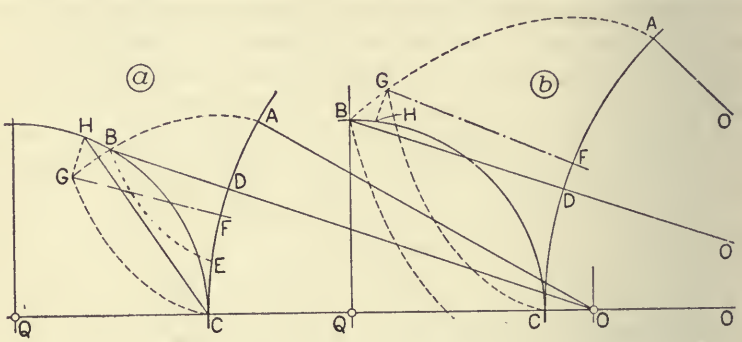

FIG. 569.—Relations for pointed-tooth pinions.

more than big enough to insure continuous action. In case (*b*), with four teeth, arcs CA and CB are 45 and 90 deg. respectively: now D falls below middle point F, or actual tooth is not big enough to reach to B, hence so small a tooth number is impossible, as stated under Fig. 566.

The construction thus far quantitatively necessary is no more than arcs CA and CB, radial ODB and middle point F. Action arc and addendum will become determinate if we can fix on locus CB the position of contact point H that corresponds with tooth apex G. For this note that in order to describe tooth arc BG the small circle must roll from D to F on the pitch circle. Add this length DF to CB as BH and get the desired point H, necessarily on the addendum circle with G. Now CH is the arc of action, the radial distance from H to CA is the addendum, and CH is at the maximum obliquity, measured from a vertical tangent through C.

Prob. 260. For the cycloidal system having 12 teeth in the smallest pinion make a diagram equivalent to Fig. 568, but on the base of log n as in Fig. 576. Compute for eight well-spaced ordinates and put in the full n scale.

Prob. 261. For the system with 15 as lowest n (with radial flanks) and addendum equal to module make a diagram like Fig. 568; compute a reason-

able number of values, locate their ordinates, and interpolate the others. Put the diagram also on the reciprocal of n as base.

Prob. 262. Draw a diagram like Fig. 569, including profiles, for the case of five-tooth pinions.

(*d*) **Involute Interference.**—To the reciprocal use of two involute curves as tooth profiles limits are set by an action which is illustrated in Fig. 570. The profiles 1 and 2 are first shown in contact at pitch point C, and we now take 2 to be driving 1 upward. From position C the point of contact on profile 1 or F_1C moves in toward the base circle, until it gets to the zero point or the origin of the involute at D. Having thus come all

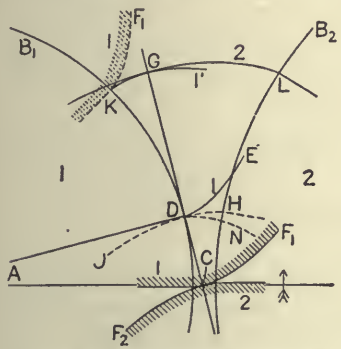

FIG. 570.—Interference of the involute.

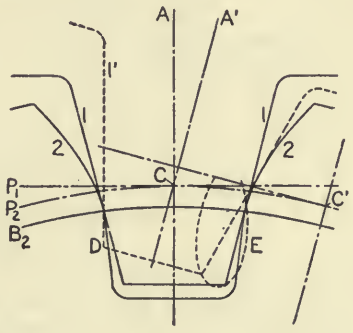

FIG. 571.—Example of undercutting.

the way in on one branch ED of curve 1, a further advance along line CD will carry the tracing point out on the other branch DN. The result of a considerable further movement is seen at G; and the very important fact is brought out that curve 2, working perfectly with KG or 1′, now cuts across the original profile KF_1.

This action of interference is exhibited in a more practical way by Fig. 571, where a true involute rack of 15-deg. obliquity is engaged with a 12-tooth pinion. In the dotted position 1′, reached by rolling pitch line P_1C on circle P_2C to contact at C′, the outer end of the straight face of tooth 1 cuts across the radial flank of tooth 2, at D. The full effect of this overlapping is shown at E, by drawing the looped curve that the other corner of tooth 1 traces on the plane of pinion 2. To make the desired relative motion possible one of two things must be done: either the flank of the pinion tooth must be undercut to the shape at E or else the face of the rack tooth must be eased off so that it will

work with the radial flank of tooth 2. The alternative to this scheme of adjustment is to change the fundamental proportions of the system, obliquity and addendum, in such degree as will keep the tooth action within the interference limit set by the tangency of describing line upon base circle.

(*e*) **Involute Range and Limit.**—Before taking up any detailed study of tooth form as related to production and service it is highly important that the geometrical limitations just described be brought to a full quantitative expression. The quantities to

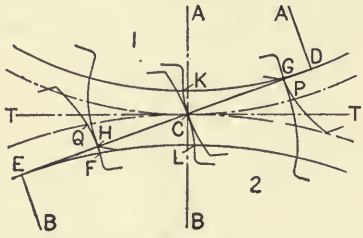

FIG. 572.

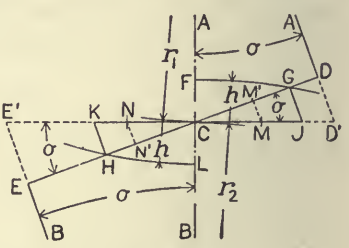

FIG. 573.

Limit relations for involute teeth.

be made definite, illustrated by Fig. 572, are the two limits of action at G and H and the two limits of range at D and E. As already stated in Art. (*b*), dimensions belonging wholly to one wheel determine the length of face action or of what will be recess if that wheel drives, at H for 1, at G for 2. Beside this, each wheel determines the limit within which its own approach or flank action must be kept, at D for 1, at E for 2.

So much of Fig. 572 as is needed for geometrical relations is reproduced in Fig. 573, with the addition of some lines belonging to a convenient scheme of measurement. Our ultimate result will be to reduce the lengths CH = a_1, CG = a_2, CD = l_1 and CE = l_2 to dimensions along the pitch circles, expressing them in terms of circular pitch p. Primarily these lengths are equivalent to certain arcs on the base circles, as CG to KG, CD to KD, etc., Fig. 572. By drawing perpendiculars GJ, DD′, etc., from line DE in Fig. 573 we get triangles CJG, CD′D, CKH, CE′E, all similar to ACD or BCE and having base (along DE) to hypotenuse (along D′E′) as radius of base circle to radius pitch circle. Then if lengths on DE represent arcs of base circle the corresponding lengths on D′E′ will represent arcs of pitch circle, to be compared with the pitch as here laid out in length MN.

But instead of thus reducing the DE-lengths to D′E′, which means that numerically each must be divided by cos σ, it is more convenient to reduce p itself from MN to M′N′ = p' = p cos σ, then use it for a divisor into the lengths along DE.

The geometrical part of Fig. 573 that belongs to one wheel is redrawn in Fig. 574, to illustrate a derivation similar in general effect to that made under Fig. 567, but with simpler relations. Imagining lines HO, CO and DO extended to center O, we have two right triangles CDO and HDO. Writing first general relations, then giving values definitely in modules by substituting $r = \frac{1}{2}n$, we have from triangle CDO,

$$\text{limit } l = r \sin \sigma = \frac{\sin \sigma}{2} n; \tag{124}$$

$$\text{base-circle depth } r - b = r(1 - \cos \sigma) = \frac{1 - \cos \sigma}{2} n. \tag{125}$$

From the second triangle, using the fact that in the first $b^2 = r^2 - l^2$, also that $R = r + h$, we get,

$$a + l = \sqrt{R^2 - b^2} = \sqrt{l^2 + 2rh + h^2} = \sqrt{l^2 + nh + h^2},$$

$$a = \sqrt{l^2 + nh + h^2} - l. \tag{126}$$

Using the negative value of the square root in Eq. (126) we should get the distance from C to the other intersection of line HD, Fig. 574, with the addendum circle, a matter of no present interest.

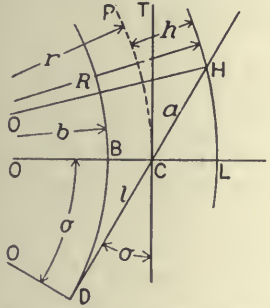

FIG. 574.—Relation for arc of recess.

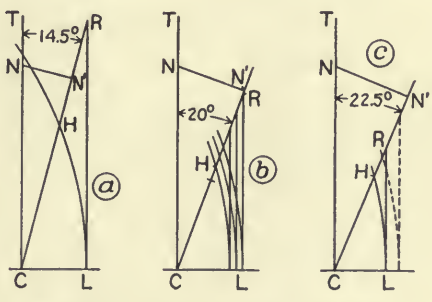

FIG. 575.—Range of arc a_r, three involute systems.

(*f*) **Limits of Involute Systems.**—The diagrams in Fig. 575 and the chart in Fig. 576 show particulars of three involute systems, of which numerical particulars are further collected in Table 7. These systems differ in respect to the fundamental quantities obliquity and addendum, or under the second alter-

native named at the end of Art. (*d*). For actual use the first system has the shape of profiles so adjusted from the true involute as to permit much smaller tooth numbers than are directly possible under the showing of Fig. 576. In general terms the systems are as follows:

I. Curves *a*, *d*, and *g* of Fig. 576 with Fig. 575 (*a*): the geometrical background of the old 14.5 deg. standard involute system, long used for disc-cutter production. This is discussed, as to application and modification, in Sect. 36.

II. Curves *b*, *e* and *h* of Fig. 576 with Fig. 575 (*b*): the 20-deg. involute, with full-module addendum, also with 0.9 and 0.8 module. Having these extra curves of a_r we can interpolate on the chart for the Fellows stub-tooth system, which is described in Sect. 37 (*b*).

III. Curves *c*, *f* and *i* of Fig. 576 with Fig. 575 (*c*): the 22.5-deg. involute with addendum $h = 0.25\ \ p = 0.7854$ mod. This represents about the limit of what can be done in the way of shortening up the arc of action so as to avoid interference on wheels of small tooth number, since on the smaller wheels a_r is now but little more than half the pitch. The dotted lines in Fig. 575 (*c*) show full-module addendum.

TABLE 7

System	I	II	III
1. Obliquity angle σ ..... =	14° 28.65′	20° 0′	22° 30′
2. Sin σ ..... =	0.25000	0.34021	0.38268
3. Cos σ ..... =	0.96825	0.93970	0.92390
4. In Eq. (126), $(1 - \cos\sigma)/2$ =	0.01588	0.03015	0.03805
5. In Eq. (125), $\sin\sigma/2$ ..... =	0.125	0.1701	0.1923
6. Reduced pitch $p' = p\cos\sigma$ =	3.042	2.952	2.902
7. Addendum h ..... =	1.0	1.0, 0.9, 0.8	0.7854
8. Arc of recess for rack, $a_r = h/\sin\sigma$ ..... =	4.000	2.924	2.052

For all linear dimensions, lines 4 to 8, the unit is the module d/n or $1/p_d$. Only the full value of a_r is given in line 8 for case II. The second term under the radical in Eq. (126), namely $nh + b^2$, has the definite values, II*b*, $0.9n + 0.81$; II*c*, $0.8n + 0.64$; III, $0.7854n + 0.617$.

The diagrams in Fig. 575, enlarged from Fig. 574, are drawn in true form and all to the same scale for the three systems just outlined. They show the range of variation of action arc *a* or

a_r, in comparison with reduced pitch CN′ or p', from CH for a ten-tooth pinion to CR for the rack, thus depicting conditions at the extreme ordinates of Fig. 576. In the latter diagram the base is tooth number n on a logarithmic scale, which is found to give an excellent distribution of the ordinates. The first two groups of curves, *a-b-c* and *d-e-f*, show respectively $a_r = CH/p'$ and $l_a = CD/p'$, both dimensions in a unit representing pitch

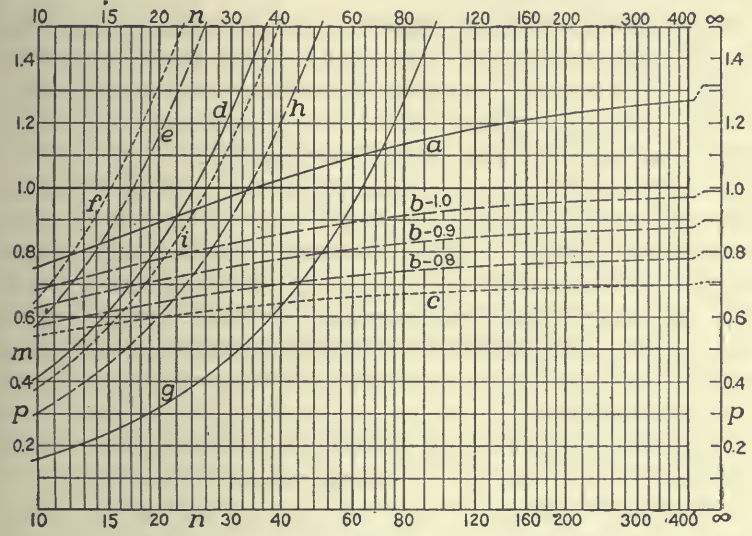

FIG. 576.—Limit diagram for involute teeth.

p, or expressing ratio to p. For the third group *g-h-i*, which gives the base-circle depth (r-b) from Eq. (125), the unit of the diagram represents one module. Where this dimension goes above unity (or sooner for stub teeth) the radial portion of the tooth flank disappears.

As an example of the use of Fig. 576, consider the question of the smallest true involute pinion that can run with the rack without interference. In system I recess arc a_r for the rack (curve *a* at right-hand edge) is about equalled by l_a (curve *d*) at $n = 31$; in system II the corresponding lowest tooth numbers are 17, 15 and 14; while in system III an 11-tooth pinion can be used. Again, a typical and more general question is, With 20-deg. obliquity and 0.875 module addendum, what is the smallest pinion that will work properly with an 80-tooth wheel? On the ordinate for $n = 80$ interpolation between *b*-0.9 and *b*-0.8 makes

a_r come about 0.80 for the given addendum, and this corresponds with $n = 14$ on curve e. The point where the a_r curve crosses l_a shows the pair of equal pinions that will just escape interference: the numbers are, 22 for I, 13, 11 and 10 for II, apparently 8 for III.

Prob. 263. Compute a and l for a wheel of $n = 36$ teeth, system I.

Prob. 264. Pair of gears, system I, 30 and 64 teeth: get arc of action, parts and total, from Fig 576

(*g*) **Flexibility of the Involute.**—Our discussion, conducted thus far almost wholly on geometrical grounds, has shown several points of advantage for the cycloidal system, in the way of broader contact, longer working flank, better distribution of rubbing effect and freedom from interference troubles. But when we come to questions of accurate production and of ease in securing and maintaining conditions for correct working, those geometrical advantages of the cycloidal curves are outweighed by the greater simplicity and flexibility of the involute. The matter of production, with the simplicity of the single curve and the great importance of the straight-line rack profile as a fundamental form, is taken up in Sect. 37. The matter of flexibility can appropriately be considered here.

Cycloidal teeth will work together properly only when their basal pitch circles are really in rolling contact, or when the center (axis) distance of the wheels is just right. If they are somewhat separated, as in Fig. 577 (*a*), the follower will receive an irregular motion: at a little way (on either side) from the middle position drawn there is a range where the short-curvature portion near primary pitch point C_1 and C_2 cannot enter into simple tangency with a face curve of longer radius; beyond that the difficulty as to tangency and a common normal disappears, but the point C′ where this normal crosses the line of centers will not be quite constant. With involute curves, on the other hand, Fig. 577 (*b*), change of axis distance means change to a new set of pitch circles but does not affect either the velocity ratio or its constancy during the cycle of tooth-pair action.

The form of an epi- or hypocycloid is determined by two data, size of pitch circle and size of rolling circle; and only curves generated by the same rolling circle will work together properly. For the involute there is but one determinant, diameter of base circle, and all involutes are geometrically similar, varying only in size: any involute will roll truly with any other, but whether

teeth will do so depends upon whether they embody similarly located portions of the curves. These statements are illustrated by Fig. 578, where drawing (*a*) shows profiles for 16, 32, 64 and 128 teeth on the same pitch and base circles, or with uniform working obliquity: as printed the teeth are of approxi-

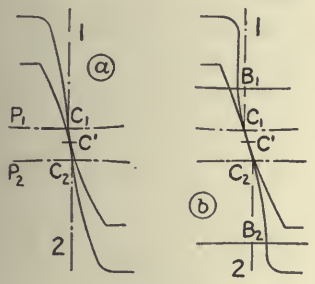

FIG. 577.—Separation of gears, profiles from Figs. 555, 556.

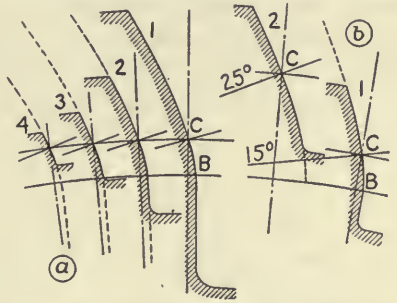

FIG. 578.—Different profiles from same involute.

mately 1.5, 3, 6 and 12 diametral pitch. At (*b*) the 32-tooth profile has two positions on the same involute, for obliquities of 15 and 25 deg. respectively, with change of pitch circle.

The reason why correct interaction of involute curves is not destroyed by change of center distance is made clear in Fig. 579.

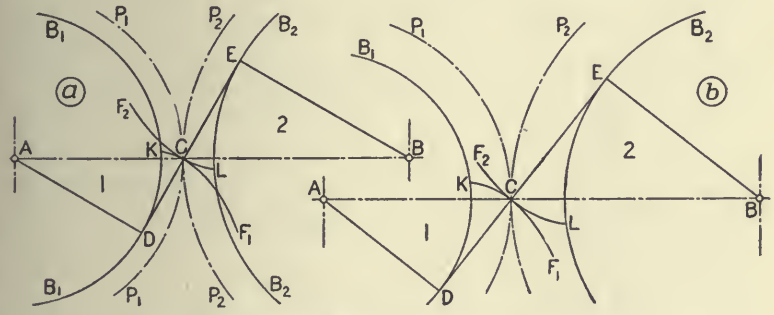

FIG. 579.—Varying center distance and pitch circles.

We may think of the tangent line as wrapped upon the base circles beyond D and E so as to give tensile constraint: then if centers A and B are separated or brought together the profiles traced by a particular point C of this line will continue in contact. Keeping C on the line of centers while the distance varies we see how it fixes the pitch circles, dividing AB in the constant

ratio of AD to BE. With this change of effective pitch circles goes a change in prevailing obliquity and a shift of working profile along the curve, such as has been shown in Fig. 578 (*b*). As a first, minor application of this principle, it is permissible to vary the center distance of any ordinary pair of gears within the narrow limits set by backlash and working depth: this meets the question of inaccuracy in original setting and of change due to wear in service. A broader application, evolving into a tooth system which easily avoids interference, is discussed in Sect. 38.

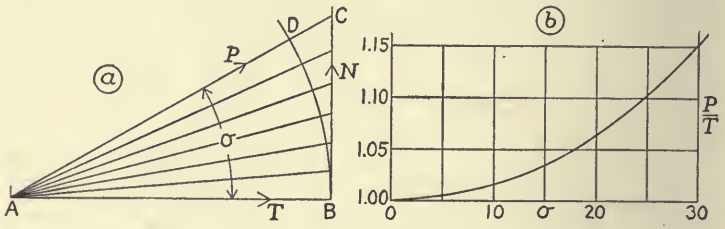

FIG. 580.—Obliquity and tooth pressure.

(*h*) **The Question of Obliquity.**—Figure 89 has shown the relation between tooth pressure P and bearing pressure H. In considering the effect of obliquity upon these forces, attention must be concentrated upon total force P and not upon normal component N. The latter increases rapidly with angle σ, indicating by itself a strong desirability of holding σ down: but total force P, as related to a required constant transmission T, varies slowly. Figure 580, where differences appear directly at (*a*) and in magnified plot at (*b*), shows an excess of only about 10 per cent over T when the obliquity is 25 deg. If then tooth action can be improved by increasing obliquity, the incurred penalty of pressure increase is not a serious matter even up to as high a limit as 30 deg. for angle σ.

On general considerations it would appear that the constant pressure direction of the involute system is a little more favorable to smooth running at high speed than is the oscillation of the pressure line with cycloidal profiles.

SECTION 34. ANNULAR GEARING

(*a*) **The annular or internal gear** may be conceived as derived from the spur gear by a passage through the rack—itself of infinite radius or zero curvature—from positive to negative

curvature of a toothed bar. Besides increasing the action range which is determined by the addendum circle of the annular wheel, this reversal introduces a new type of secondary interference, which tends to occur as the two wheels approach equality of diameters. The problem of such interference is of interest chiefly from the viewpoint of geometrical analysis, because pairs with a large spur gear are of infrequent application; but it does arise in connection with mechanisms like Figs. 312, 313

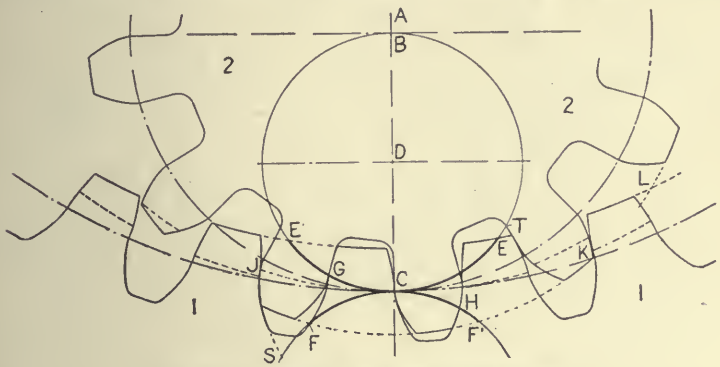

FIG. 581.—Internal gearing, cycloidal teeth.

and 329. Solution is effected by geometrical methods of the kind thus far used, and it is appropriate to complete the analysis along those lines before going forward to the question of tooth generation and production.

(*b*) **The Cycloidal Example** in Fig. 581 has a 12-tooth pinion within a 30-tooth wheel, these medium proportions serving to suggest secondary interference, at J and K, although it is not actually incurred. Noting first characteristics of the more usual kind, we see that epi- and hypocycloid interchange places on face and flank in the passage from external to internal teeth, and that now epicycloids work together in one "half" of the tooth-pair action, hypocycloids in the other half. So much of this action as is defined by the locus ECF or E′CF′ does not differ from what has been seen with external contact of pitch circles, but the arc EC (or E′C) of pinion approach or flank action is now even longer than it would be with a rack. The short dotted arcs drawn across the teeth of each wheel belong to the analysis for secondary contact which is begun with Fig. 582.

(*c*) **Secondary Describing Circles.**—Consider a single wheel in Fig. 582, with pitch-circle arc FCK (center O, radius R) and

the describing circles (centers A and B, radii r_a and r_b) as primary curves. Draw at random inclination a common chord-line DCE, then place the epicycloid at ND and the hypocycloid at TE, each in relation to this line as a normal from C. A newly apparent fact to which full value must now be given is that lines KD and KE, from the second intersection K on the pitch circle, are likewise normals to the respective curves.

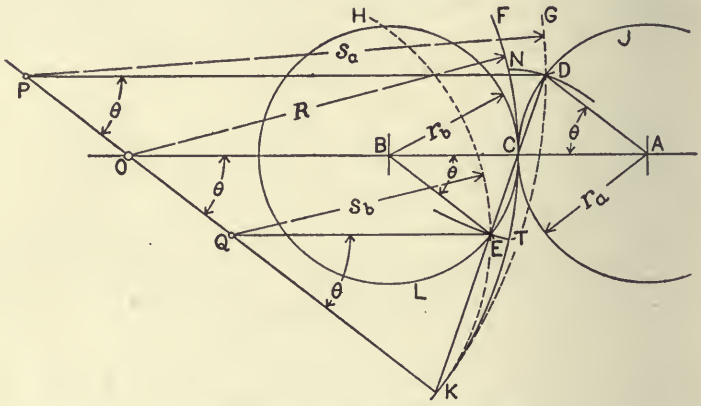

FIG. 582.—Secondary describing circles.

Because triangles OCK, ACD and BCE are within sectors of circles they are isosceles; and having equal angles at C they are similar, whence lines OK, AD and BE are parallel. Now draw DP and EQ parallel to ABO, getting two more triangles PDK and QEK of the same form; then an arc from K on center P will go through D, one on center Q will go through E. The radii of these arcs are,

$$\left.\begin{array}{rl} \text{For the epicycloid,} & s_a = R + r_a; \\ \text{For the hypocycloid,} & s_b = R - r_b. \end{array}\right\} \qquad (127)$$

Now arc CD $= r_a\theta$, arc KC $= R\theta$ and arc KD $= s_a\theta = (R + r_a)\theta =$ KC + CD; but from the known method of generating epicycloid ND, arc CD = arc CN; therefore arc KD = arc KN. This indicates that point D can have been reached and curve ND traced not only by the rolling of DC on NC but also by that of DK on NK. Similarly it can be shown that arc KE = arc KT, or that circle KEH is a secondary generator for the hypocycloid. It appears then that there are complementary describing circles for each form of cycloidal curve, as is more fully illustrated by Fig. 583.

In Fig. 583 the pitch circle on diameter MN is supposed to stand still while the four describing circles R_1 and S_1, R_2 and S_2 roll upon it, from initial common tangency at F. Curve FD has been coincidently traced by the rolling of R_1 from F to C and of S_1 from F to K, and curve FE by the rolling of R_2 from F to C and of S_2 from F to K′. Special proportions and positions are responsible for several coincidences in this figure, namely, for

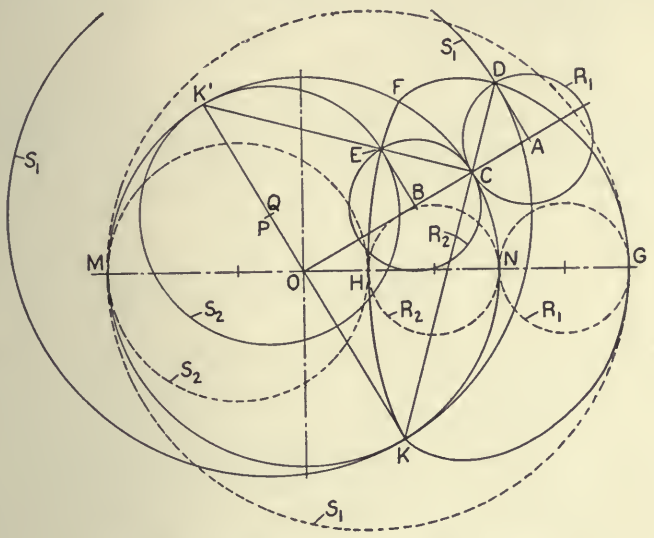

FIG. 583.—Relations among describing circles.

the location of K at the other end of the two curves and for the agreement of centers P and Q: here $r_a = r_b = \frac{1}{3}R$ and DAC = EBC = 90 deg.

The relations expressed by Eqs. (127) are given fuller meaning by drawing the four rolling circles in the dotted positions, with all centers on the middle line MN of the two generated curves.

(*d*) **Relations for Contact Loci.**—Turn now from the curve-generating function of the describing circles to their service as loci of contact, viewing the conditions and action of ordinary outside pairing as represented by Figs. 549, 555, 557, etc. If a single wheel is considered by itself, as in Fig. 582, curves like CDJ and CDL may be viewed as belonging strictly and only to the one pitch circle KCF. When another wheel is brought into contact at C two more such curves come with it; and the criterion of correct relative tooth form is that corresponding loci shall

coincide. In other words, the single describing circle on one side of C in Fig. 549—there working with the two wheel discs—is now to be regarded as the common form of two paths of contact point, each a geometrical property of its own wheel. By contact point is here meant the point on the profile at the foot of a perpendicular from the pitch point.

Characteristic conditions as to nonagreement of loci are shown in Figs. 584 and 585, where with pitch radii of 8 and 10 mod. are

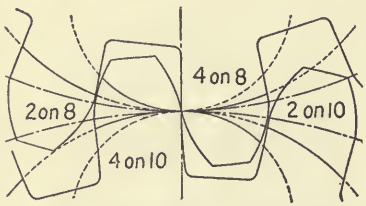

FIG. 584.—Excess freedom.

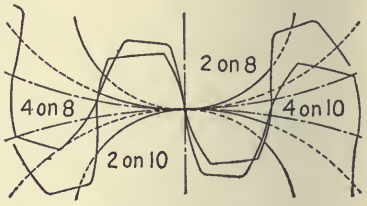

FIG. 585.—Overlapping.

Conditions with contact loci not in agreement.

used describing radii of 2 and 4 mod., the particular combinations being indicated by numbers on a tip and a root of each wheel. The first case has smaller circles for the faces (excess curvature) and larger circles for the flanks (deficient curvature); and the result is excess freedom for entrance or meshing, with failure of proper contact. In the second case conditions are reversed, widening both tips and roots of teeth; and now the superimposition of profiles shows an excess of size which will prevent meshing. A relation of this latter type between secondary loci in an annular pair is what will cause interference of the kind whose possibility is suggested in Fig. 581.

Here distinction is made by drawing each circle in full line across the tooth to which it belongs, dotting it the rest of the way. In Fig. 581 arcs of the secondary loci are drawn only on their teeth. Since there the secondary circle is evidently smaller on the spur gear than on the annular gear it appears that conditions are as in Fig. 584, or that really neither contact nor interference occurs at J and K.

(*e*) **Secondary Contact.**—The example in Fig. 586 has an added item of notation, in the use of subscript a to designate loci belonging to the epicycloids, with b for the hypocycloids. Consideration of this drawing in connection with Eqs. (127) will show that of necessity secondary circle CJ or S_{a1} belonging to the flanks of

annular wheel 1 is larger than CH or S_{a2} of the spur-gear faces; and likewise CK or S_{b2} is always smaller than CH or S_{b1}. This establishes what can also be seen much more directly, that there can be no secondary contact of face with flank, of the primary type at D or E. Remains the possibility of face with face, *between the pitch circles*, as at B here and at J or K in Fig. 581.

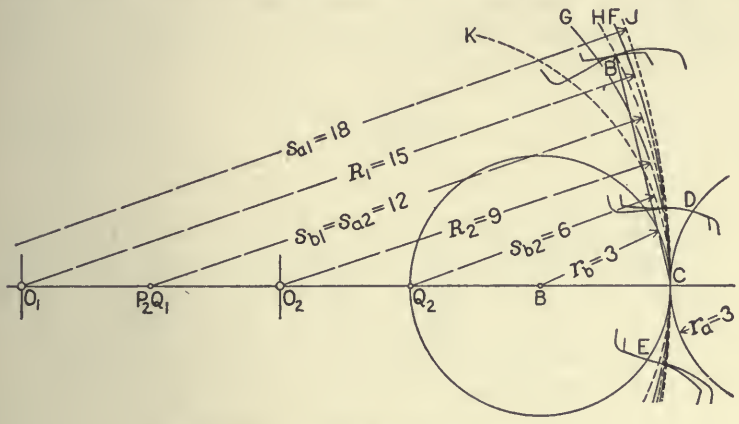

Fig. 586.—Perfect secondary action.

In Fig. 586 the proportions are such that $s_{b1} = R_1 - r_b$ and $s_{a2} = R_2 + r_a$ are equal. This determines perfect tooth contact at B and a secondary action which accompanies primary action and continues beyond it to a limit near the intersection of addendum circles, as at L in Fig. 581. For that figure the leading proportions (in modules) are $R_1 = 15$, $R_2 = 6$, $r_a = r_b = 3$, whence $s_{b1} = 12$ and $s_{a2} = 9$, again showing failure of faces to touch. The limit set by Fig. 586 is evidently that the pitch radii R_1 and R_2 shall differ by the sum of the describing radii r_a and r_b: with a larger spur gear there will be interference. In the "standard" system where $r_a = r_b = 3$, this would require that the wheels differ by not less than 12 teeth: but in the special problem of a large inside wheel there is no reason why r may not be made a good deal smaller, working tooth depth being at the same time cut down so as to avoid excessive obliquity of profile and of pressure.

Prob. 265. Investigate the annular pair $n_1 = 40$, $n_2 = 35$, testing for interference. What is the size of equal describing circles that will give correct secondary action? With 30 deg. as the maximum obliquity what addenda can be used and what will be the range of primary action?

(*f*) **Involute Internal Gearing.**—Figure 587, drawn for 18 and 30 teeth with 20-deg. obliquity, shows two series of profile positions, for the two sides of a tooth-and-space. Since initial contact for both series is at pitch point C and the spacing interval is 0.5 *p*, we get also the effect of tooth in space at each half pitch. The first question to be considered is that of primary interference, illustrated by full-line series (*a*), to which locus CT is normal within the range of primary action. Running from left to right, this action begins on the other side of center line CA, at a point

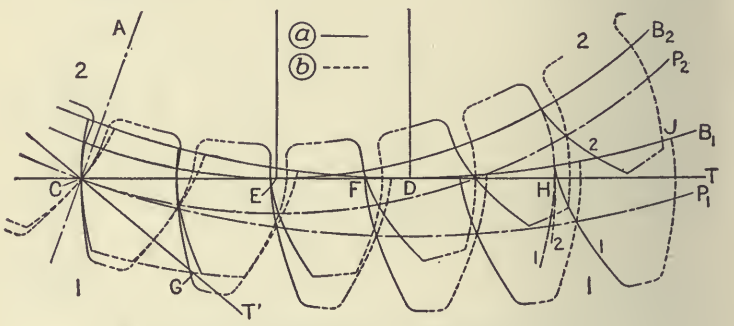

FIG. 587.—An involute pair, 18 teeth in 30 teeth.

on TC symmetrical with G; and in truly involute form it can continue only to the first tangency of describing line with base circle, at E, because there the describing point begins to trace the other branch of the involute on wheel 2—compare Fig. 570. However, by a slight adjustment of profile the tip of tooth 1 can be made to work perfectly with the flank of tooth 2, the latter then generating its own mating curve after the manner to be described in the next section. Here the base circle of wheel 1 is also its addendum circle, which shortens the addendum to about 0.9 module, in agreement with curve *h* of Fig. 576 at $n = 30$; but with adjusted profile—such adjustment introducing an element of specialization into the cutting of the annular teeth—there is no reason why full addendum cannot be used. Beyond D the second branch of the involute is being traced on both wheels, as sketched at H, consequently the actual profiles separate rapidly and widely.

The question of secondary action or interference has no such simple general solution as with cycloidal profiles, for here the contact loci coincide regardless of size of wheels, hence do not furnish any criterion of relation. Possibility of interference is

suggested by an apparent tendency of tooth corners to approach each other at J in Fig. 587, the clearance being a little less at that last position than at the two preceding. Turning to the wide-range diagram in Fig. 588 we first see that secondary tooth action of the kind in Fig. 586 is impossible, and can then find the conditions under which secondary interference will occur.

Start with the pair of profiles in contact at C, Fig. 588, and carry them back to F, somewhere between tangencies D and E, then come forward to position HD′, beyond which both curves will be described by points on rolling line CJ as well as by their primary describing point H. Now from D length DH wraps on circle B_1 or DK, from E length EH wraps on circle B_2 or DL; and because of difference in curvature the common length reaches farther around DKD′ than around ELE′. This establishes at D′ a certain distance between the profiles, which thereafter persists unchanged along line D′J, showing that *secondary contact is impossible with involute profiles.*

(*g*) **Secondary Interference.**—Consider now the pair of profiles running out from K′ and L′ in Fig. 588. On line DCH extended they touch each other. The reason for (or manner of) their separation toward M and N is seen by drawing a pair of normals toward A and noting the characteristic difference in direction of the two curves. Above or inside of line CJ this influence reverses, as shown by the normals to R, and may accumulate sufficiently to make K′ fall on or below L′N. Another view is that point K′ is pulled in toward curve L′N because the distance along arc E′SK′ is greater than along arc D′T and tangent TK′; and K′ will lie in L′N when this difference just equals the initial distance between the profiles at D′. Provided that this location of K′ is not outside of the addendum circle of wheel 2 we evidently have here the limit beyond which interference will begin.

The second line of argument in the preceding paragraph can well be applied to the question of interference with cycloidal profiles. Thus in Fig. 586 there just is contact at B because equal lengths of arc (primarily on the pitch circles, then necessarily the same on the describing circles) are measured from C along the coincident secondary loci belonging to the hypocycloid of the annular wheel and to the epicycloid of the spur wheel. Of these two loci the first is related to the outer profile of the pair at B, the second to the inner profile. Now if radius s_{b1} of the first arc is shorter than s_{a2} of the second, equal lengths will pull inward

the outer profile and push outward the inner one, with the result of overlapping and interference. For that case conditions are simpler than in Fig. 588, because each measurement is wholly along a single arc and there is a common origin at the point of tangency.

Returning to involute teeth, the data for a problem on interference include the two addenda, beside the pitch circles and the obliquity. With all these variables and a complex relation, a general solution would be intricate and laborious to a degree far outweighing the practical value of its result. It is enough to have an easy method of testing a proposed arrangement; and this can be done, without drawing any profile curves, as follows:

Draw pitch, base and addendum circles, getting the intersection of the last as a critical point like K′ in Fig. 588. From this K′ draw a tangent to base circle B_2 and if necessary to B_1 also. From a point like F step forward along the two paths FLTK′ and FKSK′. If the second distance comes out equal to or longer than the first the teeth will clear.

Prob. 266. For an annular pair let the spur wheel have $n_2 = 12$ teeth and use the obliquity $\sigma = 20$ deg. Take the base circle of the annular wheel for its addendum circle and fix the other addendum so that the total arc of action will be not less than $1.3p$. By successive trials find the lowest number of teeth in wheel 1 that will avoid interference.

(*h*) **Inside Wheel Large.**—As the spur and annular wheels approach equality of diameters the increasing tendency toward secondary interference can be counteracted only by cutting down addendum and increasing obliquity—the latter by making the rolling circles smaller as well as by using a larger angle σ for the involute describing line. These changes combine to diminish the range of primary action, which will easily become less than the full pitch, thus breaking continuity of driving effect. Since the cycloidal pair offers the possibility of a long-range and kinematically correct secondary action, it can go much farther than the involute pair toward meeting extreme requirements.

When force transmission disappears and speeds are moderate or masses small, as in a counting mechanism, it is possible to use triangular or pointed teeth which will drive positively although they do not meet the usual requirement of true pitch-circle rolling.

Prob. 267. Annular pair with 19 and 20 teeth, or diameter difference of but one module. Make a half-circumference drawing (pitch contact at

one end) for a pair with each of the following tooth forms, testing for contact and interference:

(*a*) Cycloidal teeth, with describing circles of 0.5 mod. diameter, meeting the requirement for secondary contact in Art. (*e*); use complete epi- and hypocycloids for profiles.

(*b*) Involute teeth, obliquity 20 deg., addendum 0.5 mod.

Determine the range of primary action in each case.

Devise pointed teeth for these wheels that will drive freely and yet as correctly as possible.

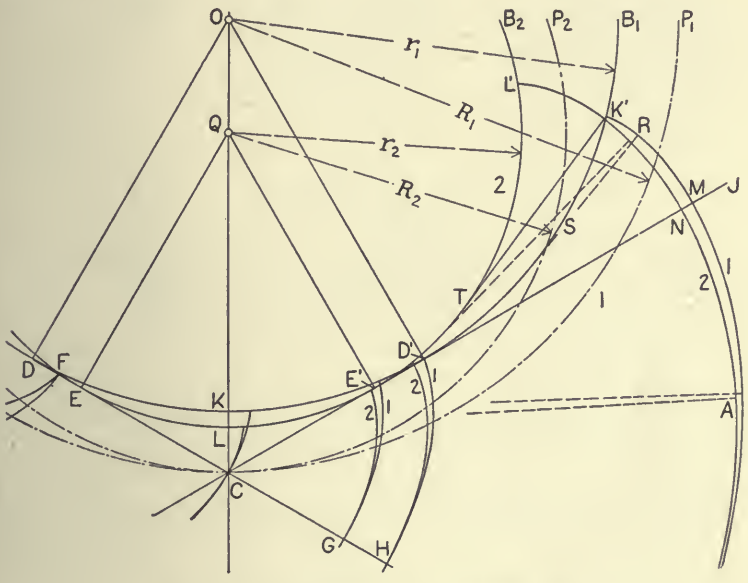

FIG. 588.—Relations for involute curves.

SECTION 35. METHODS OF TOOTH PRODUCTION

(*a*) **Use of Templates.**—The oldest method of shaping teeth, used from the beginning of the art in making wooden patterns for cast gears, is to work by hand to outlines scribed from sheet-metal templates like the one in Fig. 589 (*a*). A carefully made inside template such as (*b*) may be used for inspecting cut teeth or sometimes as a guide to hand finishing with the file; it also serves to lay out or to test a formed flat tool (of the dotted outline) which will be used for planing large teeth. An outside template turned into a cutting tool, Fig. 589 (*c*), determines the shape of the common formed milling cutter.

Template profiles may be laid out by methods of accurate draftsmanship, whether on paper or directly on the metal; but the better scheme, used by manufacturers of milling cutters, has been to embody the geometrical generating figures in a mechanism which will make a tool point follow the path of a describing point on rolling circle or rolling line—this for producing tools such as Fig. 589 (*c*).

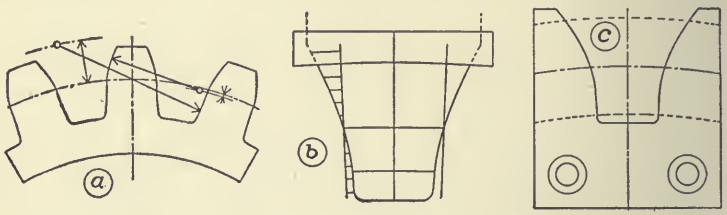

FIG. 589.—Tooth templates, various profile methods.

Various "odontographs" have been devised for drawing teeth without the trouble of geometrical curve plotting. The best scheme is that of Grant, worked out for the "standard" cycloidal and involute systems in the form of tables giving center locations and radii for circular arcs which closely approximate the true curves, as suggested on Fig. 589 (*a*). An interesting method, good especially for very large teeth, is Klein's system of tabular coordinates, here used on Fig. 589 (*b*). See Grant's "Treatise on Gearing" or any machine design handbook and Klein's "Elements of Machine Design." Grant's method is quite accurate enough for cast teeth of moderate and small size: another example of its use can be seen in Fig. 604 (*b*).

(*b*) **Use of Rotary Cutters.**—In Fig. 590 upper view (*a*) gives the profile of the cutter disc, then in view (*b*) the disc is shown *gashed* to make teeth and with the teeth *relieved* so that their cutting ends will have some clearance, or that their backs will not rub in the groove cut. This view shows also how the cutter is operated, by setting it to full depth, then feeding across the gear blank.

Since on gears of a given pitch both tooth and tooth groove vary in shape with number of teeth and diameter of wheel, strict and complete accuracy could be secured only by making an individual cutter for each size or wheel to be cut. This is commercially impracticable (except where a particular machine is being manufactured in quantity) and goes beyond technical

requirements for a wide range of service. The actual practice, established by Brown & Sharpe and followed by other tool makers, is to cover the range of tooth numbers (from 12-tooth pinion to rack) by a short series of cutters, so spaced that successive changes of shape will be about alike. For the involute system there are eight regular cutters, listed in Table 8, Sect. 36 (*a*), but the seven extra intermediates are also offered, making fifteen in

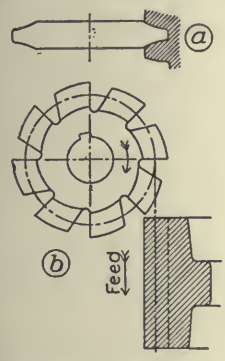

FIG. 590.—Use of formed cutter.

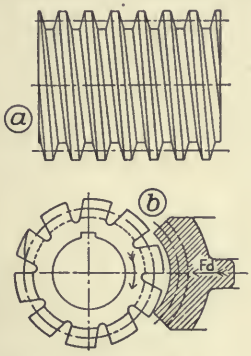

FIG. 591.—Use of hob.

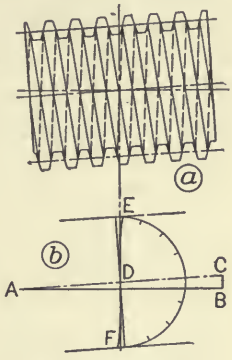

FIG. 592.—Slant of hob thread.

all: the cycloidal system, now little used, contains twenty-four cutters. This serial system (for involute teeth) is fully discussed in Sect. 36.

(*c*) **The Hob and Its Working.**—This tool, partially shown in Fig. 591, is made by threading a worm with the rack profile of the gear-tooth system, then gashing the thread and relieving the cutting teeth thus made—the second of these details, relief of teeth, does not appear in Fig. 591 (*b*). While it removes metal after the manner of the disc cutter, the hob really belongs to the class of generating tools and processes. In such processes a single tool can be made to produce all the tooth curves on wheels of a certain pitch, regardless of tooth number and with a uniform degree of exactness. The hob is not so simple in the idea and detail of its geometrical working as are the planing tools described in the next article; but it is simpler in operation because the generating motion is uniform and continuous. That motion is commonly determined by a gear train between tool arbor and work mandrel which makes the wheel revolve as if the worm were driving it. Occasionally the hob really does drive the wheel that it is cutting, but this only in a finishing operation where

the teeth (on a worm wheel) have already been roughed out by a disc cutter.

For cutting a worm wheel the axis of the hob is brought into the middle plane of the blank wheel, then the latter is slowly fed toward the hob, as indicated in Fig. 591 (*b*). For spur-gear teeth the axis of the hob must be so inclined that at the place of tangency its helical thread will have a direction squarely across the gear blank, as the dotted profiles at the back of the worm in Fig. 592 (*a*) are vertical. Diagram (*b*) shows how the angle is got,

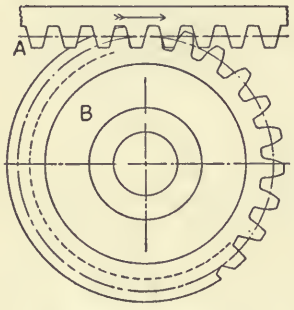

FIG. 593.—The rack as generator.

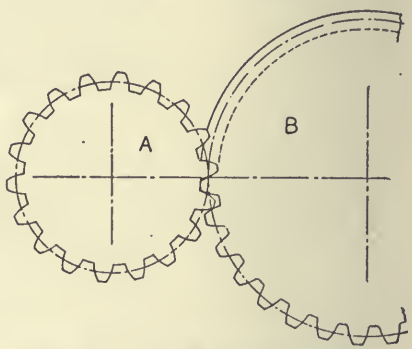

FIG. 594.—The Fellows wheel-cutter method.

by making AB = half the pitch circumference and BC = half the pitch of the hob: then helical curve EF is truly vertical at D. The hob starts just like the disc cutter in Fig. 590, being set to full depth of gear tooth then fed downward (across the gear blank) with a small advance during each revolution of the gear.

(*d*) **Generating Processes.**—The general idea of the generating method of tooth production has been briefly given in Sect. 32 (*b*), and may be restated as follows: *If on one gear we have already formed a tooth (or teeth) of workable shape, and if we roll the pitch circle (cylinder) of this first gear upon that of a second gear on which teeth are desired, then the original tooth will generate a mating or conjugate tooth on the second wheel.* For a first descriptive study of the method we use profiles whose suitability of form has already been established, leaving the more general question of workable shape to be taken up later, in Sect. 37 (*e*).

In most generating processes and machines the cutter is a segment of the rack, either directly, Fig. 593, etc., or in effect (the hob). Working on a plastic material rack A, Fig. 593,

might shape the teeth of B by rolling and pressure; but with actual materials and for accurate results it is necessary to employ a planing action, the cutter having a rapid reciprocating movement, in a direction perpendicular to the plane of the drawing, while rack and gear blank slowly roll together. With an actual rack segment, necessarily short, operation of the machine must be intermittent, as more fully described in the next article. The hob is equivalent to a continuous rack: on its own slightly helicoidal surface the profile of each gash is that of a rack; and as successive gashes or tooth faces come into action they give the effect of shifting this rack, by short stages, along the axis of the hob. Each tooth progresses along the helix and along the pitch line of the rack until it disappears and is replaced by a new tooth starting in at the other end.

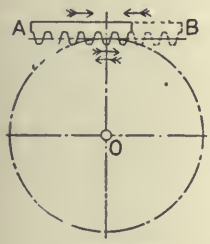

FIG. 595.

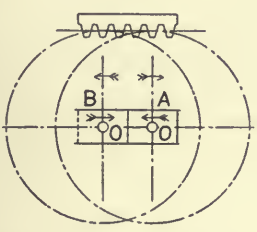

FIG. 596.

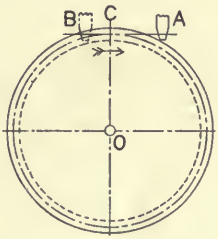

FIG. 597.

Processes of generating by rack cutter.

Continuous planing action is got by using a wheel cutter, A in Fig. 594, which again has a reciprocating motion perpendicular to the plane of the two wheels. Cutter and gear blank are, of course, connected by a gear train analogous to that between hob and blank, which is simple in its kinematic relations as to turning ratio but intricate in the transmissive arrangements needed to give desired flexibility of machine. The small rotary advance is not continuous but takes place only between cuts, just at the end of a return stroke of the tool.

(*e*) **Generation by the Rack.**—The relative motion necessary for the generation of teeth by means of the rack-segment cutter may be got in two ways, as shown by diagram in Figs. 595 and 596. The first scheme keeps the gear axis fixed at O and combines a short back-and-forth oscillation of the gear with a sidewise shifting of the cutter slide. Starting at position A (rack to left, full-line) first swing to the right to position B, then come back

to A, stop the tool clear of the gear, index the gear, and repeat the cycle: naturally, the design of a machine automatic in all these motion calls for much ingenuity. The alternate scheme, Fig. 596, keeps the rack in one place and combines shift of the gear axis with rotation upon it. In each of these arrangements, as also with the wheel cutter of Fig. 594, there is an initial period of feeding the cutter in to full working depth before the generating motion begins.

An interesting variation of this process, outlined in Fig. 597, is used on the Bilgram spur-gear planer or shaper. Here the gear blank has a continuous intermittent rotation, advancing or being indexed by one pitch between strokes of the single-tooth planer tool. Since this tool is held in the usual "clapper-box" fashion and is automatically raised out of the way when drawn backward, the whole time of the return stroke is available for indexing, which is done by a variation of the Geneva stop mechanism in Fig. 161; but even so it is a nice problem to start and accurately to stop the gear blank without shock. The tool slide has a very slow crosswise feed motion, from A to B, and the gear must be just a little overindexed—compare the problem of hobbing helical teeth as fully discussed in Sect. 41—so as to superimpose the requisite slow rolling upon its rapid rotation. Starting at A just in contact with the blank, the first work of the tool is to rough out the groove, and it does not begin to generate the finished profile until well toward C. It need not be carried clear of the addendum circle on the other side, for contact ceases when the inner edge reaches the end of its arc of action at B, as is more fully shown in Fig. 599, following.

A single-tooth cutter can be used with good effect in the process of Figs. 595 and 596, provided that it takes only a light truing and finishing cut on teeth already shaped. For grinding teeth accurately to form we come to the single edge of one rack tooth, using the flat side of a taper-edge wheel: this process is used in finishing wheel cutters, Fig. 594, and also to produce accurate hardened-steel service gears or even to make cut gears more accurate. The multiple tool of Figs. 595 and 596 may either be a solid segment or be made up by clamping together a number of single-point tools. With a wheel cutter the tool cannot swing away for clearance during return stroke, so that into the machine is incorporated a relieving device which separates work and tool, then brings them together again for the cutting stroke.

(*f*) **Length of Rack or Hob.**—On the question of needed length of rack segment or of hob consider Figs. 598 and 599. The first is merely an enlarged detail from Fig. 593 and a background for the geometrical diagram in Fig. 599. The latter shows where the tooth begins to cut, at B, and where it goes out of action, first at H on the outer side, then at G on the inner side. Since a rough

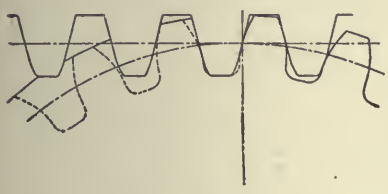

FIG. 598.

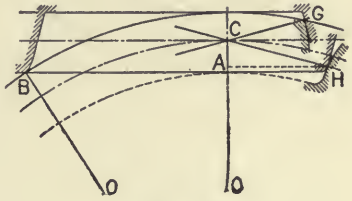

FIG. 599.

Conditions for length of rack cutter or hob.

approximation will be sufficient, disregard the excess or clearance-cutting length of the cutting tooth and call the working depth 2 mod. On this unit the diameter to A is $n - 2$, to B it is $n + 2$: then in triangle AOB,

$$\text{AB} = \sqrt{\left(\frac{n}{2} + 1\right)^2 - \left(\frac{n}{2} - 1\right)^2} = \sqrt{2n};$$

and expressed in terms of pitch p, equal to π mod., this length becomes

$$\frac{\text{AB}}{p} = 0.45\sqrt{n} \qquad (128)$$

Taking 100 to 150 teeth as the highest number likely to be cut on a hobbing machine in the coarser pitches, the cutting approach length is 4.5 to 5.5 pitches; and adding the recess from Fig. 576 we see why the larger hobs will usually carry at least 6 or 7 coils or teeth.

The geometrical study of generating action is taken up in Sect. 37, along with some special properties of the tooth curves thus produced.

SECTION 36. MILLED TEETH

(*a*) **The Cutter Series.**—The accuracy of teeth produced by a milling cutter, when that cutter is used on the wheel for which its profile is correct, depends upon the precision with which the cutter is made and upon the rigidity and the indexing precision

of the machine on which it is working. Similar considerations control the realized result of any process, but their detailed discussion lies outside the province of this book. The question now to be taken up is the effect of using a limited series of cutters, which will not be correct for most of the wheels made.

TABLE 8.—INVOLUTE TOOTH CUTTERS

Regular		Extra		Regular		Extra	
No.	Teeth	No.	Teeth	No.	Teeth	No.	Teeth
1	135– ∞	1.5	80–134	5	21–25	5.5	19–20
2	55–134	2.5	42– 54	6	17–20	6.5	15–16
3	35– 54	3.5	30– 34	7	14–16	7.5	13
4	26– 34	4.5	23– 25	8	12–13		

The series chosen for study is that belonging to the standard involute system of 14.5-deg. obliquity, No. I of Fig. 576 and Table 7, and is specified in Table 8. If of pure involute form that system runs into interference troubles at rather high tooth numbers, at 30 teeth when a pinion gears with the rack, at about 20 teeth with equal pinions. Since comparative results will meet every need, most of the work that follows is based on higher obliquity, then at the end of the section the question of adjustment to avoid interference is briefly considered.

(*b*) **Dimensions on the Addendum Circle.**—In order to test the spacing of the cutter series, dimensions for the curves in Fig. 600 have been computed by the method of Sect. 38 (*e*). The base of this diagram is tooth number n, on the variable scale got by laying out uniformly the reciprocal $1/n$, as against log n in Fig. 576. The ordinates show dimensions on the addendum circle, which is 1 mod. outside of the pitch circle: from the base to curve AA is measured the half space $s'/2$, MF on Fig. 601, between AA and BB the half width of tooth tip, $t'/2$ or FN. The total ordinate is therefore the half addendum pitch $p'/2$; and since $p' : p :: n + 2 : n$ the relation is

$$p' = p\left(1 + \frac{2}{n}\right). \qquad (129)$$

On the base here used the variation of $p'/2$, as compared with the constant $p/2 = 1.571$ mod., is represented by a straight line BB.

It is of course at the addendum circle that the distance between two successive profiles is greatest. Curve AA is so nearly straight that equal absolute changes in space width correspond very nearly with equal intervals of tooth numbers on this base: equal relative changes require a narrowing of interval as n rises and s' gets smaller, approximately what is realized. These curves represent true involutes of 14.5-deg. obliquity, without the relief against interference which slightly narrows tooth tip t'; but that will very little affect the successive differences,

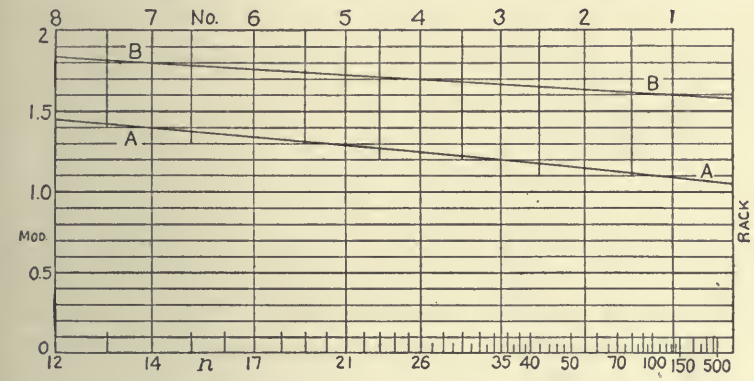

FIG. 600.—Addendum-arc dimensions of formed cutters.

which for the eight regular cutters average 0.0505 mod. in $s'/2$ and 0.0173 mod. in $t'/2$, at tooth corner F. It has been tacitly assumed that the cutter profile is made correct for the lowest number of teeth in its range. By analyzing an example we shall see why this is done, and at the same time get an idea of the irregularity of driving action caused by using cutters that are not quite correct.

(*c*) **Variation of Milled Teeth.**—In order to avoid complication by interference the 20-deg. involute will now be used, but with tooth numbers from Table 8. Accurate templates of the tooth groove or of the cutter are shown by Fig. 601 (*a*), for wheel A (full line) with $n = 21$ and for wheel B (dotted line) with $n = 26$, Nos. 5 and 4 of the series. In the first example these profiles are used on wheels of higher tooth number, A on 25 and B on 34 in Fig. 602; then to see the effect of contrasting conditions they are put on smaller wheels, A on 18 and B on 22 in Fig. 603. First, however, we draw Fig. 601 (*b*) to see how tooth shape is

affected. Template A appears in dotted line at I, showing how it would determine one side of each of teeth II and III. The full-line profiles of these teeth are correct for their respective numbers 25 and 18. It is evident that when used for a higher number a cutter gives the tooth excess curvature, making it narrower at both tip and root; used on a lower number the cutter

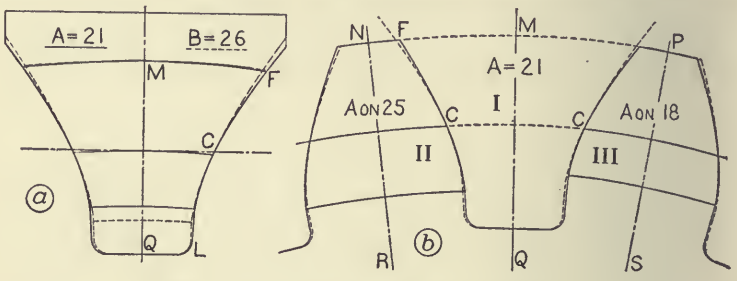

FIG. 601.—Templates and tooth profiles.

gives the opposite effect of deficient curvature and excess width at tip and root. In this figure the three pitch-circle arcs are not continuous but are struck from three different centers on line MQ.

(*d*) **Action of Incorrect Teeth.**—The general scheme of Figs. 602 and 603 is to draw the pair of tooth profiles at exact and equal

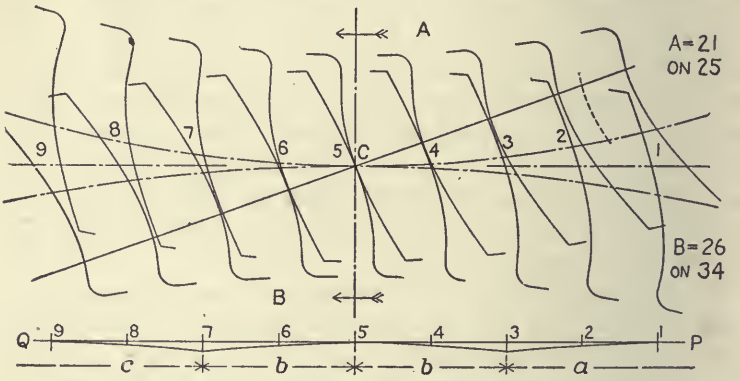

FIG. 602.—Action of milled teeth, cutters on higher numbers.

pitch-circle intervals, just as in Figs. 555 and 556. This shows geometrically what would occur with pure rolling and leaves for physical interpretation the separation and overlapping of profiles that is made evident. In tracing out the complete cycle of tooth action it is necessary to take into account three pairs of

teeth and to cover a range of two pitches: in carrying pair *b* from position 1 to position 9 we must consider also pair *a* preceding and pair *c* following it. At the bottom of each figure is a diagram showing the actual irregularity of motion of the driven wheel or follower: the base is the developed pitch circle of either gear, with equal intervals from the figure above; the ordinate (magnified) is the departure of the pitch point on follower B from true agreement with driver A, measured downward when B lags behind, upward when it is pushed ahead. While

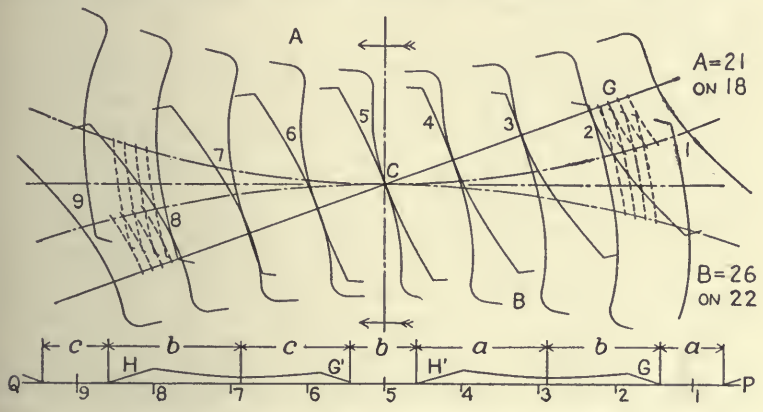

FIG. 603.—Action with cutters on lower numbers.

pair *b* is the master pair whose motion from 1 to 9 is being considered, dimensioned lengths indicate the pair that is actually determining motion during successive portions of the whole range.

Beginning with pair *b* at 1 in Fig. 602, it is seen that pair *a* is in contact at 5. Advancing, the slackness shown at 6 and 7 allows wheel B to swing slowly backward from uniform motion, pair *a* continuing to drive. At about 3 pair *b* comes into action and begins to push wheel B ahead, to ideal relation when *b* is at 5. Then B swings back again until pair *c* comes into action when *b* is at 7. The cycle is very simple but it becomes evident that not more than one pair of teeth can be in contact at once and that the arc of action is strictly limited to one pitch.

With teeth that are too broad, like III of Fig. 601 (*b*), the action is less favorable and far more complex. In Fig. 603, when pair *b* is at 1 pair *a* has contact at 5. But now there is the equivalent of interference by tip of B with flank of A, so that when pair *b*

comes into contact at G there is a rather abrupt pushing of B ahead—always provided that the teeth have enough backlash to permit this without jamming. Pair *b* continues to drive, letting B swing slowly back until *a* comes into contact at about 3-7 and takes up a driving action terminated by its sliding out at H. Then *b* has a short period of normal drive about 5 until *c* comes into contact at G, and so on. Naturally such an action as this, especially with its abrupt changes at the beginning and end of contact (at G and H), is very much to be avoided; and it is at once apparent why cutters are made correct for the lowest tooth number of their range.

Now Fig. 602 is an extreme case, in that both cutters are used at their maximum departure from correctness. On the average of selection the probable error would be only half as great, and by using the "extra" cutters it will be yet further reduced. Indeed, by judicious selection errors can be made to compensate: thus from this example, A = 21 on 25 with B = 26 on 22 will give almost perfect action. But when we combine the effect of clearing for interference (now to be shown) with that of using cutters not correct for the tooth numbers, it is easy to see that with high speed and heavy tooth pressure there is strong demand for a more accurate method of readily producing teeth on wheels of any size.

(*e*) **Clearing for Interference.**—In carrying the old 14.5-deg. involute down to a 12-tooth pinion, as shown in Fig. 565 and required by Table 8, interference is obviated by easing off the outer end of the face profile of the other gear, as analyzed for the rack in Fig. 604. Drawing (*a*) is on the scheme of the last two figures, but only recess action need be shown. Angle σ being actually taken as 15 deg., position 3 falls right at the limit of involute action, where CE is tangent to the base circle; for the half pitch is $\frac{1}{24}$ of the circumference on this pinion. Having a radial flank on wheel 2 the obvious thing is to change the rack profile to a cycloid from point E outward, thus continuing a true action to R. An interesting minor point as regards pure involute action is illustrated at position 6: the limit of rack recess (at H) has not quite been reached, but because the straight rack profile is now working with the second (dotted) branch of the pinion involute, the real branch on tooth 2 has moved away from tooth 1.

Now if the rack tooth has been cut back to curve EF, as determined by the 12-tooth pinion, this clearing will be excessive for

all larger wheels, which means that beyond position 3 the face of tooth 1 will fail to touch the flank of 2; therefore E as here located (at one-half pitch) becomes the fixed limit of rack action on that side of C. Further, not only the rack but any wheel must have its tip cleared, for curve *a* of Fig. 576 shows that even another 12-tooth pinion would have a recess arc of about $0.76p$, while interference begins at $0.5p$. The conclusion is that if the

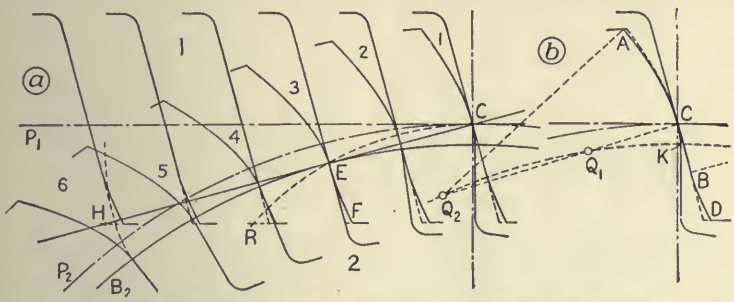

FIG. 604.—Adjusting rack-tooth face to twelve-tooth pinion.

only change in the 15-deg. involute is to adjust its face curves to the radial flank of the 12-tooth pinion, then its range of action is thereby limited to $a_a = a_r = 0.5p$ or $a = p$. In effect it becomes a stub tooth, the outer end of the tooth tip having only a fictitious value, but perhaps coming into action after wear of the initially active portion of the surface has taken place.

The profiles really used for disc cutters have been slightly modified, by trial and experience, from the geometrical combination of involute and epicycloid just described; but the definite quantitative statement of these changes is not a matter of public knowledge. The logical scheme of changing flank as well as face of the rack tooth to the form in Fig. 604 and then letting this generate all the other profiles is described in Sect. 37 (*f*), under Fig. 616.

Figure 604 (*b*) shows the use of Grant's odontograph scheme for drawing the profiles of this example—see Sect. 35 (*a*). The full-line pinion curve is made up of arc KC from center Q_1 and arc CA from center Q_2, both centers being on the base circle; and the dotted curve is from the true-involute template used for Fig. (*a*). The rack curve BD is also a circular arc, from a center on the normal at B.

SECTION 37. GENERATED TEETH

(*a*) **Generating Action.**—The best way of showing the geometric action of a cutting tooth is to hold the gear fast and roll the cutter upon it, as is done in Figs. 605 and 606 in a manner sufficiently suggested by the matching of pitch lines at the left of each drawing. Here we first use the 20-deg. involute rack to generate the 21-tooth template A of Fig. 601, then let this pinion tooth generate a mate to itself on another wheel of the same size.

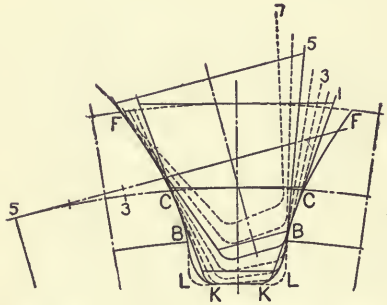

FIG. 605.—Pinion generated by rack.

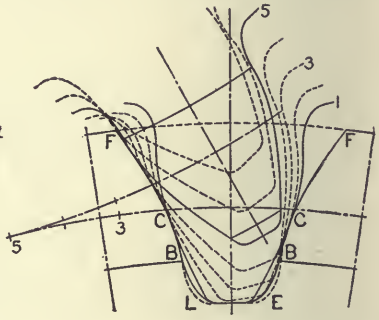

FIG. 606.—Pinion generated by pinion.

For clearness and for convenience of reference the cutting-tooth templates are drawn separately in Figs. 607 (*a*) and (*b*). Since we are using profile forms already established and of known relationship, the fact that an already laid out involute template will serve as an "envelope" to the successive positions of the rack tooth in Fig. 605 is now only illustrative and a test of accuracy in drawing; in other words, it is not necessary to prove that involutes will generate involutes. A new matter of decided interest is the shape given to the bottom of the tooth groove in this process of generation.

The rack-tooth template, Fig. 607 (*a*), has the full addendum of one module and the likewise standard radial clearance c of $\frac{1}{20}$ of the circular pitch p or of 0.157 mod.: then for root depth d, as dimensioned on Fig. 56, the formula to accompany Eq. (6) is

$$d = \frac{1.157}{p_d} = 1.157 \text{ mod.} \qquad (130)$$

The cutter tooth carries this clearance, of course, as an excess length beyond its own addendum FF, while the line or arc LL shows the top of its working depth.

Instead of the radial flank of the geometric tooth, dotted in as BL on Fig. 605, curve BK is actually generated. This root curve is carried on template (*b*), Fig. 607, and in Fig. 606 is seen to clear nicely the tip of the mating tooth. It is also dotted in at BL on Fig. 606, where the important fact appears that the new curve generated by the pinion tooth lies well inside of that due

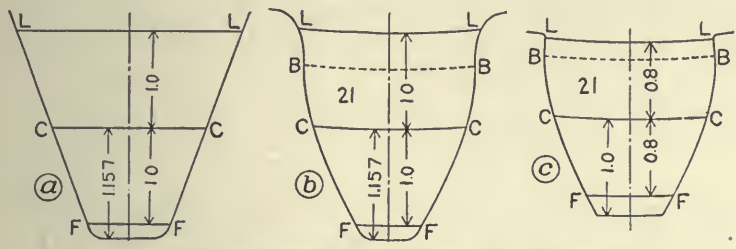

FIG. 607.—Tooth templates for Figs. 605, 606 and 609.

to the rack tooth; consequently the rack, even with the excess tip removed, will not quite roll freely with this final pinion. It is evident that we have here come to the critical requirement of the wheel-cutter or Fellows system of Fig. 594, and that profile (*b*), Fig. 607, does not have enough tip or clearance excess to meet that requirement.

(*b*) **The Fellows System.**—This system uses for addendum the module belonging to a smaller tooth: the six sizes commonly given are,

Designation.....	4/5	5/7	6/8	8/10	10/12	12/14
Addendum.....	0 800	0.714	0.750	0.800	0.833	0.857

In the "designation" the first number is actual p_d for the gear, the second is the p_d whose reciprocal is used as addendum; then taking the designation as a fraction we get at once the value of actual addendum in terms of full module.

A typical Fellows cutter tooth, made by shortening template (*b*), is given in Fig. 607 (*c*), taking $h = 0.8$ mod. as a representative average of the proportions just stated and using with it the prescribed excess $c = 0.25h$, so that the tip of this particular cutter tooth is just like that of a regular full-length gear tooth of the same obliquity. Considerations governing this ratio of excess are illustrated by Fig. 608, which is closely related to Figs. 572 and 573. Desiring to express conditions that will insure the rolling of wheel with rack, we first lay down the straight pitch line CP_0 and addendum line AH of the latter. Intersection with

the contact locus at H fixes the limit of action of face of rack tooth with flank of wheel tooth: and in order to generate that flank the cutter tooth must have this same range of action. Trying out our template (*c*), by drawing its addendum circle BH with excess AB equal to 0.25 of the gear addendum CA, we find a very close agreement with the H limit already imposed. Referring to Fig. 576 for the same information we read $a_r = 0.80p$ on curve *b*-0.8 for the rack and $a_r = 0.79p$ on curve *b*-0.1 for $n = 21$.

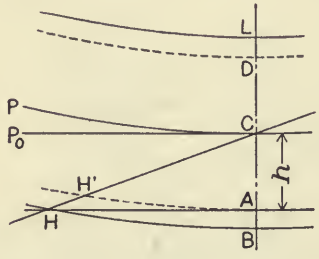

FIG. 608.—Conditions for cutter-tooth excess.

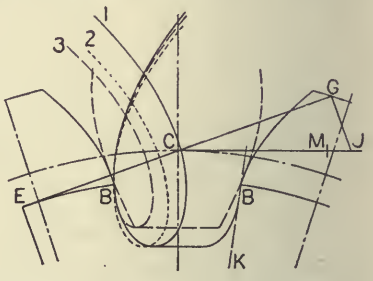

FIG. 609.—Ten-tooth pinion, Fellows method.

Comparing this $a_r = 0.80$ (for the 0.8 addendum rack) with the limit curve *e* of Fig. 576, we see that interference begins when tooth number *n* falls below 14. To find out how serious it may become, consider the system's limiting pinion of 10 teeth, drawn in Fig. 609. Three curves of tooth-corner path are given: No. 1 belongs to the cutter tooth, Fig. 607 (*c*), and determines all of the tooth profile below base circle BB; No. 2 is for the rack-cutter tooth of the same excess length as with 1, and would of course cut more deeply into the tooth; No. 3, for the actual rack tooth, fits perfectly the flank curve 1 already generated. As to the amount of undercutting, it is so slight that the actual flank can hardly be distinguished from the radial line drawn at BK. Note the construction from Fig. 573, to compare recess arc CJ with half-pitch CM.

(*c*) **Limitations of the Wheel-cutter System.**—From the preceding discussion the following conclusions are reached as to the form of tooth for a generating cutter:

Since the rack tooth is the broadest tooth in the system and has the longest range of action on the flank of a mating wheel, it needs excess length only for the sake of clearance and its corners may properly be rounded as in Fig. 607 (*a*).

The wheel-cutter tooth uses its excess length of face for generating purposes, the excess must be greater than considerations of clearance alone would require, and none of it may be sacrificed by rounding the corner. With this sharp corner the root fillet tends to show a notched outline instead of the smooth curve drawn in Fig. 609.

As the tooth tip gets narrower with longer addendum, Fig. 607 (*b*), the cutting excess needed to insure clearing of the rack and of large wheels becomes relatively greater and soon reaches its limit of possible effect: because of this the wheel-cutter method is practically limited to the stub-tooth system for which it has been developed.

From Fig. 608 it appears that the Fellows cutter ought not to have less than 20 teeth. This consideration, coupled with the difficulty of producing solid cutters of large diameter—not only because of distortion in hardening and tempering but also because of shrinkage strains that are likely to make the cutter crack—restricts the system to about the maximum pitch given above, or to $p_d = 4$.

For internal gearing quantitative relations shown by or based on Fig. 608 will have to be somewhat modified.

Prob. 268. Make a construction of the kind in Figs. 605 to 607 for a 12-tooth pinion with full addendum of 1 mod., as follows: Use a cutter tooth like Fig. 607 (*b*), belonging to a 20-tooth wheel, but do not bother with its profile inside of base circle BB; generate the whole profile of the tooth and also lay out its involute curve geometrically, testing agreement; using the simplest form and manipulation of template, get the clearing curve for a rack tooth like Fig. 607 (*a*) out to FF, seeing how much it undercuts the pinion flank as already generated. Find what excess length would have to be put on the cutter tooth in order to make a clearance for this rack. Use $p_d = \frac{2}{3}$, considering it a 4-pitch tooth drawn six times full size.

Prob. 269. With same general dimensions as in last problem, shorten the cutter tooth to one like Fig. 607 (*c*), then try it out after the manner of Fig. 609, drawing curves 1 and 2 but not No. 3.

(*d*) **Cuts per Profile.**—An important matter is the number of positions or cuts that a generating tool takes in its whole operation of shaping one tooth profile. With the hob this is positively limited by the number of gashes or of cutting teeth in a coil. Suppose that a 29-deg. hob (of 14.5-deg. obliquity) having ten gashes is cutting a 40-tooth wheel: from Fig. 576, curve *a*, the respective arcs of action are 1.31 for the rack and 1.03 for the wheel, making a total of 2.34 pitches. While the wheel turns

through one pitch the cutting tooth takes ten successive positions, so that the whole profile will be generated in 23 or 24 cuts. On a large wheel this will make the cuts or flats quite distinct enough to be easily visible. In a planer generator the feed is independent, and can be adjusted to the proper compromise between smoothness of profile and speed of production. As an example, the Fellows machine is geared to make about 720, 960 and 1,400 strokes per revolution of cutter, for coarse, medium and fine feed. With a 4-in., 5-pitch cutter (20 teeth) this gives 36, 48 and 70 strokes per pitch; and taking the average arc of action at 1.4 to 1.5 pitches (from Fig. 576, curve *b*-0.8), we have from 50 to 100 strokes per profile. For a given degree of smoothness this convex cutter will need a few more strokes than will a straight-edged rack tooth. A machine having the type of oscillating action represented by Figs. 595 and 596 should be so geared that on the return swing the cuts will fall between those already made, flattening the ridges that were left. In designing the feed of any of these machines care must be taken not to give too heavy a cut to the end teeth that take the task of roughing out.

(*e*) **Problem of the Generating Tooth.**—Under the general statement in Sect. 35 (*d*) of the principle of tooth generation only a limited application has thus far been made, in that none but the involute profile has been used. That statement calls for a primary tooth of "workable shape," and we shall now investigate the meaning of that specification as affecting choice of tooth form. The procedure will be to select two examples of contrasting characteristics, then, in analyzing them, to develop methods, relations and general conclusions.

For a first example, in Fig. 610, let a 20-tooth pinion (upper gear No. 1) have the straight-line profile AB, taken at an angle of 14.5 deg. with the radius to its own pitch point C_1. Starting at C_1, step off a number of equal intervals in both directions along AB and from each division point draw a normal like DE to intersect at E the pitch circle ECH of this wheel No. 1. The fact that below C_1 all but the first of these normals fall outside the pitch circle will be found to mean that this straight line C_1A is an impossible form for the face profile of the tooth.

The requirement of a common normal through the pitch point enables us to lay out the conjugate profile to AB on a second wheel, here represented by pitch circle FCJ of the diameter for 40 teeth. The first and more complete method is to transfer triangle

EC_1D to CHK, imagining the two wheels to roll until E and F come together at C: this determines point K of the locus of tooth contact; and by making CJ equal to arc CH and carrying triangle CJK back to FC_2G we get a point G of the new tooth curve. Between normals FG and ED is the angle of relative turning of the two wheels which, with the two-to-one ratio of radii in this figure, is three times the angle of arc C_2F or one and

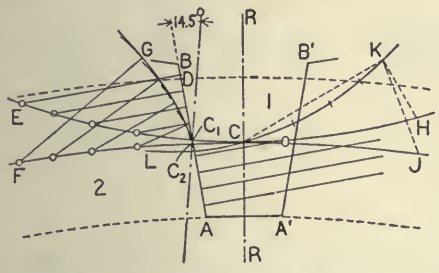

Fig. 610.—Straight-line profile—face impossible.

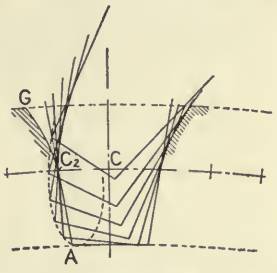

Fig. 611.—Generating action of straight tooth.

one-half times the angle of arc C_1E—this in accordance with the angular relations in Sect. 22. A series of such constructions will determine enough points for the mating profile, or it can be got by the second and simpler method of striking a series of arcs from the centers like F, each with its radius like ED, then drawing an envelope to those arcs.

So far as this determination is concerned, all that can be said in regard to the lower portion of face C_1A is that when there is no intersection with the pitch circle we have nothing to work upon. The generating method of Fig. 605, etc., applied in Fig. 611, shows a serious case of what we have heretofore called *undercutting* of the tooth, but which might now better be called *overcutting* of the profile: that is, successive positions of the first tooth cut over any curve that would fit earlier positions. This effect is exaggerated because tooth 1 has the excess, clearance-cutting length c; but even with regular working length the path curve of its corner will cross the face of tooth 2 above pitch point C_2.

In contrast with the straight face profile in Fig. 610, and even with the geometrically generated epicycloid and involute, we assume in Fig. 612 a primary tooth which has increasing curvature along the face, from pitch point C_1 to tip A, in such degree that the fourth and last normal drawn crosses the pitch circle a

little inside of number three. The most striking effect of this condition appears in the sharp reverse turn of the contact locus at L, which positively limits the range of tooth action. Instead of overcutting of the profile we have now the separation which

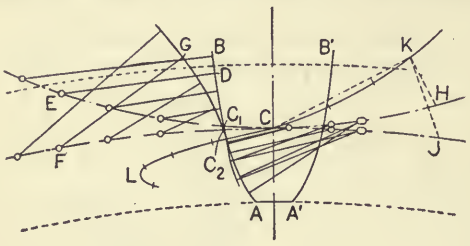

FIG. 612.

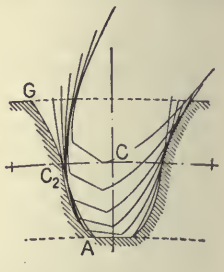

FIG. 613.

Curvature increasing toward tip of primary tooth.

is seen in Fig. 613, due to the fact that a flank curve on tooth 2 that would touch the tip of tooth 1 is cut away by the previously generated part of curve C_2A.

The principle made evident by these examples is that a workable tooth profile must be of such form that the intersections of

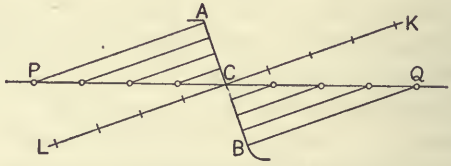

FIG. 614.—Characteristics of the involute rack.

normal with pitch circle will move along the latter in a smoothly progressive fashion, neither spreading out nor crowding together unduly.

(*f*) **Typical Rack Profiles.**—The fundamental characteristics of any system of gear teeth can best be shown by means of the rack profile in conjunction with the locus of tooth contact, as in Figs. 614 to 616. The first two, representing the two common systems, have a series of normals drawn from equidistant points of the profiles, and corresponding points are marked off on each locus KCL. Two things remain unchanged in all wheels of a set having the same pitch, namely, the complete form of this locus KCL and the lengths along pitch arcs CP and CQ that are related to a given set of points on the locus. The most general

statement of requirement for interchangeability is that the halves CK and CL must be alike, which for the rack makes profile-halves CA and CB also alike. The maximum simplicity of the involute, already fully exhibited, is here clearly apparent.

Of the other possible forms but one will be considered, developed from the adjusted involute in Fig. 604. From that figure

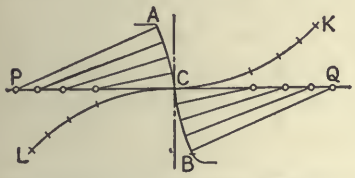

FIG. 615.—Cycloidal characteristics.

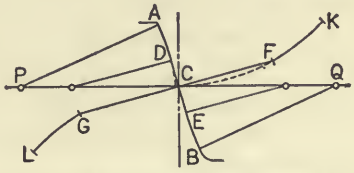

FIG. 616.—Compound system, from Fig. 604.

we take the straight line CD and cycloid DA; then as a step forward the flank CEB is made symmetrical with CDA. If the cutter profile as described in Sect. 36 (*e*) is generated by this rack tooth, such incompleteness of action as may there be implied will be avoided.

SECTION 38. THE FLEXIBLE INVOLUTE

(*a*) **General Conditions.**—Thus far we have been concerned almost altogether with gears of the common interchangeable type: such gears belong to a system in which any two will work together with the same addenda, an unchanged meshing depth, a constant (involute) obliquity and a practically invariable pitch circle on each wheel.

In Sect. 33 (*g*) it has been shown that involute curves, depending for their form upon base circles only, are not restricted to a definite distance between centers of wheels, but that this distance (and with it the active or effective obliquity and pitch circles) may be varied without disturbing profile action. Building upon this idea and the right to choose addenda as convenient, we shall now develop a flexible involute system in which interference troubles are avoided but there is no departure from the simplicity of rigorously correct curves.

Methods and relations to be used, as also the kind of problems that will arise in the practical production of teeth, can best be indicated by the preliminary study of a simple example, in Arts. (*b*) to (*d*), ending with the exhibit of a general solution (to be worked out later) in Fig. 621.

This introductory example brings out the need of conveniently and accurately finding the thickness of tooth at any working pitch circle. A general coordinate scheme for the involute curve and tooth is presented an Art. (*e*), then illustrated and applied in Arts. (*f*) to (*h*).

The matters of action range, closely related to Sect. 33 (*e*), and of radial dimensions of teeth are taken up in Art. (*i*).

The general problem of adjustment of proportions is considered in Arts. (*j*) to (*o*), using the 12-12 and 12-48 pairs as specific and representative examples. This rather intricate matter can best be understood by carefully working through all the indicated computations.

The adjustment problem and methods are summed up in Art. (*p*).

The remaining three articles present several detached problems, related by the fact that they involve variation of pitch circle and the use of Table 9.

(*b*) **Rack and Pinion with Unequal Addenda.**—Starting with a 14.5-deg. rack, working depth 2 modules, consider the question of a correctly mating 12-tooth pinion, pitch circle CP and base circle KD in Fig. 617. Interference will just be escaped if the rack's addendum line DE passes through limit point D. To get dedendum CE note that radius $r = \text{AC} = n/2$; then with $\sin \sigma = 0.25$ limit length $\text{CD} = l = n/8$; and in triangle CDE depth $\text{CE} = d = l/4 = n/32$. For $n = 12$ we have therefore $d = 0.375$ mod., leaving addendum $h = 1.625$; and the pinion blank must be turned to a diameter of 15.25 mod., as against 14 for the standard tooth. The cutting condition is defined by stating that the cutter is set back a distance ϵ (epsilon) = 0.625 mod., which means that its mid-line at M is this far from the generating or primary pitch circle CD.

Mechanically the generating pitch circle is fixed by the fact that the gear train of the machine makes the blank turn once while the rack advances through a distance of n pitches: that is, this circle must have such a diameter that the fraction $1/n$ of its circumference will just equal the pitch of the rack. Fulfilling this requirement necessarily brings it into conformity with the geometrical pitch circle as laid out on a drawing. In order to escape for the time any question of generated clearance curves, the profiles are not carried below the limit of working depth in Figs. 617 to 619.

(*c*) **Variable Pitch Radius.**—Having two 12-tooth pinions generated by a rack cutter after the manner of Fig. 617, set them together at the same working depth of 2 mod. in Fig. 618, or at a center distance of AB = 13.25 mod. The first thing to be noted is that from C_1 and C_2 the working pitch point now goes to

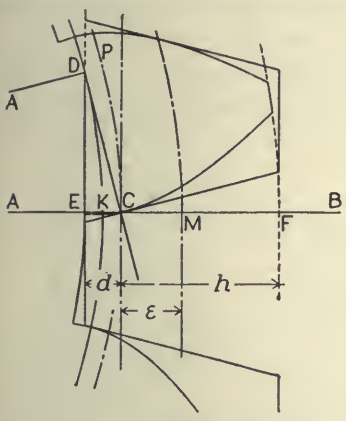

FIG. 617.—Long-addendum pinion with its rack.

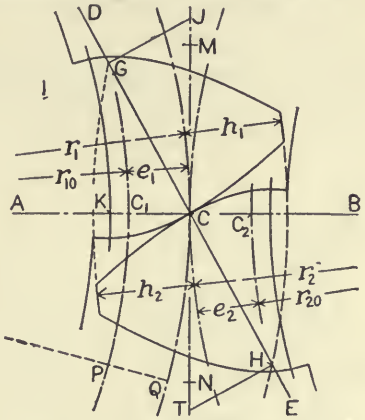

FIG. 618.—Pair of twelve-tooth pinions.

C, obliquity changing from the primary $\sigma_0 = 14.5$ deg. (fixed by the rack cutter) to the effective $\sigma = 28.7$ deg. For the shift of pitch point or change of pitch-circle radius we adopt the symbol *e*, using it however with subscript on the individual wheels, then letting plain *e* stand for the sum $e_1 + e_2$ or for the distance between primary pitch circles. To have $e_1 = e_2 = \epsilon$, as here, is a very special condition. In general, as can be seen by referring to Fig. 579, if two such excess-addendum wheels of different diameter are mated point C will divide the distance C_1C_2 in the ratio of diameters, so that

$$e_1 : e_2 :: n_1 : n_2. \tag{131}$$

The change in rolling radius from r_0 to r is accompanied by an increase in linear pitch—illustrated by means of radial line PQ in Fig. 618—from $p_0 = 2C_1P = 3.142$ to $p = 2CQ = 3.469$ (for the particular case). Converting action range GH to the tangent line as length JT, by the method of Fig. 573, we find that the ratio $a/p = JT/MN$ is only 1.16, a rather narrow margin above the limit unity. This shortening of action is a matter to be considered along with the increase of working obliquity.

(*d*) **Tooth Thickness and Backlash.**—Now while on the mid-length pitch lines at M in Fig. 617 the pinion tooth is only a little thinner than the rack tooth, the increase of effective pitch makes the working thickness (at C, Fig. 618) considerably less than half of this pitch. The resulting big backlash appears in Fig. 619, which is Fig. 618 reduced in size and with the other side of each tooth groove drawn in; here the cross marks on line TT, inside of JL, show a shorter length which is the rack-cutter pitch from Fig. 617 and a greater length reproducing enlarged pitch MN from Fig. 618. One remedy for the backlash is to thicken the teeth,

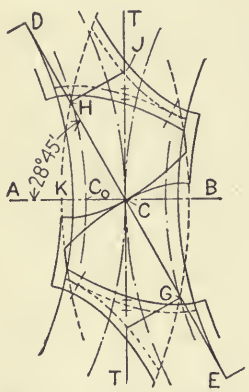

FIG. 619.—Pinion teeth thickened.

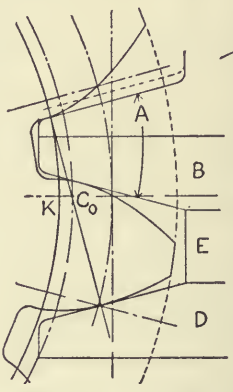

FIG. 620.—Use of special cutters.

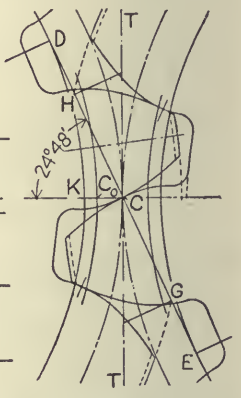

FIG. 621.—Teeth from standard cutter.

to the extent shown by the dotted profiles. Without any other change this would require that the cutter tooth be thinned as shown at A in Fig. 620: an alternate and more practicable scheme would be to use a one-tooth combination tool, composed of cutters B and D with a changeable block E clamped between them. Of course the teeth would have to be roughed out before being generated; but since the adjustable cutter would be needed only on smaller wheels this might not be a serious difficulty.

Another way of eliminating backlash is to set the pinions closer together, shortening their teeth (cutting down addendum and working depth) so that they will clear in the spaces generated by the standard rack cutter of Fig. 617. A solution along this line, into which enter some fairly complex relationships, is shown in Fig. 621. Some of its elements of change are as follows: The cutter is set deeper than in Fig. 617 or its setting distance ϵ is

less; the pitch-circle offset distance e is less than ϵ; radial clearance is diminished, for while the dotted lines show an addendum corresponding with the standard clearance of $0.05p_0$, the teeth are actually made a little longer. The net result is a smaller obliquity and, in spite of less working depth, an action range at least as long as in Fig. 619.

Two dimensions of special importance in this problem are, the thickness of tooth at any effective pitch circle and the arc of

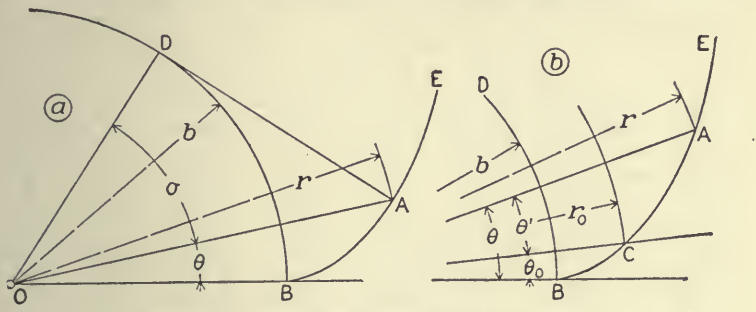

Fig. 622.—Coordinates for the involute curve.

action. The latter can readily be handled by an extension of the methods of Sect. 33 (*e*). For the former we must now develop the simplest possible method and illustrate its use.

(*e*) **For the involute curve** it will be most convenient to use the polar coordinates BOA = θ and OA = r in Fig. 622 (*a*), starting at origin B on the base circle. Instead of expressing a relation between them by an equation we simply formulate a method of calculation, then enter computed results in Table 9. At any point A the lengths of tangent DA and arc DB are equal. Taking r as the independent variable and making r/b the "argument" of the table, a sample computation is given in Form 33.

Form 33

1. Ratio r/b		1.10
2. Log b/r = log cos σ		9.9586073
3. Angle σ	24° 37′ 12″	= 24 6200°
4. Log tan σ = log arc $(\sigma + \theta)$		9.6611096
5. Log arc $180/\pi$ deg		1.7581226
6. Log angle$(\sigma + \theta)$, degrees		1.4192322
7. Angle $(\sigma + \theta)$		26.2562°
8. Angle θ, by difference		1.6362°

As apparent, angle AOD is the obliquity σ of the curve at A, or its angle with radius OA, a matter of interest but not of direct

use in further calculation on the tooth. The useful angle θ is best expressed in decimals of a degree. If there is occasion to start at some point C of the curve, Fig. 622 (*b*), we pass over its r_0, still going into the table with r/b to get θ, then subtracting θ_0 for θ'. Data belonging to certain prominent values of σ_0, named in Sect. 33 (*f*), are collected under the heading "Special" at the bottom of Table 9.

TABLE 9.—POLAR COORDINATES OF THE INVOLUTE

$\frac{r}{b}$	θ		σ	$\frac{r}{b}$	θ		σ
1.002	0.0048	0088	3° 37′	1.08	1.1806	1096	22° 12′
1.004	0.0136	0115	5° 7′	1.085	1.2902	1126	22° 50′
1.006	0.0251	0134	6° 16′	1.09	1.4028	1154	23° 27′
1.008	0.0385	0152	7° 13′	1.095	1.5182	1180	24° 3′
1.01	0.0537	0169	8° 4′	1.10	1.6362	1206	24° 37′
1.012	0.0706	0183	8° 50′	1.105	1.7568	1232	25° 10′
1.014	0.0889	0197	9° 32′	1.11	1.8800	1255	25° 43′
1.016	0.1086	0208	10° 11′	1.115	2.0055	1279	26° 15′
1.018	0.1294	0220	10° 48′	1.12	2.1334	1302	26° 46′
1.02	0.1514	0231	11° 22′	1.125	2.2636	1323	27° 16′
1.022	0.1745	0242	11° 55′	1.13	2.3959	1345	27° 45′
1.024	0.1987	0252	12° 26′	1.135	2.5304	1365	28° 14′
1.026	0.2239	0261	12° 56′	1.14	2.6669	1386	28° 42′
1.028	0.2500	0270	13° 25′	1.145	2.8055	1405	29° 9′
1.03	0.2770	0278	13° 52′	1.15	2.9460	1424	29° 36′
1.032	0.3048	0287	14° 19′	1.155	3.0884	1442	30° 2′
1.034	0.3335	0296	14° 45′	1.16	3.2326	2938	30° 27′
1.036	0.3631	0304	15° 10′	1.17	3.5264	3008	31° 16′
1.038	0.3935	0311	15° 34′	1.18	3.8272	3074	32° 4′
1.04	0.4246	0318	15° 57′	1.19	4.1346	3137	32° 49′
1.042	0.4564	0325	16° 20′	1.20	4.4483	3197	33° 33′
1.044	0.4889	0333	16° 42′	1.21	4.7680	3255	34° 16′
1.046	0.5222	0340	17° 4′	1.22	5.0935	3310	35° 57′
1.048	0.5562	0346	17° 25′	1.23	5.4245	3362	35° 37′
1.05	0.5908	0352	17° 45′	1.24	5.7607	3413	36° 15′
1.052	0.6260	0359	18° 5′	1.25	6.1020	3461	36° 52′
1.054	0.6619	0365	18° 25′	1.26	6.4481	3508	37° 28′
1.056	0.6984	0372	18° 44′	1.27	6.7989	3554	38° 3′
1.058	0.7356	0378	19° 3′	1.28	7.1543	3598	38° 37′
1.06	0.7734	0383	19° 22′	1.29	7.5141	3641	39° 10′
1.062	0.8117	0389	19° 40′	1.30	7.8782	740	39° 43′
1.064	0.8506	0394	19° 58′	1.32	8.618	756	40° 45′
1.066	0.8900	0399	20° 16′	1.34	9.374	770	41° 44′
1.068	0.9299	0405	20° 33′	1.36	10.144	783	42° 40′
1.07	0.9704	0410	20° 50′	1.38	10.927	796	43° 37′
1.072	1.0114	0415	21° 7′	1.40	11.723	808	44° 25′
1.074	1.0529	0421	21° 23′	1.42	12.531	819	45° 14′
1.076	1.0950	0426	21° 40′	1.44	13.350	829	46° 1′
1.078	1.1376	0430	21° 56′	1.46	14.179	840	46° 46′
1.08	1.1806		22° 12′	1.48	15.019	850	47° 30′
	Special			1.50	15.869	2.163	48° 11′
1.0328	0.3162		14° 29′	1.55	18.032	2.213	49° 49′
1.0353	0.3524		15°	1.60	20.245	2.258	51° 19′
1.0642	0.8540		20°	1.65	22.503	2.297	52° 42′
1.0824	1.2327		22° 30′	1.70	24.800		53° 58′

How this table of the involute curve may be used for getting points on a tooth profile can best be shown by explaining the method used in computing points for Fig. 600. Taking cutter No. 8, $n = 12$, a condensed statement is as follows: pitch radius $r_0 = n/2 = 6$; base radius $b = r \cos \sigma_0 = 5.809$; addendum radius $R = r_0 + 1 = 7$; $R/b = 1.2050$, whence $\theta = 4.608°$ and with $\theta_0 = 0.316°$ we get $\theta' = 4.292°$. Using p', s' and t' for arc lengths on the addendum circle in modules and $p°$, $s°$ and $t°$ for corresponding angles, $s° = 0.5p° + 2\theta' = 15 + 8.584 = 23.584°$, $t° = 0.5p° - 2\theta' = 6.416°$; and with $p' = 7p/6$ it is easy to calculate the lengths s' and t' in modules.

Prob. 270. What equation or formula will express the operation used in computing Table 9?

Prob. 271. Compute values for one of the ordinates of Fig. 600, other than that for cutter No. 8.

Prob. 272. By computation from Table 9 lay out the tooth profile for a pinion of 40-in. diameter, $p_d = 0.5$, $\sigma = 22.5$ deg., $h = 0.8$ mod., equal-addendum pair. Get two points on flank, one at base circle, and three on face, spaced about equally. Draw arcs on which to measure the computed dimensions, from radial center line of tooth. Construct for arc of recess and compare with Fig. 576.

Prob. 273. Similarly by computation make a template for a gear of 20-ft. diameter, $p_d = 0.5$, $\sigma = 20$ deg., $h = 1$ mod. Here arcs cannot be drawn; devise your own method to determine by how much the computed arc distances (practically the same as chords) are to be swung away from perpendicularity to center line of tooth. Compute arc of action of this wheel with an 18-tooth pinion by Eq. (126) and compare with Fig. 576.

(*f*) **Angular Thickness of Tooth.**—In problems of tooth thickness and backlash it is better to work in terms of angular than of linear (arc) dimensions. To see how Table 9 will be applied, consider the pinion tooth of Fig. 617, redrawn in Fig. 623 central in a space of the generating rack and with radial lines LQ, L'Q' through its primary pitch points P, P'. By means of the table we can get the angles subtended by arcs TS and T'S' at any radius r; then the scheme is to divide their sum by the pitch angle of the gear and plot the resulting ratio or fraction in Fig. 624, representing the total variation in angular thickness of tooth by a single curve LF.

In Fig. 624 the length of base DQ or D'F' is the 2 modules of working depth; and along it are laid off, first ϵ from the rack mid-line at M to the generating pitch line PC_0, then the pinion addendum h_p and the rack addendum h_r as indicated. Ordinates

for the tooth curve are measured from upper base line D′F′ to the scale at left, the numbers on dotted lines designating values of r/b for which points are computed. In order to show thickness of tooth the lower base DQ must now be located as follows:

At middle point C_m the rack-space width C_mM is $0.5p$. With a slant on each side measured by tan $\sigma_0 = 0.25820$ the rate

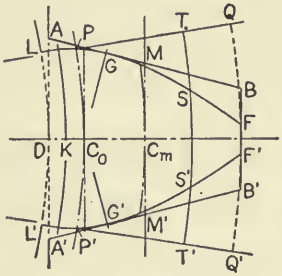

FIG. 623.—Pinion tooth.

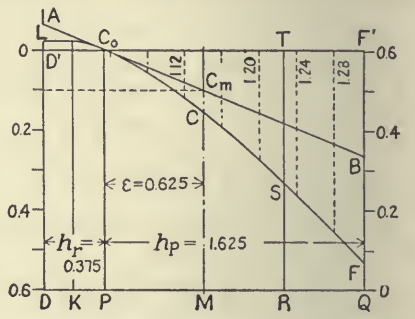

FIG. 624.—Diagram of angular thickness.

k of width variation, per module of radial displacement of cross arc, is

$k = 0.51640$ mod. or $k = 0.51640 \div \pi = 0.16438$ pitch. (132)

If $\epsilon = 0.625$ mod. the rack-space width at the generating pitch line is $0.50000 + 0.10274 = 0.60274$ pitch; and this thickness of the tooth at pitch circle PP′, Fig. 623, is the distance from D′F′ down to DQ. From this lower base the angular thickness of tooth is now read upward, to scale at right.

FORM 34

1. With 12 teeth radius r_0 =	6 mod.
2. Base-circle radius $b = r_0 \cos \sigma_0 = 6 \times 0.96825$ =	5.8095
3. At radius $r = 6.625$ ratio r/b =	1.1405
4. From Table 9 angle θ =	2.6808
5. Deduct $\theta_0 = 0.3162$ at C_0, get θ' =	2.3646
6. Since this is half the change in tooth thickness, divide by the half-pitch angle 15 deg. and get whole change/pitch =	0.15764
7. Deduct from the thickness 0.60274 at C_0, get at C	0.44510
8. Then the tooth deficiency, the half backlash in this case of equal pinions and effective pitch point C, is	0.0549
9. Incidentally, the working obliquity is read as σ =	28° 45′

Consider the question of tooth thickening in Fig. 619. To meet the requirement that two teeth shall complement each other in filling out one pitch at the working pitch circles of radius 6.625

mod., the difference between tooth ordinate CM and rack-line ordinate C_mM, *as it appears in this diagram*, must be eliminated. The procedure for accurate determination is shown by Form 34.

In Fig. 624 the necessary change in tooth thickness would be made by lowering base DQ through 0.0549 on the ordinate scale, making CM = 0.5. On the cutter the tooth would have to be narrowed by 0.0549 pitch or 0.1727 mod. Using Eq. (132), the amount by which a standard cutter would have to be withdrawn in order to get the same effective narrowing of tooth is found to be $0.0549 \div 0.1644 = 0.334$ mod.; and when added to the original $\epsilon = 0.625$ this gives the excessive value $\epsilon' = 0.957$.

(*g*) **Variation of Backlash.**—In order to see how the element of tooth thickness and backlash enters into the question of interchangeability, we shall now consider a tentative system based upon the following data:

All teeth are to be generated by the standard rack cutter in Fig. 617, having 14.5-deg. obliquity, a working depth of 2 mod., and thickness t equal to space width s on the middle pitch line.

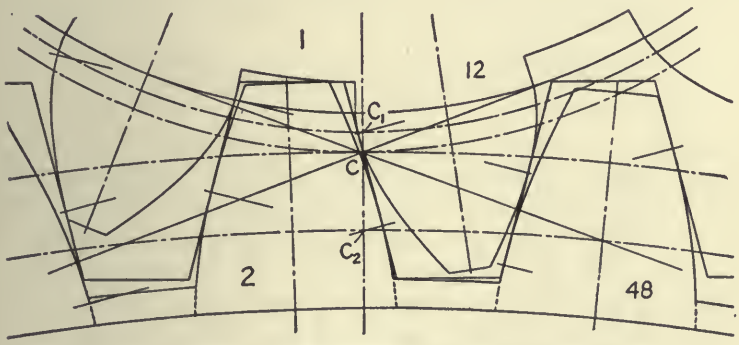

FIG. 625.—Twelve-forty-eight pair, with rack as intermediate.

This cutter is to be set at the same excess distance $\epsilon = 0.5$ mod. for all gears, and the gears are to have a constant excess in center distance of $e = 1$ mod., with 2 mod. working depth. The mean diameter of any wheel, to mid-height of tooth, will then be

$$d_m = n + 1, \tag{133}$$

and the center distance in any pair n_1 and n_2 will have the value

$$r_1 + r_2 = \tfrac{1}{2}(n_1 + n_2) + 1. \tag{134}$$

Gears thus generated are shown in Fig. 625, each in working contact with the rack profile, which functions as a geometrical intermediate between the teeth. The gears roll with the rack at their primary pitch circles C_1 and C_2 and with each other at effective pitch circles C: why and how they are deficient in thickness is clearly apparent. This drawing gives fuller meaning to the backlash diagram in Fig. 627.

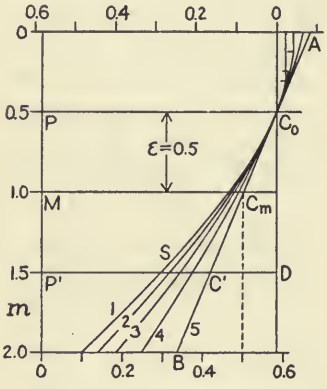

FIG. 626.

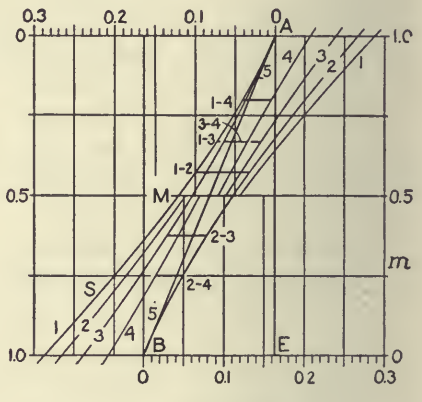

FIG. 627.

Angular thickness and backlash, series of pairs.

Guided in choice of representative numbers by Fig. 600, take wheels of 12, 16, 24 and 48 teeth, with the rack, these giving equal intervals on the $1/n$ base of that diagram. For simplicity they are numbered 1 to 5 and a set of curves like LF of Fig. 624 is drawn for them in Fig. 626. At $\epsilon = 0.5$ the common primary thickness $C_0P = t_0 = 0.5822p_0$, by application of Eq. (132). In Fig. 627 the middle portion of Fig. 626, covering the full range of possible effective pitch point from P to P′, is enlarged to double size in both directions, so that A here corresponds with C_0 original and B with C′. Then the gears are mated or paired by drawing a set of the same curves from B as an opposite C_0 or primary pitch point and taking all the combinations of two: 1 with 2, 3 and 4, 2 with 3 and 4, and 3 with 4 are indicated by short heavy cross lines at the locations of effective pitch contact. The double numbers 1-1, 2-2, 3-3 and 4-4 (for equal pairs) are on mid-line M, while the rack pairs 1-5, 2-5, 3-5 and 4-5 show zero backlash at A or B. Effective contact is, of course, located by Eq. (131), that for No. 1-4

being at 0.2 of the way from A to B, as in Fig. 625; and the length of cross line, between curves, shows total amount of backlash.

Computation for an unequal pair requires only a slight extension of the form in the last article; thus for the 12-48 pair of Figs. 625 and 627 the effective radii are 6.2 and 24.8 mod., and since r/b must be the same on both wheels it is sufficient to work for but one at first, as in Form 35. Values computed after the manner thus shown are given in Table 10 for all the wheel pairs of Fig. 627.

FORM 35

1. From the pinion $r/b = 6.2 \div 5.810 =$	1.0672	
2. Obliquity $\sigma = 20° 26'$ and angle $\theta =$	0.9139	
3. Deducting $\theta_0 = 0.3162$, $\theta' =$	0.5977	
Now separating the two wheels,		
4. Half-pitch angle $p°/2 = 180/n =$	15.0°	3.75°
5. Tooth narrowing $2\theta'/p =$	0.0398	0.1594
6. Deduct these from 0.5822, get $t =$	0.5424	0.4228
7. Combined angular thickness at C, sum of t's	0.9652	
8. Total backlash f	0.0348	

TABLE 10

	1 12	2 16	3 24	4 48	5 Rack	
1 12	0.0742	0.0659	0.0537	0.0348	0	f
	26° 39′	25° 31′	23° 28′	20° 26′	14° 29′	σ
2 16		0.0592	0.0494	0.0326	0	f
		24° 19′	22° 46′	20° 7′	14° 29′	σ
3 24			0.0425	0.0300	0	f
			21° 39′	19° 36′	14° 29′	σ
4 48				0.0239	0	f
				18° 29′	14° 29′	σ

Prob. 274. Make computation (*a*) for one of the equal pairs 16-16, 24-24 or 48-48 in Form 35; (*b*) for one of the unequal pairs other than 12-48; (*c*) for some unequal pair not in Table 10, with same value of e.

(*h*) **On the question of interchangeability** the showing already made as to how backlash varies in a system based on excess center distance is sufficient to justify a general statement, namely, that such a system cannot have the simple and complete interchange-

ability realized with invariable pitch circles. Fixing a constant excess, as in the scheme just discussed, it easy enough to adjust any pair so as to eliminate backlash, but only by making each pair a special case. Again, by using on the smaller wheels some such flexibility of tool as is suggested in Fig. 620 and at the same time permitting a considerable range of variation in radial clearance, a system might be built up in which any random pair would mesh closely and have continuous action; but then the excess of center distance would become an erratic variable.

A strong reason for the adoption of the diametral-pitch system was the desire for a numerically simple relation between tooth numbers and diameters or center distance. This is a great convenience, but there are many occasions when the freedom

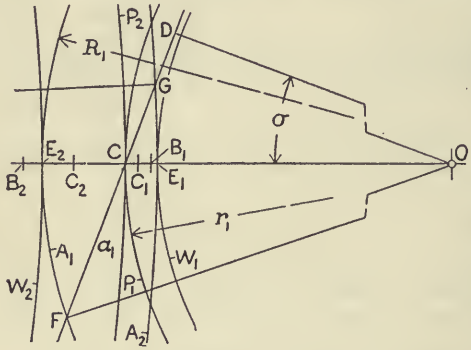

FIG. 628.—Relations for arc of action.

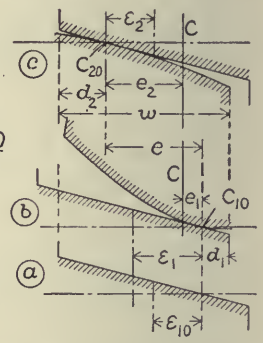

FIG. 629.—Variation in cutter setting.

to use some distance other than an integral number of modules will be a decided advantage. From the manufacturing viewpoint the really important thing is that with the benefits of the flexible system goes the ability to generate all gears of a given pitch by means of the same hob or rack cutter, with perhaps the variable tool shown in Fig. 620 for the smaller pinions.

A useful property of the flexible system is the possibility of adjustment toward equality of root thickness, or of strength of teeth.

Having very fully discussed the first dimension (thickness of tooth) named at the end of Art. (*d*), we must now take up the question of range of action, especially as related to variable addendum.

(*i*) **Range and Depth of Tooth Action.**—Conditions for the unequal pair from Fig. 625 are depicted in Fig. 628, most fully for the smaller wheel 1 whose radials can be drawn nearly to full length. This shows the primary pitch points C_1 and C_2, with effective point C through which pass the working pitch circles P_1 and P_2. In addition, points B_1 and B_2 locate the base circles, while through E_1 and E_2 are drawn addendum circles A_2 and A_1 and root or working-depth circles W_1 and W_2.

As regards unit of radial dimensions, we have carried over the module into this flexible system without as yet questioning whether it is affected by the variation of working pitch. Really, of course, both diametral pitch and module are definite quantities only as related to the rack cutter and to the primary or generating pitch circle of the gear. Primary radius r_0, as $OC_1 = r_{10}$ in Fig. 628, continues to be $0.5n$, and for system I of Table 7 base-circle radius b is $0.48413n$. Further, since limit length l = CD and range a = CF or CD are in effect arcs of base circle—see Fig. 572—they will be expressed in terms of the angular pitch by using the divisor 3.042, from line 6 of Table 7, upon their module lengths. The formulas for l and a are to be brought over from Sect. 33 (*e*) in their general terms, as

$$l = r \sin \sigma \tag{135}$$

and

$$a = \sqrt{R^2 - b^2} - l, \tag{136}$$

leaving r and R for separate definition.

The matter of radial dimensions is more fully illustrated by Fig. 629, for the same 12-48 wheel pair with total excess $e = 1$ mod. At (*a*) the cutter profile is positioned as in Fig. 625, and the same setting is retained at (*c*) for the large wheel, with $\epsilon_2 = 0.5$ mod. At (*b*) the cutter is drawn back to a larger ϵ_1, making the pinion tooth enough thicker just to take up backlash, and shortening the addendum of rack along with root depth of pinion. From dimensions written on the drawing the following relations can be seen:

The cutter having a full working depth of $w_0 \doteq 2$ mod., the primary or cutting root depths are,

$$d_1 = (1 - \epsilon_1),\ d_2 = (1 - \epsilon_2). \tag{137}$$

Since the root depth of each wheel now determines the addendum of the other—subject to an excess of length that cuts into

clearance as in Fig. 621—the addenda from primary pitch circles are,

$$\begin{aligned} h_{10} &= d_2 + e = 1 + e - \epsilon_2, \\ h_{20} &= d_1 + e = 1 + e - \epsilon_1. \end{aligned} \tag{138}$$

If taken from the effective pitch circles these are diminished by e_1 and e_2 respectively, to

$$h_1 = 1 + e_2 - \epsilon_2,\ h_2 = 1 + e_1 - \epsilon_1. \tag{139}$$

The effective working depth is

$$w = h_1 + h_2 = h_{10} + h_{20} - e = 2 + e - (\epsilon_1 + \epsilon_2); \tag{140}$$

for full depth, as in Fig. 625, the sum $\epsilon_1 + \epsilon_2$ is equal to e; when the cutter is set back this sum exceeds e and by the same amount effective working depth w is less than full w_0.

Finally, the overall radii are,

$$\begin{aligned} R_1 &= r_{10} + h_{10} & R_2 &= r_{20} + h_{20} \\ &= (\tfrac{1}{2}n_1 + 1) + (e - \epsilon_2); & &= (\tfrac{1}{2}n_2 + 1) + (e - \epsilon_1). \end{aligned} \tag{141}$$

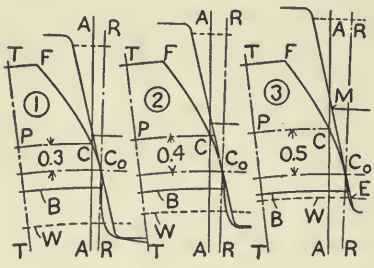

FIG. 630.—Pinion teeth, various cutter settings.

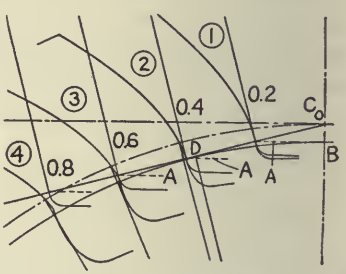

FIG. 631.—Undercutting in generation.

(*j*) **General Problem of Adjustment.**—Having established necessary methods and relations for quantitative work, we now turn to the general problem introduced by Fig. 621. A study of concrete examples continues to be the best way of developing ideas and procedure; and since conditions are simplest in the equal pair we at first take up the 12-tooth pinions of Figs. 618 to 621. The method now to be followed, illustrated by Fig. 630 and with results plotted in Figs. 632 and 633, is to choose different values of excess distance e, set the cutter so as to eliminate backlash in each case, and see how other conditions or dimensions vary.

The determining dimension $e_1 = e/2$ is marked on each individual drawing of Fig. 630; radius AA through effective pitch point

C is vertical and tooth center line TT is at 7.5 deg. (half the angular thickness) from AA; and the generating rack profile has its radial or normal line RR turned slightly from the vertical, so as to give contact at primary pitch point C_0.

One limit of possible proportion, here overpassed by case (3), is set by the fact that if the teeth get too short arc of action will become less than pitch. In the other direction a positive (but not very definite) limit is imposed by the danger of undercutting in generation, fully active in case (1). To supplement the showing of Fig. 630 this matter is considered by itself in Fig. 631, for various depth of cutter setting: the several cases are designated by the distance of rack addendum line A below the generating pitch line through C_0. In case (2), which runs just past the interference limit, there is no visible indication of undercutting; and even in case (3) it has not become very serious. For this 12-tooth pinion we may then put this limit at about $d = 1 - \epsilon = 0.5$ mod.

Variation of the several dimensions on Fig. 630 can better be seen from the plots in Fig. 632, on total e as base and related to the tooth drawings by correspondence of letters. In condensed description of the diagram, which is plotted from Form 36,

C_0 is primary pitch point, at constant radial height $r = 6$ mod.;

C is effective pitch point, at $r + \frac{1}{2}e$;

M is mid-height of rack tooth, at ϵ_1 above C_0;

E shows bottom of working depth, from $d_1 = 1 - \epsilon_1$;

F shows addendum; for equal wheels $h_1 = e_1 + d_1$.

Overall radius R, up to line FF varies but slowly, root radius to EE much more rapidly.

Curve GG shows the half action arc (of one wheel) in terms of pitch; where it crosses half-pitch line KK, running to the right of limit L, continuity of action ceases.

At N on line EE, where $d_1 = 0.5$, is marked the other limit, from Fig. 631. Evidently there is but a narrow range of possible proportions when equal 12-tooth pinions are generated by the standard cutter.

It is case (2) of Figs. 630 and 632 that is drawn in Fig. 621.

(*k*) **Use of Special Cutter.**—The effect of using a special thinner-tooth cutter, chosen with $t = 0.47p$ at its middle line, is shown by Fig. 633. Since now the cutter need not be so much with-

drawn, this change gives longer teeth and makes possible a higher range of e and of σ, but it also fixes the undercutting limit N considerably higher. In the range (horizontal) between limits N and L there is an increase of more than 40 per cent from Fig. 632 to Fig. 633; but inasmuch as requirements are inadequately met by Fig. 632 this slightly greater flexibility is hardly a suffi-

FORM 36

PART I, FIG. 632

Case	1	2	3	4
1. Excess center distance e	0.6	0.8	1.0	1.2
2. Effective radius r	6.3	6.4	6.5	6.6
3. Angle θ', r_0 to r	0.962	1.359	1.788	2.244
4. Fractional change of thickness	0.0641	0.0906	0.1192	0.1496
5. Cutter setting ϵ_1	0.390	0.551	0.725	0.910
6. Addendum radius R	7.210	7.249	7.275	7.290
7. Working depth w	1.820	1.698	1.550	1.380
8. Effective obliquity σ	22° 46′	24° 48′	26° 39′	28° 20′
9. Limit length $l = r \sin \sigma$	2.438	2.685	2.916	3.132
10. $a + l = \sqrt{R^2 - b^2}$	4.270	4.335	4.379	4.404
11. Action arc a	1.832	1.650	1.463	1.272
12. Action ratio $2a/3.042$	1.204	1.085	0.962	0.836

PART II, FIG. 633

13. Excess center distance e	0.8	1.0	1.2	1.4
14. Cutter setting ϵ_1	0.368	0.542	0.727	0.922
15. Addendum radius R	7.432	7.458	7.473	7.478
16. Working depth w	2.064	1.916	1.746	1.556
17. Action ratio $2a/3.042$	1.282	1.158	1.031	0.902

cient advantage to compensate for the greater working obliquity. Since the practical problem is not to make small pinion pairs but to get small pinions that will work well with large gears, and since (as our next example will show) the standard cutter solves that problem to full satisfaction, there is no real need of a special cutter, at least down to 12 teeth. The question of very small pinions is discussed in Arts. (*q*) and (*r*).

The computations for Figs. 632 and 633 are outlined in Form 36, where Part I is fairly full but Part II gives only the more important results of cutter change. Using 0.47 instead of 0.50 pitch for the middle thickness of the rack tooth requires that setting ϵ be $0.03 \div 0.1644 = 0.183$ mod. less than in line 5; and succeeding quantities are modified accordingly.

(*l*) **Example in Unequal Pairs.**—Take the 12-48 pair of Fig. 625, for which the backlash with $e = 1.0$ and each $\epsilon = 0.5$ mod.

was found in Art. (g) to be 0.0348 pitch. This corresponds to a change $\Delta\epsilon = 0.0348 \div 0.1644 = 0.212$ mod. in cutter setting, so that for close meshing sum $(\epsilon_1 + \epsilon_2)$ will exceed e by this

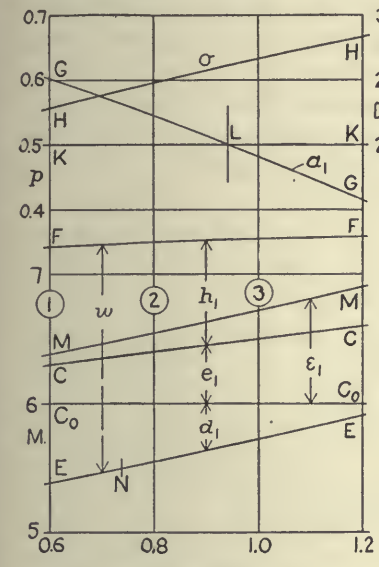

FIG. 632.—Standard cutter.

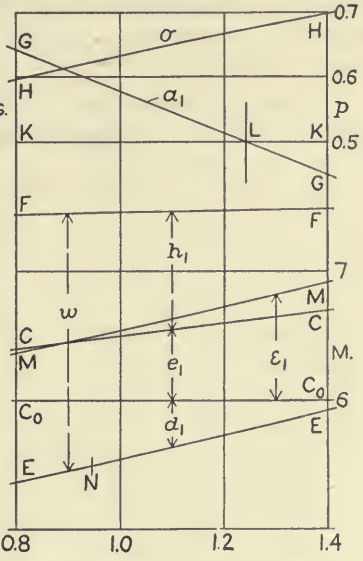

FIG. 633.—Thinned cutter.

Effect of variation in excess distance.

amount, or be 1.212. A new element of flexibility now appears, in that the division of this total cutter drawback between the gears may be varied over a considerable range. At first, however, in order to see the effect of change in excess center distance

FORM 37

Case	1	2	3	4
1. Excess distance e	0.4	0.6	0.8	1.0
2. Working obliquity σ	17° 10′	18° 20′	19° 25′	20° 26′
3. Total backlash f	0.0063	0.0136	0.0228	0.0348
4. Extra cutter drawback $\Delta\epsilon$	0.0383	0 0827	0.1387	0.2119
5. Working depth w	1.962	1.917	1.861	1.788
6. Cutter setting ϵ_1 on pinion	0.238	0.383	0.539	0.712
7. Cutter setting ϵ_2 on gear	0.200	0.300	0.400	0.500
8. Pinion addendum radius R_1	7.20	7.30	7.40	7.50
9. Gear addendum radius R_2	25.162	25.217	25.261	25.288
10 Pinion interference limit l_1	1.79	1.92	2.05	2.17
11. Pinion-flank action arc a_2	2.47	2.09	1.71	1.32
12. Total action arc a/p	1.62	1.51	1.40	1.28

e, that dimension is varied under a particular condition, namely, that gear 2 shall be cut with $\epsilon_2 = 0.5e$ and that all the extra

drawback or thickening of tooth be applied to the pinion. Computed dimensions are plotted in Fig. 634 and the more important numerical quantities are given in Form 37. Down to backlash f the calculation follows Form 34 and through line 5 all terms are definite with e, or are not affected by variation in cutter setting.

The first, lower group of curves in Fig. 634 is begun by taking a horizontal line C for the working pitch point, from which diverge

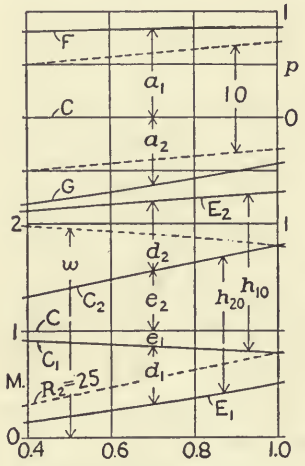

FIG. 634.—Variation of distance e.

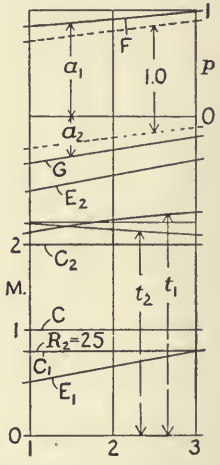

FIG. 635.—Varied setting, e = 1.0.

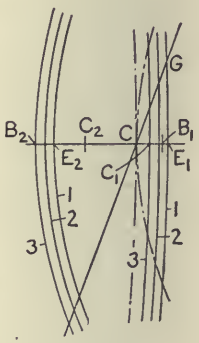

FIG 636.—Change of action range.

lines C_1 and C_2 for the generating pitch points. The general arrangement is, pinion below, gear above line E_1 showing root of pinion and addendum of gear. Carrying forward the result from Fig. 631, that for a 12-tooth pinion undercutting will begin if d_1 exceeds 0.5 mod., the plot of d_1 places this limit between cases 3 and 2; and lines 10 and 11 of Form 37 show about the same location for the interference limit. The other limit, where action arc falls below pitch, lies considerably beyond (to the right of) the range of this diagram.

Two dotted comparison curves are drawn, one for working depth w in relation to the 2-mod. line, the other (at 1 mod. below C_2) to show by how much actual R_2 exceeds the value for a gear of standard type. Upper curves F and G give action arcs in terms of pitch: with pinion as driver arc a_2 is approach and a_1 is recess. Parallel comparison lines, at unit distance apart, make visible the margin of action range.

(*m*) **Variation in Cutter Setting.**—Assuming addendum circles to be accommodated to cutter positions and consequent root circles as in Fig. 629, consider the effect upon tooth dimensions of shifting the zone of cutter action from pinion toward gear. The case investigated is No. 4 of Fig. 634, with $e = 1.0$. In Fig. 635 the ordinates show successive cases, No. 1 being identical with the last ordinate of Fig. 634. The others are determined by equal intervals of change toward the state $R_2 = 25$; that is, generating root depth d_1 on the pinion is first 0.288 from Fig. 634, then 0.144, then zero, with the primary effect shown by equidistant lines E_1 and E_2. In the third case there is left a working root depth of only 0.2 mod. on the pinion, the addendum (or root depth on gear) becoming 1.588. This shift of working zone accelerates the tendency already apparent in Fig. 634, to shorten approach arc a_2 and put most of the action range into recess, on pinion face. As can be seen from the diagram in Fig. 636, which reproduces and multiplies a portion of Fig. 628, relations between addendum circles and action line are such that increase of a_1 is considerably less rapid than decrease of a_2, so that total a slowly diminishes. Numerical statement of these quantities is made in the first five lines of Form 38.

FORM 38

Case	1		2		3	
Gear	1	2	1	2	1	2
1. Primary root depth d	0.288	0.500	0.144	0.644	0.000	0.788
2. Overall radius R	7.500	25.288	7.644	25.144	7.788	25.000
3. Root radius $R - w = r_d$	5.712	23.500	5.856	23.356	6.000	23.212
4. Face-action arc a	2.58	1.32	2.79	0.95	3.02	0.56
5. Total action a/p	1.281		1.228		1.176	
6. Generating thickness t_0/p	0.6170	0.5822	0.6407	0.5585	0.6643	0.5349
7. Working thickness t/p	0.5772	0.4228	0.6009	0.3991	0.6245	0.3755
8. Root-point ratio r_d/b	below	1.0118	1.0080	1.0078	1.0328	below
9. Root-point angle θ_d	0	0.0689	0.0385	0.0372	0.3162	0
10. Angle difference $\theta_0 - \theta_d$	0.316	0.247	0.278	0.279	0	0.316
11. Change $2(\theta_0 - \theta_d)/p$	0.0211	0.0660	0.0185	0.0744	0	0.0843
12. Root thickness td/p	0.6381	0.6482	0.6592	0.6329	0.6643	0.6192
13. Arc pitch p_d at root	2.991	3.076	3.066	3.057	3.142	3.038
14. Root thickness t_d, mod	1.909	1.994	2.022	1.935	2.087	1.881

The remainder of Form 38 is concerned with a matter that can generally be handled graphically, as in Figs. 637 and 638, but for which the computational method must be available: it is specifically illustrated by Fig. 637. Beginning with line 6, for $\epsilon = 0.5e$ the generated thickness t_0 on arcs C_1 and C_2 would be 0.5822 pitch, as for Fig. 625 in line 6 of Form 35; but in order to eliminate backlash this is increased by 0.0348 on the pinion. The 0.144 mod. shift of working zone changes tooth thickness by $0.144 \times 0.1644 = 0.02365$ pitch, adding to pinion, subtracting from gear. The complementary working t's, on arcs C of Fig. 637, come from

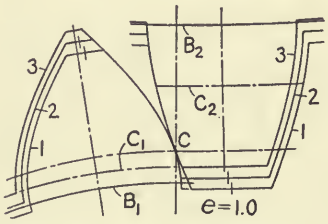

FIG. 637.—Teeth for form 38.

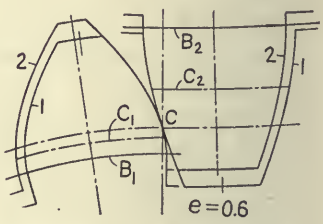

FIG. 638.—Case 2 of form 38.

the t_0's by the deductions 0.0398 and 0.1594 in line 5 of Form 35.

Now by means of Table 9 determine the change in angular thickness from generating radius r_0 down to root radius r_d, assuming radial flanks where root lies below base circle. Adding the result (line 11, Form 38) to t_d/p (line 6) we get the angular thickness at root. Multiplying this by the arc pitch at r_d the final result is arc thickness on root circle in modules.

(*n*) **Tooth Proportions.**—An advantageous result attained in Form 38 and Fig. 635 is the agreement of overall radius R_2 with that of a standard gear, the advantage consisting in ability to use the same pattern for gears in different pairs: to vary patterns for pinions or (more probably) to forge the blanks to particular diameters, is a simple matter. But a third limitation upon practical proportioning (beside interference and undercutting) now appears in the tendency of pinion teeth to become too narrow at the tip (too pointed) with very large addendum.

Teeth for the three cases of Form 38 are carefully drawn in Fig. 637, with contact at working pitch point C: on them are shown generating pitch circles C_1, C_2 and base circles B_1, B_2. Evidently the last case goes too far, not only in thinning of tip but also in making pinion tooth thicker at root than gear tooth: note the

proportions in line 14 of Form 38. Going back to Fig. 634 we see, however, that $e = 1.0$ mod. is more than is needed for avoidance of undercutting, even under the initial condition as to location of working zone. With that zone shifted from pinion toward gear a value of e as low as 0.6 mod. will give excellent working; and this conclusion is embodied in Fig. 638, where the first profiles represent case (2) of Form 37 and there is but one step to the state $R_2 = 25$.

As to the important question of root thickness the second cases of Figs. 637 and 638 are tested and compared in Fig. 639, drawings (*a*) and (*b*) respectively. First the teeth of pinion and gear

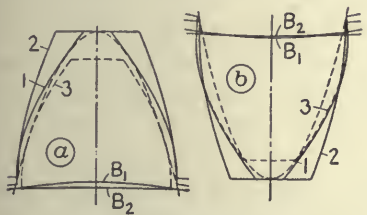

FIG. 639.—Comparison as to root thickness.

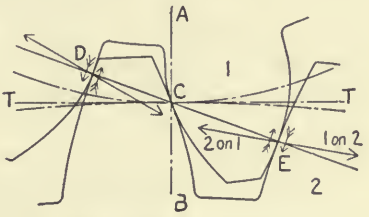

FIG. 640.—Effect of tooth friction.

are drawn together on the same center line, agreeing at addendum and with base-circle arcs which (partly as a characteristic of the particular tooth-number combination) happen to fall very close together. Next on each combination is laid down the parabola outline of a cantilever beam of uniform strength, which is barely contained within the rather too pointed outline of the pinion tooth of solution (*a*): omission of clearance curves makes the showing not quite complete, but essential equalization is realized in either solution. Finally the pinion tooth of Fig. 621 is dotted in (on the same base circle B_1) for comparison, showing very close agreement with solution (*b*) except as to addendum. Leading general proportions of these two solutions are collected in the following comparison:

	r_1	r_2	R_1	R_2	w	a_1	a_2	a/p
(*a*) Fig. 637–2 ..	6.20	24.80	7.64	25.14	1.79	2.79	0.95	1.228
(*b*) Fig 638–2...	6.12	24.48	7.52	25.00	1.92	2.85	1.52	1.435

(*o*) **Action Range and Tooth Friction.**—In connection with the unequalization of approach and recess in the type of proportioning to which we have come, it is of interest to consider the general influence of tooth friction. This is shown in Fig. 640, on teeth

of less extreme proportions. The pinion (now above) being driver, there is contact and pressure at D in approach and at E in recess, with the relative sliding indicated by pairs of short motion arrows. Because of frictional resistance to sliding the lines of total pressure are inclined from normal DE by the "angle of friction," in such direction that the pressure of each tooth upon the other will have a component opposed to the relative motion of that other. The result is to increase obliquity of pressure line during approach, to decrease it during recess, by an amount which will range from 1 to 5 deg. according to conditions of lubrication. Since smaller obliquity means slightly smaller total pressure—see Fig. 580—there is a slight advantage in having more of recess than of approach. This benefit will easily be neutralized if the unbalance is carried so far as to cause a high velocity of sliding toward end of recess. In any case the variation of friction action within the limits of practicable proportion is an insignificant element of the problem of design.

(*p*) **Use of the Flexible Involute.**—The large freedom in choice of proportions that is offered by the flexibility of the involute system of profile curves is excessive. Introduction of the principle of the single generating cutter restrains and directs this freedom into the practically useful channels of a system of gear teeth. There have been sporadic instances of the use of long-addendum gears, cast, made by special cutters or produced by hobbing; but a general application of the idea is first to be found in the Maag system and machine, which was brought to full development in Switzerland about 1915 and introduced into the United States in 1918. The machine has the motion cycle outlined in Fig. 596.

This flexible system, with its two main variables of excess center distance and relative cutter setting, can never be reduced to the simplicity of the older system with fixed pitch circles and uniform addendum. It is, however, not nearly so cumbersome to handle as might be inferred from the wide range of the trial solutions used for our illustrative problems. The very full analysis made is intended to show principles, relations and methods. When these are understood there is much room for condensation and short cuts.

The whole scheme may be summed up as follows:

The controlling considerations are, to avoid interference and to keep at a reasonable distance from its beginning; to avoid

undercutting in generation, on pinions; to secure a sufficient range of action; not to have pinion teeth too narrow at tip. These may be said to set boundaries within which a solution must lie. Desirable elements of a result are, minimum working obliquity and equalization of strength of teeth.

The basis of the method here presented is Table 9, in form and use the simplest means to exact quantitative relations. The foundation of the tooth system is the primary obliquity $\sigma_0 =$ 14.5 deg. of the generating cutter and its 2 modules of full working depth. The primary variable is generally excess center distance e, since that fixes the working pitch radii; it may also take the form of an assumed working obliquity.

The first step is to compute backlash, with primary cutter setting or with combined cutter drawback equal to e, most simply with each $\epsilon = 0.5e$ as in Fig. 624 or 626: this condition is clearly illustrated by Fig. 625.

Next by Eq. (132) find increase $\Delta\epsilon$ in combined drawback, with resulting value of working depth w.

Then apportion working zone w between the two wheels, using the relations in Art. (i) and computing range of action by Eq. (136). If the larger wheel is not too big this part of the problem may be handled graphically, as in Fig. 636. Considerations and methods in Arts (m) and (n) govern the problem of adjustment.

It is highly desirable to have a method for going right to a nearly correct solution, without making a full and laborious trial as in Fig. 632 or 634. Probably the best tool for this purpose will be a collection of large-scale accurately drawn curves like LF of Fig. 624: the first problem following will produce an element of such a collection.

Prob. 275. For an assigned tooth number n lay out as precisely as possible a curve of angular variation of tooth thickness, like LF in Fig. 624, running reference line D′F′ through the primary pitch point C_0 with a 14.5-deg. rack cutter. For scales let 1 module = 100/30 in. (30 scale) and 1 pitch = 10 in. Extend curve from 1 mod. inside of C_0 to 2 mod. outside. Cross rule at intervals of 0.2 mod. and of 0.5 pitch. Make an accurate tracing on 9 by 12 in. sheet of good tracing cloth, with standard lettering as directed. Do not put in any lower line such as DQ.

Prob. 276. For the 12-48 pair of Forms 37 and 38 find the effect of decreasing e under the requirement that R_2 shall equal 25 mod. in every case. Get all the quantities needed for a diagram like Fig. 634 and lay it out.

Prob. 277. Make a complete investigation of the pair 10-60 (or some other as assigned) under the assumption that $R_2 = 31$ (in general $\frac{1}{2}n + 1$):

the result will be a computation like Form 37 (more items) and a plot like Fig. 634.

Prob. 278. Take a lathe back gear (see Fig. 126) with 32 and 95 standard teeth of 6 diam. pitch in first pair B-C and design a second pair D-E with 13 and 92 teeth of nominal 5 diam. pitch: let large gear E have standard overall diameter. Determine all needed dimensions, working by so much as is needed of the method of Forms 37 and 38, with Fig. 634.

Suggestion. Bases for other problems of this sort can be found in Figs. 213–217, 227–229 and 232– 235

(*q*) **Smallest Involute Pinion.**—By means of Table 9 the problem parallel to that of Fig. 566 can be definitely solved in simple fashion, with the same result that five teeth is the minimum number for continuous action in an equal pair. Since action is limited by tangency D, Fig. 641, and since the total action

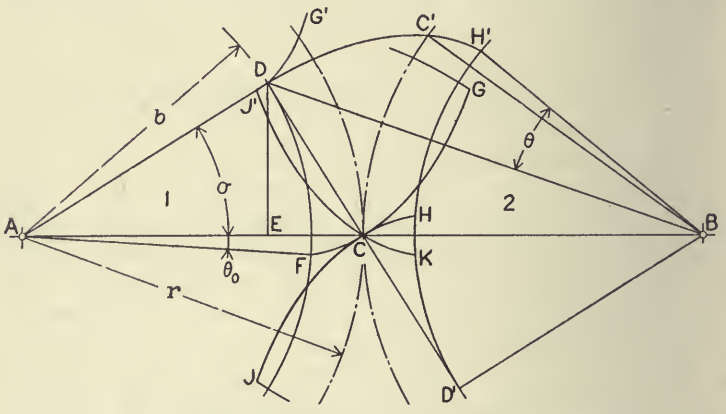

FIG. 641.—Relations for five-tooth involute pinions.

arc must be at least one pitch, the minimum distance AB between centers is that which makes angle FAD equal to half the pitch angle: then with a small tooth number the question of continuous action is whether or not tooth tip J′ reaches to or past D. Since the half-pitch angle is $180/n$ and Fig. 641 is drawn for 5 teeth, profiles DG′ and FCG must be 36 deg. apart. To locate radii AF and AD in proper relation to line-of-centers ACB, we find by interpolation in Table 9 that $\sigma + \theta_0 = 36$ deg. when $\sigma = 32°8.5'$ and $\theta_0 = 3.86°$. The reading $r/b = 1.1810$ from the table or the construction of triangle ADC determines the working pitch circle and makes the rest of the figure a mere matter of drawing. The short overlap of tip J′ beyond D shows that the action range is sufficient.

In contrast with the graphical method of Sect. 33 (*c*) we can here get relations wholly by computation, as outlined in Form 39, where all linear dimensions are in terms of base-circle radius b as unit. The first five lines of this schedule have already been explained. The further scheme of calculation is to get BD or R' as the hypotenuse of triangle DBE, then from Table 9 to find angle θ or H'BD for this R'/b on wheel 2, and finally the angle C'BD. If this is less than the half-thickness angle $90/n$ tooth tip J' will extend beyond D; if greater, as for four teeth, the tooth will not reach to D.

FORM 39

1. Tooth number n	6	5	4
2. Half-pitch angle $180/n$	30	36	45
3. Working obliquity σ	27° 38′	32° 8.5′	38° 9′
4. Angle ordinate θ_0 at pitch point	2.37°	3.86°	6.85°
5. Pitch-circle radius r	1.1287	1.1810	1.2716
6. Center distance AB $= 2r$	2.2574	2.3620	2.5432
7. AE $= \cos\sigma$	0.8859	0.8467	0.7864
8. EB $= 2r - \cos\sigma$	1.3715	1.5153	1.7568
9. DE $= \sin\sigma$	0.4638	0.5319	0.6177
10. BD $= R$	1.4477	1.6054	1.8622
11. Angle θ = H'BD	13.67	20.49	32.48
12. Angle $\theta - \theta_0$ = C'BD	11.30	16.63	25.63
13. Half-thickness angle $90/n$	15.00	18 00	22 50
14. Difference	−3.70	−1 37	+3.13

Deficiency of contact range in the four-tooth pair is shown by Fig. 642, together with the generated clearance curve, below base circle, that would be produced if true rolling were externally maintained. Figure 643 serves as an illustrative example, to answer the question, What will happen if center distance is varied with these extreme, pointed-tooth pinions? It is drawn for the 5-tooth pair of Fig. 641, first decreasing the working obliquity to 27.5 deg., then increasing it to 38 deg.: with change of pitch circle goes a very considerable variation in thickness of tooth and in its height above base circle. In the first case the clearance curves are not generated, but it is evident that there will be a good deal of undercutting, with deficient range of true action. A small increase of center distance is the best expedient for eliminating the slight interference that exists in Fig. 641. The increase of tooth length that accompanies this change in some degree compensates for the shortening of action range due to greater slant

of line DD′; but Fig. 643 (*b*) shows how the separation may be carried so far that arc of action becomes less than pitch.

(*r*) **Small Pinions with Rack.**—Conditions governing the use of very small pinions with the rack are sufficiently illustrated by

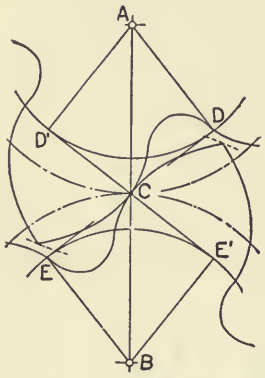

FIG. 642.—Four-tooth pair.

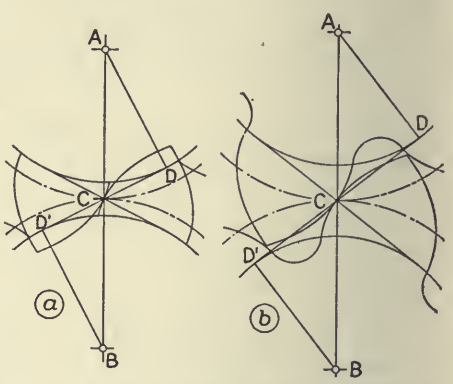

FIG. 643.—Variation of center distance, five-tooth pair.

the examples in Figs. 644 and 645. The first shows a standard rack in mesh with a five-tooth pinion which it has generated at the same relative setting as in Fig. 617, or with its addendum line through interference limit D: according to the relation devel-

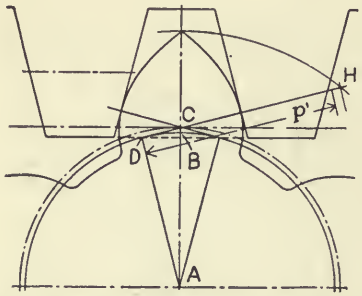

FIG. 644.—Standard cutter with five-tooth pinion.

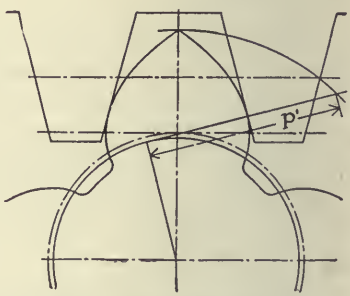

FIG. 645.—Thin-tooth rack with four-tooth pinion.

oped in Art. (*b*) the working root depth is then $\frac{5}{32}$ and drawback ϵ is $\frac{27}{32}$ mod. Curvature of profiles brings the tooth to a point before the full working depth of 2 mod. is reached, and range of action has a very narrow margin over one pitch. For an effective pair the rack tooth should be somewhat thinned, making

pinion tooth correspondingly thicker and longer. By a large change in this direction it is even possible to get continuous action with a four-tooth pinion, as shown in Fig. 645, but evidently that pair lies beyond the range of practical proportions.

It is of interest to see how the pinion of Fig. 644 compares with those in Fig. 641. From Form 39, with DE = 0.5319 and EB = 1.5153 under $n = 5$, we have tan DBE = 0.3510 and DBE = 19.35 deg., while H'BD = 20.49 and CBK = 3.86 deg. are already known; then total base angle H'BK is 43.70 deg. In Fig. 644, with $\epsilon = {}^{27}\!/_{32}$, generating (and working) thickness of cutter tooth is $0.5000 - 0.1644 \times {}^{27}\!/_{32} = 0.3613$ pitch, leaving 0.6387 as thickness of pinion tooth. On a pitch angle of 72 deg. this gives an angular thickness at pitch circle of 45.99 deg. Adding the increase 0.63 from pitch to base circle, the angle to compare with 43.70 in Fig. 641 is here 46.62, showing that in order to generate the equal pinions the rack must cut a little deeper than for the condition in Fig. 644.

Now the purpose in using very small pinions will be to get a high turning ratio with a larger gear of only moderate size. With the primary cutter setting of Fig. 625 there will be a good deal of backlash, to be taken up by thickening the pinion tooth. Since margin of action range is small no working depth ought to be sacrificed by withdrawal of cutter on pinion, as in Fig. 621, etc. Consequently the use of a special cutter, like the one in Fig. 620, becomes really necessary with very small pinions. It is easily possible to proportion a five-tooth pinion (for a high-ratio pair) that will have sufficient action range with its tip slightly flattened or truncated. In the pair of Fig. 644 or Fig. 617 the flat on tip of pinion tooth (for a working depth of 2 mod.) disappears when number n goes below 8.

(*s*) **Unequal Numbers with Same Diameter.**—The type of planetary differential used in Figs. 327 to 329 calls for two gears of equal overall or addendum diameters but with slightly different numbers of teeth Conditions in such a combination are represented by Fig. 646, where 1 is the rolling pinion of double width while 2 and 3 are the differential gears: the tooth numbers in this example are $n_1 = 15$, $n_2 = 21$, $n_3 = 20$. Gears 1 and 2 have standard proportions and profiles, so that the center distance is 18 mod., with pitch point C. For pair 1-3 the pitch-circle radii are then

$$r'_1 = {}^{15}\!/_{35} \times 18 = 7.714, \quad r_3 = {}^{20}\!/_{35} \times 18 = 10.286,$$

fixing pitch point C′. Since base circle B_1 is unchangeable the line of action will be a tangent to it through C′, determining base circle B_3 and the new obliquity σ', here 19.7 deg. as against the primary obliquity $\sigma = 14.5$ deg. Gear 3 now has long-addendum teeth, similar in a general way to those that we have been discussing.

This arrangement is likely to be used under conditions of speed and tooth pressure such that the profiles need not be exactly correct. Sketch (*a*) suggests the method for milling teeth on

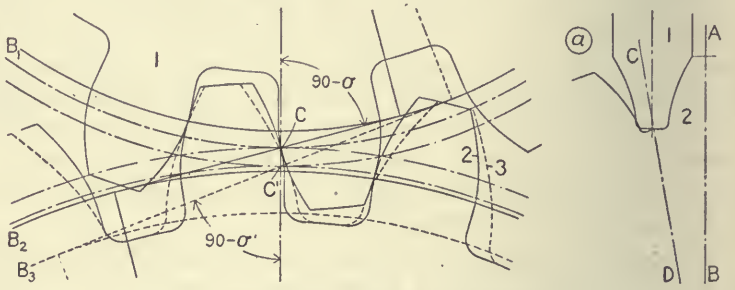

FIG 646.—Gears for planetary differential.

gear 3, using the cutter nearest to the true curve, setting it off center as indicated, and taking two cuts in each tooth space. In order to make proper selection for this operation it will be well to have a set of templates for the series of standard cutters, then find by trial which one best fits the plotted profile of the special gear and determine graphically how cutter and gear blank shall be set in the machine.

Prob. 279. Make a set of templates for the "regular" involute cutters in Table 8, on 2 diam pitch or with the module = 0.5 in.; on each draw center line AB representing its middle plane and a cross line at C tangent to pitch circle. Draw the profiles by Grant's circle-arc method.

Prob. 280. For the combination in Fig 646 (or for some other as assigned) lay out tooth profile of gear 3, find by trial the cutter that will most closely fit it, then determine graphically the offset *a* from axis of gear to middle plane of cutter. With tooth numbers as low as in this example (Fig. 646) it will be necessary to use the thin cutters made for bevel gearing, in order not to cut into flank on other side.

SECTION 39. PIN AND CHAIN GEARING

(*a*) **Pin Gearing.**—An interesting variety of gearing, historically the oldest type and much used in clockwork, is introduced by Fig. 647. The primary geometrical forms are, a

point C (or D or G) on the pitch circle of the pinion and the dotted epicycloid CF (or cycloid CJ for the rack) which this point describes on the mating wheel. Point C serves as the center of a pin, and the actual tooth profiles are equidistants or parallels to the primary curves: to say that curves are parallel means that they have a persistent common normal of necessarily constant length.

Since it is evident that the toothed wheel can have only face action, the pin center must be considered as a flank reduced

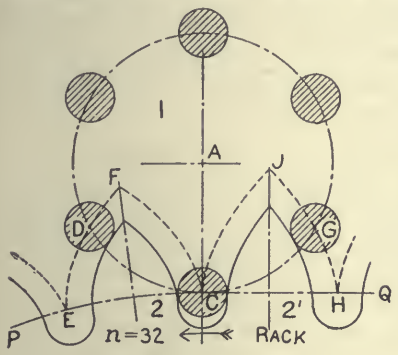

FIG. 647.—Pins on smaller wheel.

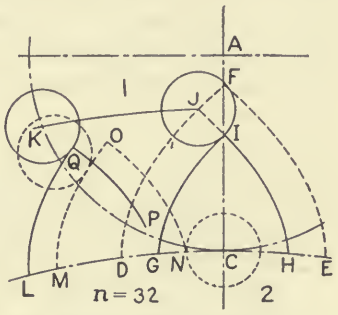

FIG. 648.—Limit of tooth action.

to minimum. With the pinion driving, action is limited to approach, with the wheel driving, to recess, ending or beginning at pitch-circle contact C. Relations determining the other limit of action are illustrated by Fig. 648. Having a tooth DEF-GHI on the line of centers, draw common normal IJ to locate the highest contact position of the pin, then by arc JK from wheel center B carry J over to K, on the pitch circle which is also locus of primary contact. Tooth GHI goes to LQP, showing in LM the amount by which arc of action exceeds pitch. With accurate work point Q will be on straight line CK.

Drawn with the same proportions and scale as Fig. 648, Fig. 649 shows that a four-tooth pinion is just beyond the limit of continuous action for a 32-tooth wheel (No. 2), just within it for rack 2′. In practice the wheel is not likely to have so few as 32 teeth, therefore the range of usual sizes lies between the cases here given; and the pinions usually have 6 or 8 teeth. Figure 650 gives an 8-48 pair with typical clockwork proportions, pin diameter and tooth thickness approaching equality, but with the pin a little smaller and with some backlash. View (*b*)

shows the "lantern" form of the pinion, the wire pins being pressed into holes in two discs.

Geometrically the pins might equally well be on the larger wheel, the profiled teeth on the pinion: then the primary curves would be epicycloids generated by a describing circle larger than the pitch circle, while for the particular case of the rack they would be involutes of the pitch circle. Beside the advantage of keeping the heavier pinwheel construction on the smaller diameter, there is the further fact that in clockwork the drive is always from spring or weight-drum toward escapement, or from wheel to pinion in each pair; then the recess action of Figs. 647 and 650 gets the benefit of slightly lowered friction effect, as described in Sect. 38 (*o*).

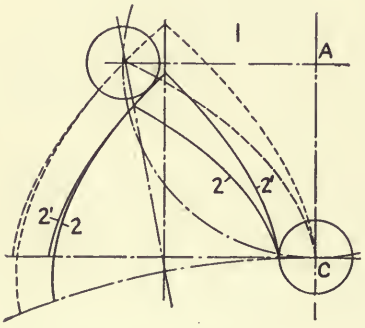

FIG. 649.—Four teeth on pinion.

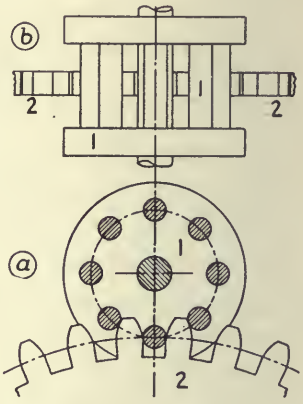

FIG. 650.—Lantern pinion from clockwork.

Simple pin gearing is, of course, not suitable for a train in which forces of any size are to be transmitted, because so much rubbing action is concentrated on a small portion of the pin surface; equally of course, from the viewpoint of wear one would put the pins on the larger wheels. It is always possible to use rollers on (around) the pins, but practical considerations limit this type of construction to one or two special cases of gear wheels, although it is the regular thing in the better type of transmission chain.

(*b*) **Special Cases.**—When gearing of the type in Figs. 647 to 650 changes to the internal form the primary tooth curves on the larger wheel become hypocycloids. In the special case where the pinion is half the size of the annular wheel these hypocycloids

arc diameters and the tooth spaces become straight grooves with parallel sides. Because the pin now has double contact, *touching both sides of the tooth space*, it can drive in both *approach and recess;* and with this doubled range of action two pin teeth

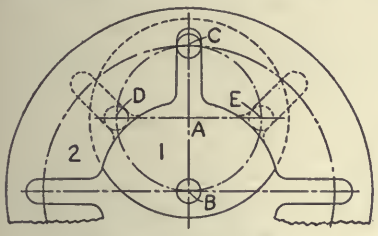

FIG. 651.—Two pins.

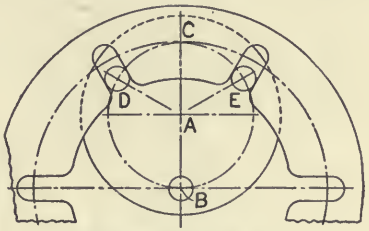

FIG. 652.—Three pins.

Two-to-one internal pair, radial slots for teeth.

arc enough for continuous action, as shown by Fig. 651. Here the addendum of the annular wheel is determined by the dotted positions of the pinion teeth, since it must be long enough to embrace the 180-deg. pinion arc DCE which is equal to the pitch of gear; maximum obliquity is seen to be 45 deg. By using three teeth, Fig. 652, the limit of obliquity is lowered to

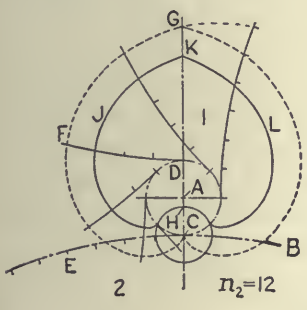

FIG. 653.—One-tooth pinion.

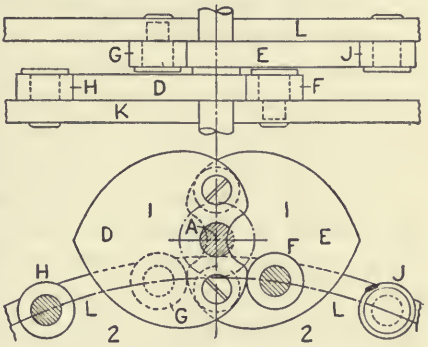

FIG. 654.—Grisson gear.

30 deg. This arrangement is chiefly of kinematic interest, for even with rollers to diminish wear it would not be nearly on a transmissive equality with a pair of ordinary toothed wheels.

In Fig. 653 point C on the larger pitch circle CB traces epicycloid CFG on the plane of small pitch circle CD. By making the pitch equal to the circumference of this small circle and putting a pin on center C we get a cam form HJKL which is really a one-tooth pinion. While contact is, of course, not continuous, it

persists through much more than half the pitch; so that by combining two such pairs into a unit, as in Fig. 654, an effective continuous drive is secured. An important constructional difficulty is seen in the way that the rim of wheel restricts diameter of pinion shaft; the latter must be forged solid with its pinion, and the necessary undercutting of the shaft end in order to get a profile that will clear the roller (in the position of Fig. 653) is a decided element of weakness.

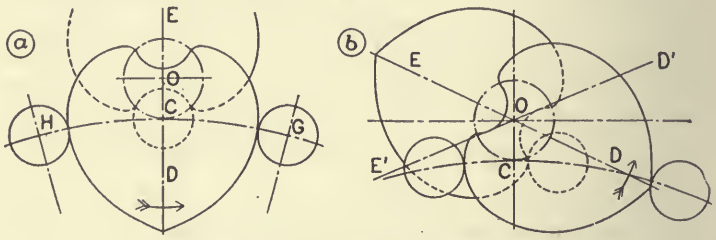

FIG. 655.—Action of Grisson gear.

Figure 655 (*a*) shows the double contact, of one profiled tooth with two pins, that exists during a large portion of the pinion revolution: this does not, however, have the effect of doubling the contact range as in Figs. 651 and 652, but is like the contact of a common gear tooth with both sides of its space. It is also made apparent that with respect to the pin rollers the pinion tooth is a cam of constant width. At the position in view (*b*) lobe D is just going out of double contact, but at D′E′ lobe E will come into that state. Because arc HG (or CB in Fig. 653) is the locus of contact, obliquity is very small in this pair; and with rollers to diminish friction it can be reversed to better effect than a high-ratio pair of common toothed wheels; by reversal is here meant to make the large wheel driver, or to use the pair for speeding up.

(*c*) **Pin-and-sprocket Chain Gearing.**—A pin rack with small pinion is shown in Fig. 656, not as a practically useful form in itself but rather as an introduction to the study of chain gearing; evidently the pinion profiles are involutes to their pitch circle. Approximately the gear chain, skeletonized in Fig. 657 and fully shown in Figs. 658 to 664, may be viewed as a flexible rack that winds on or off its sprocket wheel; but its division into links substitutes polygon for circle and considerably modifies the action from a simple rolling of pitch lines. Changes and differences may be briefly described as follows:

First of all, we now pass definitely from the arc pitch of spur gears to the chord pitch that appears in Fig. 657, then with p = pitch AB and n = number of teeth in wheel, diameter d is no

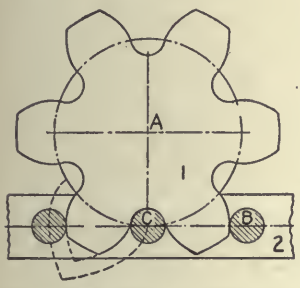

FIG. 656.—Pin rack with pinion.

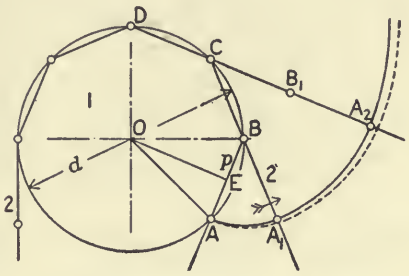

FIG. 657.—Outline of chain and wheel.

longer np/π but must be got from triangle AEO. Since angle AOE = $360/2n = 180/n$,

$$d = 2\text{AO} = p \div \sin \frac{180}{n}. \qquad (142)$$

Further, linear pitch p is now the primary quantity, to be given in simple fractions of the inch, instead of taking the decimal value that comes from simple, exact numbers for diametral pitch.

The effective radius of the wheel, or the distance from center O to the center line of the chain, is continually fluctuating between the limits OA and OE. The slight disturbance thus introduced is generally negligible, but it is an argument against using too small numbers of teeth in wheels for high-speed service.

As chain unwinds from wheel in Fig. 657 the path of a pin center is a "broken arc," portion AA_1 having center at B, A_1A_2 at C, and so on. This differs appreciably from the involute on the pitch circle, which is dotted in for comparison.

Finally, the pitch of chain pins is not nearly so fixed and unchangeable as that of teeth on a gear. Because of wear in the joints it lengthens progressively during the life of the chain, and this change has an important influence upon the design and the working of sprocket-wheel teeth.

(*d*) **Action of Chain with Wheel.**—Passing from these matters of pitch relationship to a general view of chain-and-tooth action, the broad fact appears that from the nature of the chain-and-wheel pair there really is no principal tooth action of the gear-

wheel type. Consider Fig. 656, letting the portion of the rack to the right of C represent the tangent or unwound chain. If the pinion drives, turning clockwise so as to pull the rack toward the left, tooth rolling is in recess, on the left side of C. If the rack drives, pulling toward the right, tooth action is again on

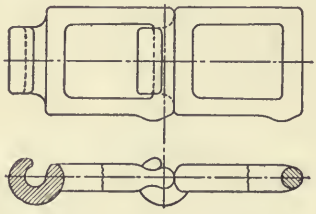

FIG. 658.—Detachable link chain.

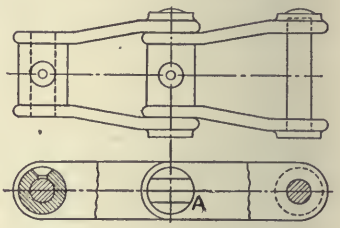

FIG. 659.—Pintle chain.

the left-hand side of C. But on that side the chain is wound upon the wheel, its pins being simply bedded in the tooth spaces, with no motion relative to the wheel; so that with true equality of pitches there is none of the combined rolling and slipping under full pressure that is characteristic of gear teeth and is necessary to their desired transmission of motion. Only in certain second-

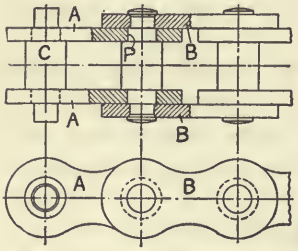

FIG. 660.

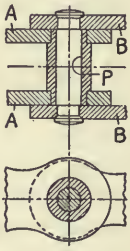

FIG. 661.

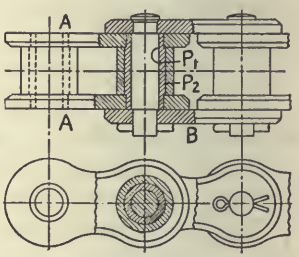

FIG. 662.

Development of pin and roller chain.

ary movements due to inequality of pitches will there be a small sliding of pin upon tooth. In Fig. 656 the question of pin B and its tooth is evidently one of *clearance*, not of pressure action.

(*e*) **Construction of Pin and Roller Chains.**—Before analyzing the action of chain gearing we shall view a series of examples, selected to show types of chain and lines of development. The first two, Figs. 658 and 659, represent the rougher grade of construction, used chiefly in conveyors and the like, sometimes to transmit power in moderate amount and at low speed. The

detachable chain is much used for light service of all kinds: when the pair of links is folded to an acute angle pin will slide out of hook. The pintle chain is a stronger form, with better protection of the hinge joint: note the tee head of the riveted pin (at A), which insures that turning shall take place on the long, not the short, bearing surface. These links are commonly "malleable iron" castings, but cast steel may be used for strength: and if the chain is to work in gritty substances the very tough and highly resistant manganese (cast) steel is the proper material.

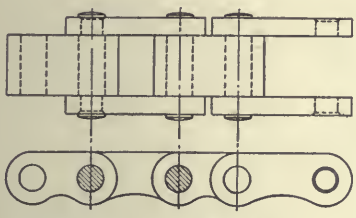

FIG. 663.—Block chain.

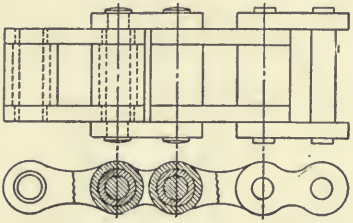

FIG. 664.—Double-roller chain.

An early form of side-link pin chain is shown in Fig. 660: the pins are fast (riveted) in the outer B links and all hinge action comes upon the narrow bearing surfaces in the A links. The pin-and-bush arrangement with a long bearing, Fig. 661, is a great improvement as regards wear in the chain joint: now the hollow pin or bush is fast in the inner A links. For high-grade service the roller chain, Fig. 662, has come to be the accepted type. The roller has a bearing of such large diameter that its effect in reducing friction cannot be very great, except for the better lubrication within this bearing: but as has just been pointed out there can be very little of the kind of rolling in Fig. 654 or in a cam-and-follower pair, and the function of the roller is rather to present continually new surface for the impact and pressure effects of chain working.

The block chain, Fig. 663, stands beside Fig. 661 in its action within joint and with teeth, but has fewer parts and does not require so many teeth on the sprocket wheel; it is, however, commonly used only for lighter service, where roller effect is not needed. The double-roller chain, Fig. 664, fits the same type of wheel and equals Fig. 662 in its action with teeth.

In the manufacture of these link-and-pin chains the flat links are stamped or punched out of strip steel, the holes being either punched small and then reamed or else drilled to exact size.

The thick block links in Fig. 663 are sawed from a bar cold-rolled to the proper cross section, then drilled. Pins, bushes and rollers are case hardened or may be of a moderately self-hardening steel: blocks should be of the latter material.

Prob. 281. In a block chain let p stand for combined pitch, kp for link alone, then get an expression like Eq. (142) for diameter of pitch circle of wheel.

(*f*) **General Action of Pin Chain.**—Passing over the very simple case in which chain fits wheel exactly, we first consider the

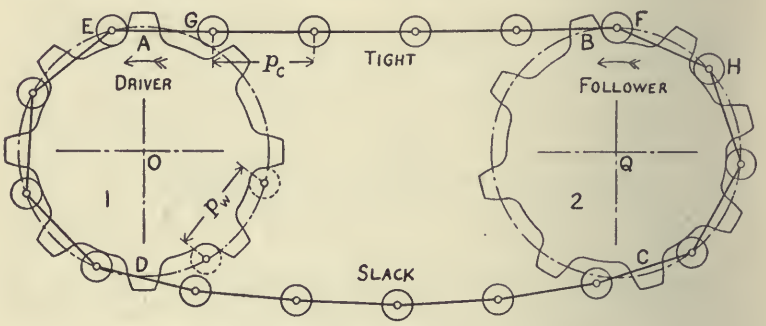

FIG. 665.—Chain with pitch of excess length.

apparent effects of unequal pitch (in chain and wheel), then advance to a closer study of tooth action and form. In Figs. 665 and 666 the pins are relatively small, running even below the proportions in Figs. 658 and 659; and the teeth are also made thin or narrow, so as to leave an abnormal side clearance in which pitch differences can accumulate without interference. When p_c is longer than p_w, Fig. 665, all the pressure comes upon single teeth A and B, on the tight side at driver approach and follower recess. In order to get to the bottom of tooth space pin G must slide or roll forward on its tooth, pushing ahead all of the chain already wrapped on wheel 1; and as pin F leaves wheel 2 it will let the latter slip backward, within the chain, until the next tooth comes against H.

With the chain pitch short, Fig. 666, full tight-side tension is carried around each wheel to the active teeth C and D, on the slack side at follower approach and driver recess. Now pin F will push wheel 2 ahead as it goes in, while pin E (as it comes out of contact) will let the chain slip backward on wheel 1. Of course, neither of the actions shown by these figures can affect overall

velocity ratio, which is determined by the number of links wound on or off; they merely make the transmitted motion irregular.

On the question of how truly real performance is represented by the conditions depicted in Figs. 665 and 666 we can make some

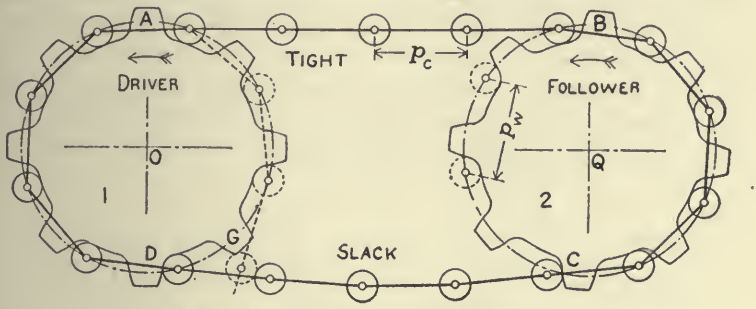

FIG. 666.—Chain with short pitch.

preliminary general statements as follows before proceeding to a detailed study of the interaction of pin and tooth:

With cast links, as in Figs. 658 and 659, and probably with cast teeth on the sprocket wheels, some irregularities in pitch are pretty sure to occur; and in such service considerable backlash is allowed, perhaps as much as 10 or 15 per cent of the pitch. Further, these chains commonly run on wheels having only a moderate number of teeth, so that cumulative pitch differences may be allowed.

With accurately made transmission chain and cut teeth, backlash is held below 5 per cent; and in the best practice—see Figs. 669 and 672—it is eliminated altogether. If the backlash is low a shortage of chain pitch must be kept very small, for it is likely to accumulate to a positive interference such as appears even in Fig. 666 when the chain (in dotted line) is wrapped around the wheel down to tooth G.

Excess length of chain pitch is not thus positively restricted. If it is due to wear, from a pitch originally correct for the wheel, the condition in Fig. 655—where the pins rest upon the bottom of the tooth spaces—might be realized by having the joints slack up beyond active teeth A and B. But as we shall presently see the pins need not and do not run thus fully "bedded," and any reasonable excess is easily accommodated without disturbance of motion.

(*g*) **Action of Pin and Tooth.**—With simple and truly correct geometrical conditions, as shown in Fig. 667 (*a*), the face profile of the tooth may conform to (or lie within) but must not project beyond the arc struck from center E of the pin in next space. In order, however, to see more clearly certain elements of possible action, we now assume and study a tooth which does not meet that specification. To be made with a fairly large backlash and used with a chain having excess pitch (type of 665), this profile has the form in figures (*b*) and (*d*), its face outline being a semicircle struck from center on pitch circle; then at the point of inflection or arc juncture a radial line R is tangent to the tooth profile.

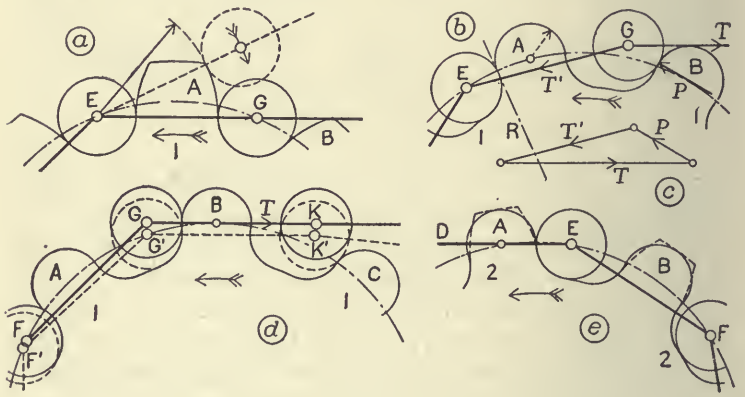

FIG. 667.—Overpitch chain with improper tooth.

View (*b*) of Fig. 667 is an instantaneous position after pin G has come into contact with tooth B, from which (assuming no friction) it receives normal pressure P: and the equilibrium of forces upon pin G is shown by triangle (*c*). As the wheel turns pressure P increases and tension T' in the advance link grows smaller, until it vanishes at the critical position in view (*d*), where tension line GK passes through profile center B, P coincides with T, and the pin is in neutral equilibrium. As the T-line swings below center B the pin will be snapped down to the bottom of its space (chain in dotted position), next pin K touching tooth C at about the same instant. In effect, with respect to direction of chain, this tooth is slightly hooked, and the action just described really lets the chain fall back a very little way.

At recess from driven wheel, sketch (*e*), this "hooking" of the tooth has no other effect than slightly to retard rise of pin and

forward creep of chain. Since this movement cannot begin until tension line DE rises above center of face arc (point A for the full-line profile), it will be a littler later than with the usual (dotted) curve. Having longer radius the latter presents a path whose lope varies less rapidly, hence is perhaps a little more favorable to smooth action. Before continuing this matter, in Art. (*i*), an action in conditions of the kind in Figs. 665 and 666 may appropriately be considered.

(*h*) **Best Condition for Conveyor Chain.**—Suppose that we have a cast chain like Fig. 658 or 659, on a wheel with cast teeth, and that the average pitches of chain and wheel are the same. With some irregularity of pitch highly probable, there is strong chance of a long link falling on a short tooth pitch; then pin will ride on tooth as in Fig. 667 (*b*), and the sudden increase of effective radius will mean a jerky acceleration of the loaded chain.

This trouble can be avoided by making the wheel a little oversize or its pitch a little overlong, with the effect of the driver in Fig. 666. Now each pin will bed quietly and surely and as has just been indicated by Fig. 667 the recess action will permit a creeping of the chain without any jerkiness. A chain thus originally short will approach pitch agreement with the driver as it wears; but if numbers of teeth and of links are properly prime to each other, so that contacts are continually changing in a big cycle, the pitches will also wear toward equality; and thus the danger of jerky action in running onto driver will tend to disappear.

Normally a conveyor will have its driving wheel at head or top; then the follower at foot is a mere guide pulley and can easily adjust itself to irregularities. If the driver is unavoidably at the foot, so that full tension must be carried around the other wheel at the head, the latter is pretty stiffly held by friction, and conditions may arise which can be met only by using a special wheel without teeth.

Because it substitutes polygon for circle no chain-and-wheel pair can give a perfectly smooth transmission of motion. With the very long links often used in conveyors, on wheels having few teeth and a consequent big variation in effective radius, the fluctuation of relative motion may become a serious matter. Instead of making the driving mechanism try to give a pulsing motion to a heavily loaded chain, it is better to produce intentionally a periodic fluctuation of the driving wheel, accom-

modated to uniform motion of the chain. This is done by means of a noncircular gear pair as outlined in Fig. 668, which shows the pitch curves and a series of intervals stepped upon them, in comparison with circles representing the average effect of the pair. One chain-pitch angle of wheel S is the motion unit, arc DE working commonly with the whole pinion circum-

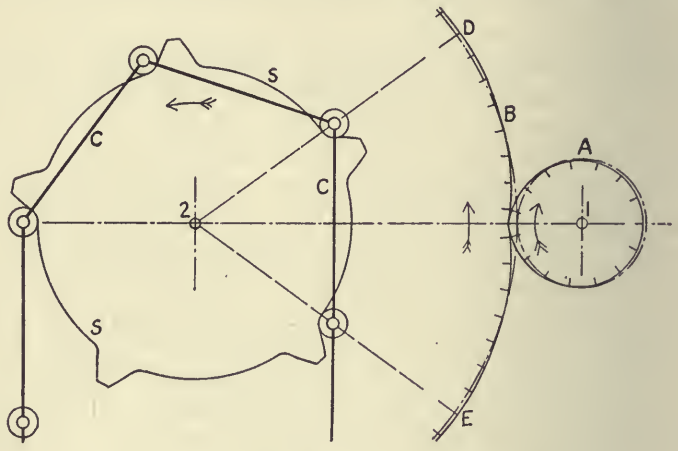

FIG. 668.—Compensating drive for conveyor sprocket.

ference as here, but sometimes with half of a two-lobe pinion. Even with the unusually small number of five sprocket teeth the departure of the gears from simple circular form is not very large. This example suggests the propriety of having an even number of pitches on the sprocket wheel, so that the two sides of the chain will have their motion cycles timed together.

(*i*) **Teeth for Transmission Chain.**—Figure 669 serves the double purpose of showing certain important general dimensions and of illustrating a particular system of sprocket wheel teeth. With pin centers A and B both on the pitch circle, link chord AB makes with tangent AT an angle equal to half the tooth angle or pitch angle θ. The upper limit of contact, at C, determines the highest common normal or pressure line of pin and tooth, in line PA (which merely happens to be tangent to roller B). Between link-tension line AL and pressure line PA is what we may call the obliquity σ for this type of gearing. The first requisite for correct and smooth working is that σ shall have a positive value, never reversing its direction as in Fig. 667 (*d*).

The Renold profile here shown is composed of a bottom arc fitting the pin without side clearance, straight sides CD having a constant angle of 60 deg. between them (the same for all sizes of wheels or numbers of teeth), and clearing arcs DE. Limit D is determined by a tangent from B, so that even with the constant angle a series of cutters is required: with profile of the

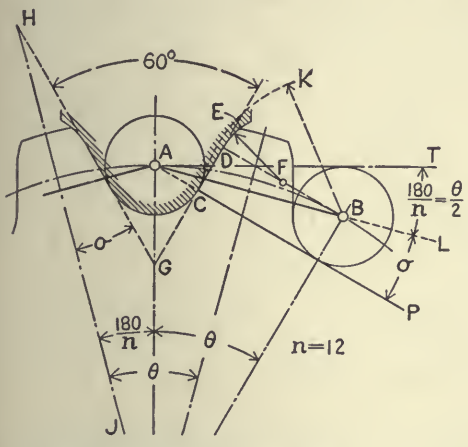

FIG. 669.—The Renold teeth.

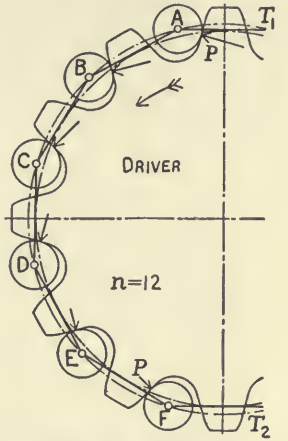

FIG. 670.—Action with long pitch.

shaded form, the number used is five, for a range from 6 to 80 teeth. From the figure it can be seen that obliquity is given by the equation

$$\sigma = 30 - \frac{180}{n} \text{ deg.} \qquad (143)$$

At the extreme case of six teeth σ will be zero and center F must go to B for the sake of clearance, the straight side disappearing: on the tooth here drawn actual profile lies well inside of the close clearing curve CK.

The 12-tooth wheel of which Fig. 669 is a detail is shown to full diameter in Fig. 670, with a chain of excess pitch length wrapped about its half-circumference. A new but exceedingly important fact now presented is that the chain automatically adjusts itself to a uniform radial distance from the primary pitch circle, or to a new and larger effective pitch circle for which its actual link-length is the correct pitch. Because contact thus tends to establish itself constantly above arc limit C, Fig. 669, obliquity σ becomes a much more definite quantity than it might be with the pin or roller fully bedded.

(*j*) **Chain Tension and Tooth Pressure.**—Evidently tension T must relax from T_1 to T_2 around the driver, build up from T_2 to T_1 around the follower; and this change is accompanied by a variation of tooth pressure which may be uniformly progressive as suggested in Fig. 670. The problem contains certain elements that might be reasoned to definite values if their group existed alone, together with others not thus rationally determinable. The first, purely geometrical constituents of the force combination have been used and shown in Fig. 667 (*c*), where conditions are determined wholly by the configuration or combined form of the bodies, or by center lines of links and surface angle

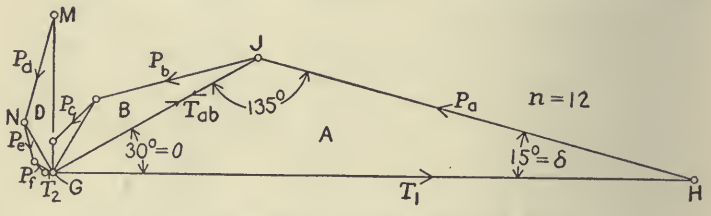

FIG. 671.—Suggested force diagram for Fig. 670.

at contact. In order to see the result from considering these forces alone, Fig. 671 is drawn for the example in Fig. 670. This starts with tension T_1 as base line GH and consists of a series of similar force triangles, GHJ for pin A, GJK for B, and so on. These grow smaller so rapidly that in the diagram tension T_{cd} has to be magnified from GL to GM; but even then the final T_2 is so small that it can hardly be seen. Working with the angles of Fig. 659 and the six contacts of Fig. 670 it can be computed that the ratio T_2/T_1 would be only 0.0024 if this analysis were true and complete.

Taking up the elements left out of account, we have first the minor influence of friction in joints and at contacts. Of far greater importance is elasticity and the indeterminateness of pressure division or apportionment among a number of contacts between two bodies: here it is chiefly a question of the cushion-like oil films in the chain system. Chains being fairly heavy bodies and even the slack side of a horizontal or inclined drive hanging in a flat catenary, gravity alone will prevent tension from running down to any such values as are reached by the reasoning of Fig. 671. And at high speed centrifugal force plays a major part, requiring a large basal or minimum tension

upon which will be imposed the differences due to transmissive action. Strength needed in a chain is therefore determined by this T_2 for gravity and centrifugal force plus the increase to T_1 for power transmission.

(*k*) **The "Diamond" Form of Tooth.**—The system of sprocket wheels developed by the Diamond Chain Company is represented by Fig. 672. One cutter, having the shaded profile, is used for all the wheels of a given pitch, so that now the tooth

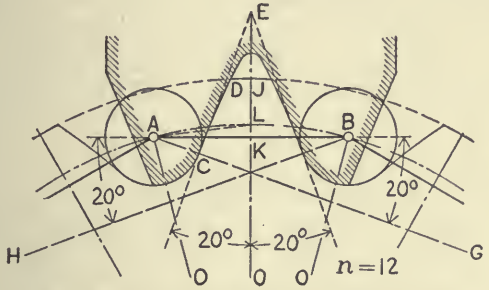

FIG. 672.—The "Diamond" form of tooth.

remains the same while the space varies in shape. As was made apparent in Fig. 669, the angle between the side of a tooth and its center line, there GHJ, is also the obliquity angle σ as we are now taking it: consequently a constant value of σ for all wheels is the outstanding characteristic of the Diamond system. This value is 20 deg., as dimensioned on Fig. 672, but the side of the tooth is not now a straight line; instead, profile CD is an arc from a center on line AG with radius $4p$. The addendum is fixed by making distance KJ $= 0.3p$: then since angle KAL $=$ 0.5 angle AOL $= 0.25\theta = 90/n$, height KL $= 0.5p \tan (90/n)$ and the addendum proper or LJ is

$$h = 0.3p - 0.5p \tan \frac{90}{n}; \qquad (144)$$

combining this with Eq. (142) we get the overall diameter, to which the wheel is to be turned, as

$$D = p\left(\operatorname{cosec} \frac{180}{n} + 0.6 - \tan \frac{90}{n}\right). \qquad (145)$$

Prob. 282. For a chain-wheel pair having the tooth numbers 13 and 75 with 2-in. pitch find diameters of pitch and addendum circles, the latter with teeth like Fig. 672.

Prob. 283. Find number of links in a chain for the preceding pair if the center distance is to be about 5 ft. Allow what you consider a reason-

able sag and get approximate lengths by assuming circular arcs and using handbook tables. Try for an even number of links, if necessary changing the distance slightly.

(*l*) **The Toothed Chain.**—This type of transmission chain is shown in relation to its wheel by Figs. 673 and 674; it consists of a number of parallel rows or layers of flat links, as seen in Figs. 675 and 677, connected by joint pins running clear through from side to side. In effect it is a flexible link belt, with the links so shaped as to form projecting teeth which have surface contact with the wheel teeth. Each link enters into two teeth, using the outer part of its profile for contact while the inner part is shaped for clearance only. A single cutter, made to the shaded profile

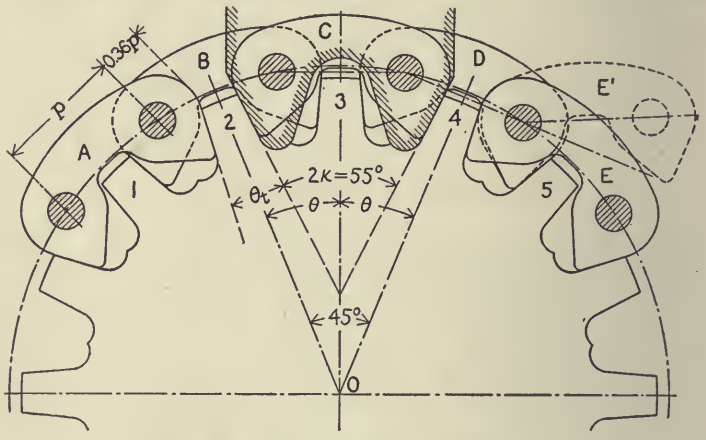

FIG. 673.—Toothed chain, Renold form.

on Fig. 673, will produce wheels of all sizes for a given chain: note that the wheel is always wholly inside of the pitch circle, its "addendum" being a minus quantity. The commerical-technical situation is that these wheels are furnished by the chain makers, who have their own rules for addendum, etc., whereas sprocket wheels are very often made by the purchasers of block and roller chains.

The overall link-tooth angle, dimensioned $2k$ on Fig. 673, is commonly either 55 or 60 deg. This angle fixes the sides of alternate wheel teeth, whose center lines are 2θ apart. Evidently the side of the tooth is at $(k - \theta)$ degrees from its center line, or the angle between sides is

$$\theta_t = 2k - 2\theta = 2k - \frac{720}{n}, \qquad (146)$$

varying in the same fashion as does σ in Fig. 669 and Eq. (143); but here the thrust angle σ (taken as in Fig. 669) is constant and equal to k, being either 27.5 or 30 deg.

With 60-deg. links the sides of the wheel tooth will be parallel when $n = 12$; with 55 deg. the similar limiting tooth number is 13.1. Partly in order to have some taper on the tooth, but largely also because of other considerations, it is specified that the smaller wheel in a drive shall never have less than 15 teeth, while 17 or 19 is the limit set for larger pitches and higher speeds.

FIG. 674.—Action of chain with pitch long.

On a 15-tooth wheel the bend of chain is 24 deg. at each pin, this being the maximum value required. By swinging back link E to E′ in Fig. 673, with the inner side of tooth point just touching wheel tooth, we see how large is the margin against an interference of the kind thus suggested.

If the chain has excess length of pitch, or the wheel is relatively small, the chain runs as shown by Fig. 674, riding higher on the teeth—compare Fig. 670. This drawing brings out clearly the fact that the driving wheel acts on the rear side of the link, while the front side of the link presses upon the tooth of the follower. As the chain rides higher a force-action condition peculiar to this type of chain is accentuated: this is that the line of total tooth pressure passes not through but below the pin center, wherefore there is a tendency to tip or rotate the link, which must be resisted by the force at the other pin. Just as for the case considered in Art. (*j*), so here there are too many uncertain elements to make a rational analysis possible: the requirements as to total tension, there set forth, will insure sufficient holding force to keep the teeth in full contact.

(*m*) **Construction of Tooth Chains.**—Figures 673 and 674 suggest a plain bearing of link on pin, but this simple design was soon found to be subject to excessive wear—the more that it is

desirable to make the links of soft steel, easily punched. Short ring bushings, each in a layer group of two or three links, have been tried, but are troublesome to make and assemble. The better scheme, seen in Fig. 675, is to use two "half" bushings in the form of strips of hard steel as long as the pins (between washers). Each link holds one bushing strip and clears the other, as seen

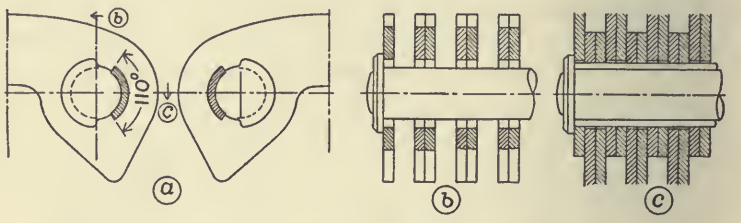

FIG. 675.—Chain joint with Dodge bushing strips.

also in longitudinal section (*c*): the clearances shown permit a bend of nearly 35 deg. in either direction from straight chain. View (*b*) is a cross section showing the pin and one set of links, view (*c*) a longitudinal section of the complete joint. Figure 676 indicates the method of producing links of needed accuracy with a minimum of machine work, by punching them with a slight excess on the working surfaces, to be milled off the tooth faces and broached from the bushing seats.

FIG. 676.—Construction of link.

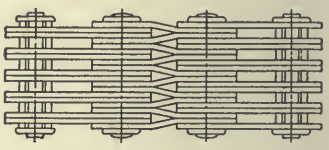

FIG. 677.—Offset coupler for odd link.

The chain will join up more easily if there is an even number of links: but an odd number can be made to work by using the special offset coupler in Fig. 677—where the end pins are shown with their ends not riveted. The bushing strips go into their seats with an easy push fit. Pins are case-hardened but have their riveting ends left soft.

An interesting and important variation from the pin joint is illustrated by Fig. 678. We first separate rocker pin A and seat pin B; these go clear through the joint, like the bushings in Fig. 676, and pin B carries riveting ends to hold the washers. The joint is assembled in view (*b*), with the slight backward bend

needed to permit sag in the top side of a drive; while (*c*) shows the bend of 24 deg. that is required for a 15-tooth wheel. Obviously there must be some holding and guiding of the pins by the clearance surfaces, introducing a small element of frictional action. The chain is intended to run always in the direction of the arrows on view (*a*).

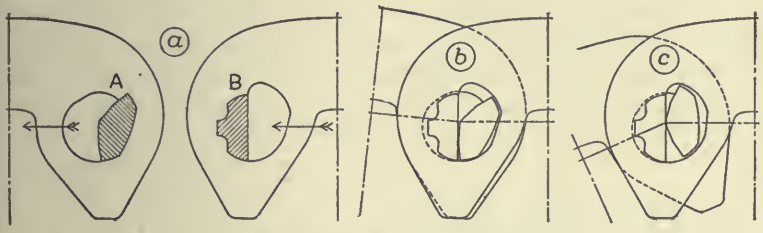

FIG. 678.—The Morse rocker joint.

(*n*) **Guiding of Chain on Wheels.**—In order to cover the whole of this subject, Fig. 679 first gives the rim section of a sprocket wheel, for link and pin (or block) chain as in Figs. 658 to 664: the straight sides in full line would be used on a forged steel pinion, the dotted profile for a cast-iron rim. Coming to the tooth chain, we have cross sections of rims in Fig. 680. With guiding by

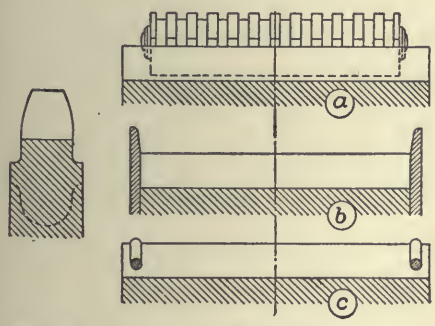

FIG. 679.

FIG. 680.
Guiding chain on wheels.

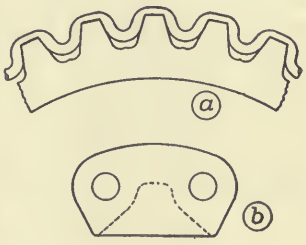

FIG. 681.

flanges one wheel, preferably the driver, is left plain and has the considerable excess width at (*a*). Using steel plate flanges as at (*b*), the guiding action comes upon the flat rivet heads; but if the driven wheel is the one flanged this action occurs at approach from the slack side, where chain tension is lightest.

A less expensive but quite effective scheme is the crimped wire flange in Figs. 680 (*c*) and 681 (*a*): it is really a hoop of stiff steel

wire, but when sprung into place and tightened by setting the crimps deeper with a blunt chisel it holds very firmly.

The other method of guiding is to use one or more rows of the blank links in Fig. 681 (*b*). Sometimes, on narrow chains, these blanks are outside, but it is more usual to run them at the middle in grooves cut through the teeth, around both wheels of course.

(*o*) **The Question of Impact.**—The tooth chain has two advantages over the pin chain: its transmitting capacity can be made much greater, by an easy extension of width, and because links are commonly shorter and lighter for the same capacity it runs

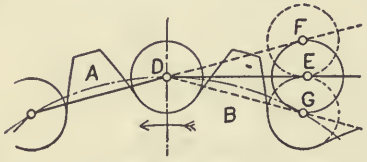

FIG. 682.

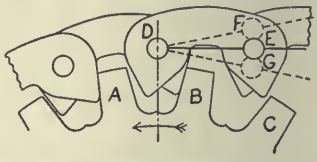

FIG. 683.

Impact of link on wheel.

more quietly or will run quietly at higher speed. Into the latter detail of action enters the question of the velocity of impact with which the link comes down upon its tooth in approach. In Figs. 682 and 683 we see link DE in the act of swinging from stretched position DF to wrapped position DG—these being taken, of course, with reference to the wheel.

Since the wheel or a line DG upon it is turning at the rate ω (radians per second) while chain line DE has no angular motion, the rate of the relative turning of link on wheel is also ω: and with a line length of l in. or $l/12$ ft. the velocity with which pin center comes to its stop position is

$$v = \omega \frac{l}{12} \text{ ft. per sec.} \tag{147}$$

Now let V be the linear velocity of chain in feet per minute and N the revolutions per minute of a wheel having n teeth. Disregarding polygon effect, the circumference of the wheel is nl or $nl/12$, whence

$$V = \frac{nl}{12}N \text{ or } \frac{l}{12} = \frac{V}{nN};$$

and with $\omega = 2\pi N/60$ we have

$$v = \frac{2\pi}{60}\frac{V}{n} = 0.1047\,\frac{V}{n}, \tag{148}$$

showing the controlling influence of tooth number and the importance of using fairly large numbers for high-speed drives.

The usual range of speed for transmission chains is from 1,000 to 1,500 ft. per min. With a 19-tooth wheel the corresponding values of v are 5.5 to 8.3 ft. per sec., indicating 6 to 8 as simple numbers to remember. . The impact is decidedly oblique, entailing considerable wedging action which tends to ease up the tension in preceding links (on the driver). To the remark in Art. (*e*) that it is the function of the chain roller to present new surface for contact the addition may now be made that the film of oil between the roller and its bearing will help to cushion the impact just described.

A practical statement of the conclusion expressed by Eq. (148) is that it is better to get a desired linear velocity by using a large wheel at lower speed (of turning) than a small wheel at higher speed. The same principle holds in regard to centrifugal force, which for a given velocity varies inversely as radius.

Prob. 284. For the chain drive of Prob. 282 find r.p.m's corresponding to a chain velocity of 1,440 ft. per min., then ω for each wheel. Get impact velocities first by Eq. (147), next directly from V by Eq. (148).

CHAPTER VIII

GEAR TEETH IN SPACE

The word space is here used in the geometrical sense, meaning that we now come to problems requiring three-dimensional or "solid" treatment. Of course spur gears are solid bodies, but with parallel axes and cylindrical surfaces all relations can be shown and all determinations made by the methods of plane geometry alone. Starting from this simplest possible case, we take up the following questions in order:

1. Form of pitch surfaces that will properly roll upon each other;

2. Layout of teeth upon pitch surfaces, or the strip sections of teeth by pitch surfaces, as in Fig. 61 (*a*) and in Figs. 692 to 696;

3. Form and generation, action and production of teeth.

General treatment of the first two questions will be found in Sect. 40. Then follows a closer study of helical teeth along the line of question (2), running into a detailed discussion of the motions and the gear trains in hobbing machines. The comparatively simple subject of bevel-gear teeth is covered by Sect. 42, as to both form and generation. The final section of this volume takes up in a general way the form and action of helical teeth, but leaves certain matters of their more intricate geometry to be considered in Part II.

SECTION 40. PITCH SURFACES AND TOOTH LAYOUTS

(*a*) **Cylinders and Cones.**—The geometrical elements essential to the transmission of rotary motion by rolling contact, for the simplest case and reduced to simplest terms, are shown by Fig. 684, in the form of two coplanar rolling circles. Appearing in side view (*b*) as mere lines GC and CH, these are conceived as indefinitely thin discs yet rigid bodies; and it is by the aggregation of such discs that actual wheels are made up. The maximum simplicity of this case consists in the fact that the two discs lie in the same plane, hence have their axes parallel; and under that condition any other disc elements, as at line G′C′H′, are exactly like the primary pair GCH.

The first and smallest departure from coplanar simplicity is made by taking common tangent TT as a hinge line and turning the planes of the two discs into the relatively oblique position GC-CH, Fig. 685 (*b*). This makes the axes intersect at O but keeps them in a common plane. Any other pair of disc elements,

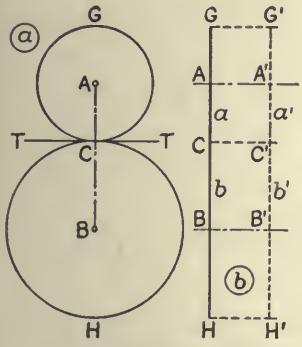

FIG. 684 — Pitch cylinders.

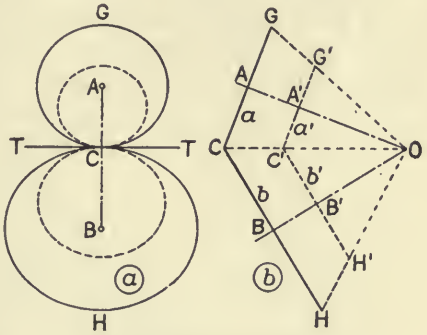

FIG. 685.—Pitch cones.

as G′C′ and C′H′, must now have their radii a' and b' in the same ratio as a and b, hence contact locus CC′ must pass through axis intersection O and the rolling "solids" are necessarily cones. The change from pitch cylinders for spur gears to pitch cones for bevel gears does not, however, infringe at all upon the property of pure rolling.

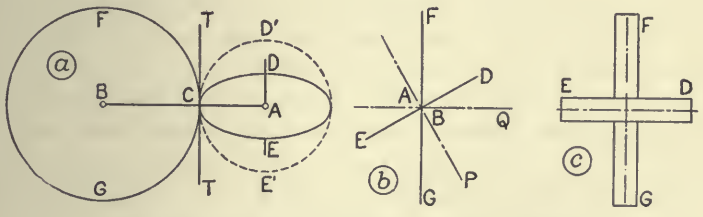

FIG. 686 — Disc planes turned on line of centers.

(*b*) **Axes not in One Plane.**—Another change from coplanar rolling, simple in the sense that it is all of one kind, is made in Fig. 686 by turning one disc plane on the line of centers ACB, with the effect most definitely shown by side or end view (*b*). The axes AP and BQ have now come to be neither parallel nor concurrent (intersecting), but lie in parallel planes which are perpendicular to the plane of view (*a*), parallel to that of view (*b*). A tangent plane, perpendicular to line AB, hence parallel to the

planes in which the axes lie, must take the place of tangent line TT of Figs. 684 and 685; and in this plane a point on the rim of one disc moves obliquely with reference to a point on the rim of the other. In other words, each circle has a combined tangential and crosswise motion with respect to the other, after a manner to be more fully considered presently. The most direct use of this arrangement is made when each disc is extended

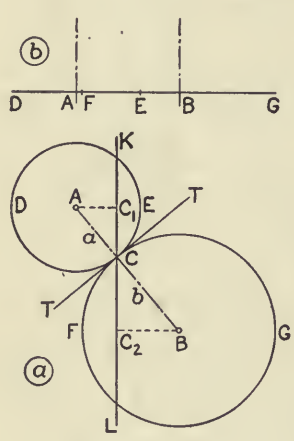

FIG. 687.

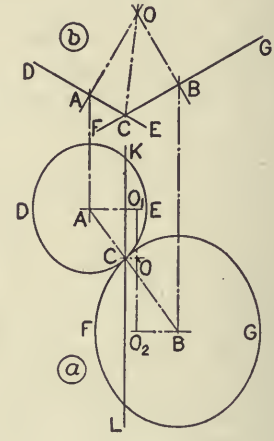

FIG. 688.

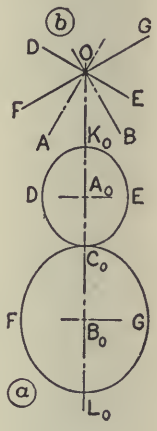

FIG. 689.

The general case of oblique rolling circles.

into a short cylinder, thus producing the fundamental form of a pair of helical or of worm gears, as outlined in Fig. 686 (*c*) for the common case of axes at right angles. Only the mid-plane disc elements of these wheels have true pitch-circle contact.

For the general case of oblique rolling circles start with the pair of coplanar discs in Fig. 687 and through pitch point C draw a line KL, somewhere (anywhere) between tangent TT and line of centers AB. On this inclined line KL as hinge axis turn the two planes into the oblique position projected in view (*b*) of Fig. 688: the discs will remain in contact at point C, view (*a*), but will roll with a combination of tangential and crosswise motion. Point O in top view (*b*), where the projections of the axes intersect, is the position of their common perpendicular, which appears as O_1O_2 in the elevation or front view. The pair of discs respectively parallel to the primary ones at this place, or with O_1O_2 as the line of intersection of the two planes, is drawn in Fig. 689 and is exactly like the combination in

Fig. 686, showing the latter to be a special case of this general type of arrangement.

(*c*) **Pitch Surfaces for Skew Bevel Gears.**—Taking the general case of axes neither parallel nor concurrent, let us now consider the surfaces evolved by aggregation of discs related as in Fig. 688. The essential requirement is that every pair of conjugate disc elements shall have their radii in the same ratio. Start in Fig. 690 (*a*) with a top view of the axes OA and OB, of which the first is the higher, the distance between their planes being shown

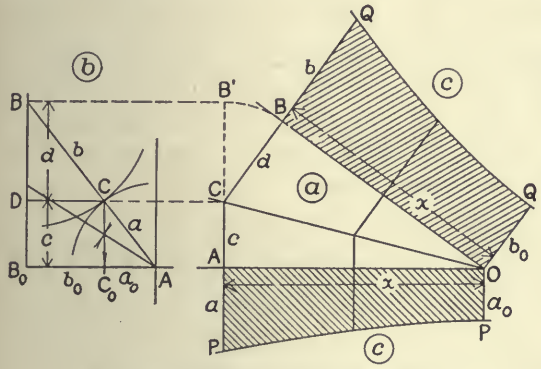

FIG. 690.—Pitch surfaces for skew bevel gears.

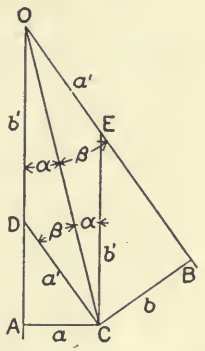

FIG. 691.—Construction for line of contact.

by the common perpendicular AB_0 as it appears in view (*b*). On this line choose a pitch point C_0 for primary discs now of the kind in Fig. 689, thus fixing a linear ratio $a_0 : b_0$ which will determine the turning ratio. In view (*a*) draw a line OC such that perpendiculars CA and CB to the projections of the axes will have their lengths c and d in the same ratio as a_0 and b_0: the method of locating this line, explained below, is illustrated by Fig. 691. Line OC, appearing as C_0C in view (*b*), lies in a plane parallel to those of the two axes.

Go now to any point C on line OC, and by swinging CB to CB′ reverse the operation through which Fig. 688 was derived from Fig. 687. Since $c : d :: a_0 : b_0$, triangles AC_0C and CDB are similar, AC and CB agree in direction, and C is the contact or pitch point of a pair of discs whose radii a and b are in the required ratio. True for any point on line OC or C_0C, this relation establishes that line as the locus of contact of two solids of revolution, on axes A_0A and B_0B (A_0 and B_0 being under O

in view (*a*)), which will roll upon each other with a similar relative motion at all points of contact.

In order to get the shape of these surfaces of revolution, use axes OA and OB as base lines and on them lay out successive lengths of radii a and b, determined as in view (*b*). Their outer ends will lie on curves PP and QQ which define half sections of the two solids. Taking OA or OB as abscissa x and a or b as ordinate y, and letting $c = mx$, $d = nx$, we get from the triangles in view (*b*),

$$y_1^2 = a^2 = a_0^2 + m^2x^2, \quad y_2^2 = b^2 = b_0^2 + n^2x^2, \qquad (149)$$

which are equations of hyperbolas referred to their principal axes OP, OA and OQ, OB. Therefore the pitch surfaces for correct rolling with axes as here related are hyperboloids of revolution. Gears having slices of these hyperboloids for their pitch forms are called skew bevel gears.

(*d*) **Location of Contact Line.**—Projected outline AOB-OC in Fig. 690 is of exactly the same general form as the similarly lettered outline in Fig. 685, which there really is in one plane. Whether then for true bevel or for skew bevel gears we have given a ratio of desired radii or a desired velocity ratio, the line of contact or its projection can be located as follows:

On axis OA, Fig. 691, lay off a length OD proportional either to the angular velocity on OA or to a particular radius from the other axis; and on OB a length OE representing to the same scale either the angular velocity on OB or a corresponding radius from OA. Construct the parallelogram ODCE, then in triangle OCD, $a' : b' :: \sin \alpha : \sin \beta$; but $a = \text{CA} = \text{OC} \sin \alpha$ and $b = \text{CB} = \text{OC} \sin \beta$, therefore $a : b :: a' : b'$ and the requirement for desired ratio is met.

(*e*) **Tooth Layout on Pitch Surfaces.**—In Figs. 692 and 693 the two wheels of a pair are represented by pitch-surface sections through their teeth, developed or laid out flat: the cone segment of a bevel gear develops as an annular strip, generally with different curvature for the two members of the pair—see Fig. 725. Here they are supposed to go together with wheel A on top of B, hence view A is taken from inside of rim, as indicated by dotted lining. The more usual case of "straight" teeth, lying squarely across the aggregation of rolling discs, is simple and obvious: but the thing now particularly to be noted is the fact, made evident by Fig. 693, that the teeth may equally well have an

inclined direction. So long as there is to be pure rolling of the pitch cylinders or cones this slanting of the teeth will not affect or conflict with that primary requirement.

When, however, a crosswise component is added to the relative motion of pitch surfaces conditions are not so simple. On helical gears, Figs. 67 and 686, choice of tooth angle ceases to be kinemat-

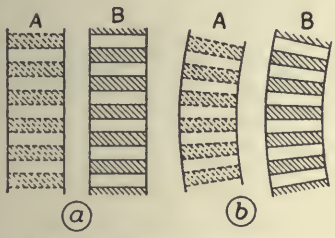

FIG. 692.—Straight teeth.

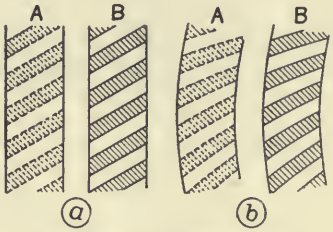

FIG. 693.—Oblique teeth.

Tooth outlines on spur and bevel gears.

ically a matter of indifference, because that angle becomes a coordinate element in determining the motion transmitted, which no longer depends upon diameters alone. From another viewpoint choice of angle becomes freer, since there is no attempt at a true surface rolling even in one direction and inclination of teeth is now an independent variable. Fundamental relations for this type of gearing are stated in the next article, introducing a fuller discussion in the following section.

On skew bevel gears true and invariable rolling is combined with cross sliding, in a way that will be more definitely shown by Fig. 698. This means that successive straight-line elements of the two pitch hyperboloids must come together just as do successive elements of cylinders or cones: now, however, the transition from one element to the next is not squarely across but oblique. In order not to conflict with the component of sliding along element lines the tooth strips must be bounded by those lines, hence may have only the single direction that corresponds with the cases in Fig. 692.

(*f*) **Velocity Relations on Helical Gears.**—The effect of variation in tooth angle is shown by the comparative examples in Figs. 694 to 696, using segments of the same pair of pitch cylinders or assuming unchanged diameters for wheels 1 and 2. In the velocity diagrams the initial fact is that each wheel has its own circumferential velocity $OA = v_1$ or $OB = v_2$. These are

resolved into components v_n normal to tooth and v_t along tooth. Necessarily v_n is common to the two wheels, therefore v_1 and v_2 are related by the right triangle AOB, with direction of OC and AB (or DE) determined by inclination of tooth. Obviously the ratio between v_1 and v_2 is the same as that between the cir-

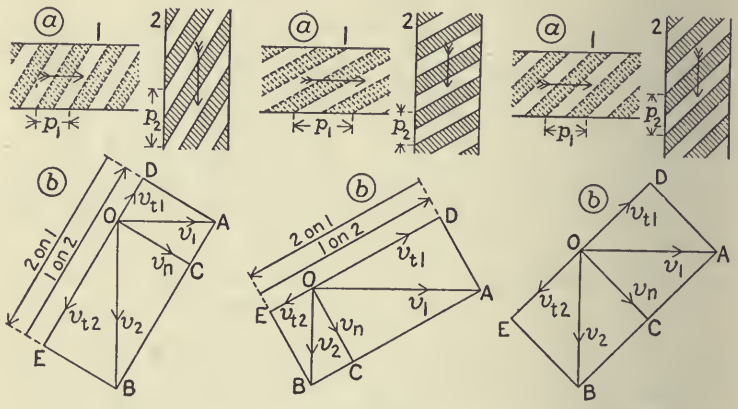

FIG. 694. FIG. 695. FIG. 696.

Helical gears with different tooth angles.

cumferential pitches p_1 and p_2, as these are indicated by dimensioning. Only when the tooth line bisects the angle between wheel planes, here in the 45-deg. case of Fig. 696, are these

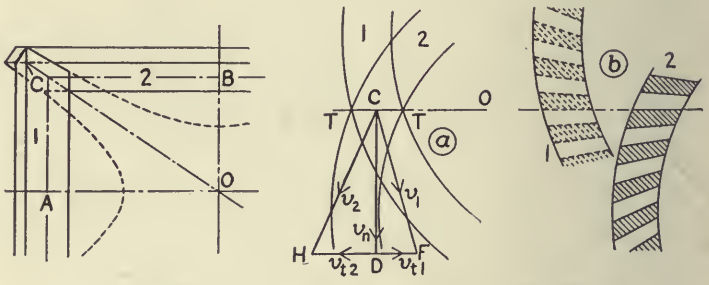

FIG. 697. FIG. 698.

Relations in skew bevel gearing.

pitches the same and the diameters completely determinative of turning ratio. Figure 694 shows the condition for more rapid drive of wheel 2, Fig. 695 that for slower drive.

(*g*) **Tooth Layout on Skew Bevel Gears.**—Figure 697 shows the pitch-surface form of a pair of gears, No. 1 being above No. 2

as in Fig. 690. Although generated by a straight-line element the hyperboloid of revolution is a twisted surface, hence cannot be developed; but for a narrow strip or segment very close approximation may be secured by using the cones that most nearly conform to the true surfaces. In Fig. 698 the two pitch strips from Fig. 697 are developed on this principle, at first together as related by contact line TT, then separated and with tooth strips shown in their uniquely possible direction. On diagram (*a*) the total velocities v_1 and v_2 are laid out and resolved

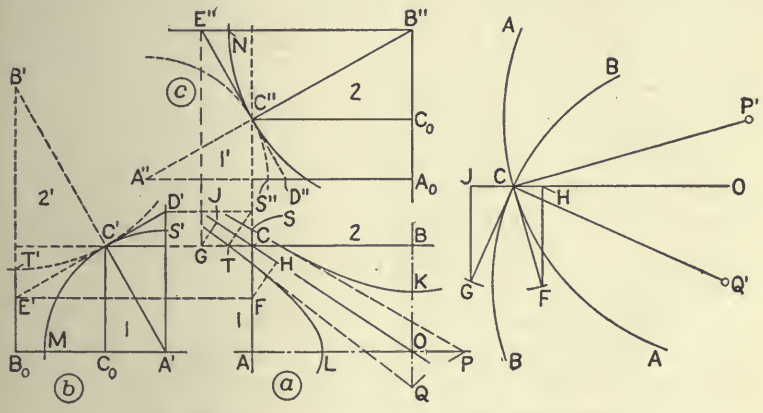

FIG. 699. FIG. 700.

Relations in skew bevel gearing.

into a common v_n and the remaining v_{t1} and v_{t2}: the sum of the latter is the relative velocity of transverse tooth sliding.

(*h*) **Geometry of the Skew Bevel Pair.**—The geometrical evolution of Fig. 698 is shown by Figs. 699 and 700. View (*a*) of the first is a plan projection like Fig. 690 (*a*), for the pair with axes at right angles: curve SK is a longitudinal section of the hyperboloid on axis OA, curve TL is similarly related to axis OB: note that O here stands for different points on a line perpendicular to plane of drawing. We consider a single pair of rolling discs, in planes CA and CB, and assume a cone to be tangent to each hyperboloid around the rim of the respective disc. These cones have their apexes at P on axis AO and at Q on axis BO, located by drawing tangents to curves SK and TL. Elements PS and QT are shown in true length and furnish radii P′C and Q′C for Fig. 700. The two cones have with each other the same kind of contact as that between the cylinders of a helical pair,

Fig. 686 (*c*), touching only at point C. Through this point passes also the tangency line OC of the hyperboloids; and the problem in solid geometry is to find the angle which each disc rim makes with OC at point C.

In Fig. 699 elevation view (*b*) is perpendicular to axis OA and (*c*) to axis OB. Partly for fuller perception, partly for utility, these two diagrams are completed to the form of Fig. 687, by first turning CB into the plane of AC, then CA into the plane of BC. Tangents C′D′ or C″D″ to disc 1 and C′E′ or C″E″ to disc 2 here show in their true lengths. Now project E′ to F and E″ to G, getting CF and CG as projections of C′E′ and C″E″, then through points F and G pass planes perpendicular to line OC. The resulting offsets CH and CJ are transferred to the same line CO in Fig. 700 and the true lengths of CF and CG are struck off equal to C′E′ or C″E″, determining in one plane the inclination of CF (for disc 1) and of CG (for disc 2) to line CO. Radials from C, normal respectively to CF and CG, give lines on which centers P′ and Q′ must lie, and determine the basal outline of Fig. 698 (*a*).

For disc 1 it would be more direct to project from D′ rather than E′ of view (*b*), as indicated by a dotted-line construction. The equivalent determination from E′ is used in order to avoid overlapping on a crowded figure.

(*i*) **Status of Skew Bevel Discussion.**—The instantaneous condition represented by the combination of directions CF and CG with line CO in Fig. 700, or by the velocity diagram in Fig. 698 (*a*), is truly correct only for the action of the disc pair from which it is derived. The inaccuracy incurred when these discs are extended into wheels may be described in two ways, as follows:

1. If the wheels are true segments of hyperboloids the surface that will roll between them, carrying successive positions of tangent line CO, is not a plane but a helicoid; this fact is developed and illustrated in Sect. 42 (*h*).

2. If cone segments are substituted for hyperboloids there is first the approximation that these do not really roll upon each other, then the further fact that the plane of Fig. 700 does not roll upon either cone with line contact.

Some further consideration of the skew-bevel pair will be found in Sect. 42 (*h*), in connection with the subject of tooth generation. The productional difficulties that have prevented its actual use will there appear very clearly. Even though these have deprived

the pair of practical importance it is worthy of full understanding as the most general case of gearing for the transmission of uniform motion.

SECTION 41. LAYOUT AND PRODUCTION OF HELICAL TEETH

(*a*) **The Question of Tooth Angle.**—Because of its extensive use in high-grade spur and worm gearing, the helical tooth has a practical importance which calls for and justifies a close study of its detail. Such a study falls into two main divisions, concerned with tooth inclination (as measured on the pitch cylinder) and with tooth form and action. The first, which we shall take

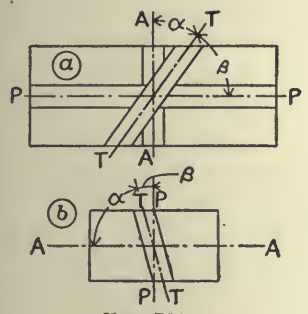

Fig. 701.

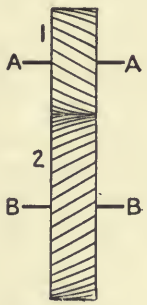

Fig. 702.

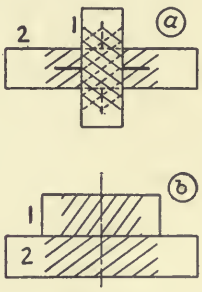

Fig. 703.

Angles and "hand" of helical teeth.

up here, proceeds geometrically from the introduction in Arts. (*e*) and (*f*) of the last section, mechanically from the beginnings in Sect. 25 (*f*). Discussion of the second question is begun in Sect. 43, then its more intricate aspects are deferred to Part II.

On the pitch cylinder in Fig. 701 (*a*) are drawn three strips or bands representing an axial or straight tooth AA, a circular or collar tooth PP and an inclined or helical tooth TT. The inclination of the last can be defined by giving either angle α or angle β as indicated. With helical gears, to be used in the pairs outlined in Figs. 702 and 703, it is logical to think of departure from the spur-gear form as the outstanding fact and to use angle α. The worm, Fig. 701 (*b*), is so much like a common screw in proportions that complementary angle β, used in stating the slope of a screw thread, may be preferable. As regards direction of helix, note that the wheel is right-hand if on a front or external view there is clockwise measurement of α from AA toward TT, and vice versa.

Figures 702 and 703 are intended to illustrate certain angular properties of helical-tooth gearing. In a slant-tooth spur pair, or with axes parallel, the two gears have the same tooth angle α but are of opposite hand, one right and the other left. In the helical pair, axes crossing, the wheels are of like hand and the angles are either complementary (for axes at 90 deg.) or are together equal to the angle between projections of axes. The latter statement does not cover every possible case or combination of angles—see relation of hob to gear in Art. (*e*)—but is adequate for practical purposes.

(*b*) **Tooth Angle and Lead of Helix.**—The process of generating a helix on the milling machine, as described in Sect. 25 (*f*), may be used to produce helical teeth by means of the formed cutter; and into the more usual hobbing process the combined rotation and axial advance there defined enters as one component of the total motion required. In any case, the angle of the helix is likely to be the primary given quantity rather than the lead, so that it is necessary to understand clearly certain relations that are illustrated by Fig. 704. This shows two developments of the cylindrical surface in which lies the helix: the first (full lines, rectangle EF) is slit at the back and rolled out to a front tangent plane, giving the true slant of the curve as seen in the front view or from outside; the second (dotted lines, rectangle GH) is slit at the front and rolled out to a back tangent plane, giving the slope as it must be considered with respect to the fitting of a gear to another behind it or of a hob to the gear that it is to cut. Using angle α with diameter d and lead l, we first state a general relation then substitute in Eq. (97) for the common mechanism on milling machine, getting,

$$\frac{\pi d}{l} = \tan \alpha; \quad l = \frac{\pi d}{\tan \alpha}; \quad t = \frac{10 \tan \alpha}{\pi d}; \quad (150\ a, b, c)$$

t being the value of the change-gear train in the direction d/f.

Evidently the incommensurable factors $\tan \alpha$ and π are likely to lead to values of gear-train ratio t that will be difficult to realize exactly. Relations are further complicated by the fact that diameter d is itself a function of pitch and number of teeth together with angle α. The formulas just stated are commonly applied to the pitch cylinder of a gear, where α will have its nominal or rated value. Lead being a fixed quantity in any case, the angle varies with diameter as most directly shown by Eq. (150*b*).

(*c*) **Relations for Helical Teeth.**—In Fig. 705 (*a*) strip ABA, one of a number of equal inclined bands unwrapped from the pitch cylinder, represents the "space" occupied by a complete

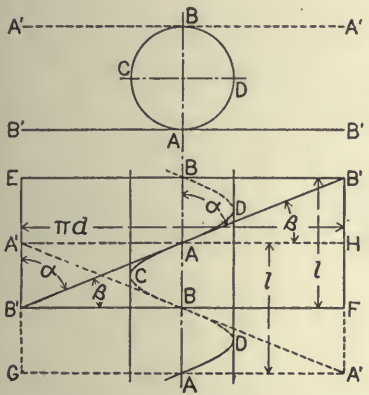

FIG. 704.

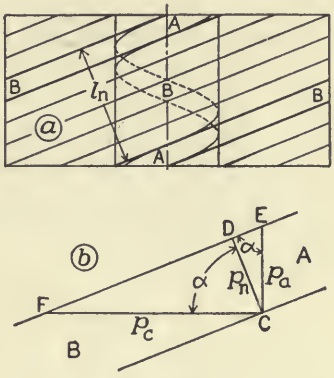

FIG. 705.

Angle, lead and pitch relations.

helical tooth coil. Considering the enlarged detail at (*b*) we see three important dimensions of this helical strip, namely,

Normal pitch or true width, CD = p_n;

Axial pitch or common screw pitch, CE = p_a;

Circular pitch, in a section perpendicular to axis, CE = p_c.

Let n (necessarily an integer) be the number of strips or coils completely covering the cylinder, so that circumference $\pi d = np_c$ and lead $l = np_a$; then from the figure $p_c = p_n/\cos\alpha$ and $p_a = p_n/\sin\alpha$, whence,

$$\pi d = \frac{np_n}{\cos\alpha}, \qquad l = \frac{np_n}{\sin\alpha}. \tag{151 a, b}$$

The quantity np_n, a normal helical length dimensioned l_n on the development in Fig. 705 (*a*), may be called the normal lead of the helix, in contrast with axial lead l.

To generalize Eqs. (97) and (150*c*) substitute m for 40 as teeth on worm wheel and s for 4 as threads per inch on feed screw—these in a mechanism like Fig. 341—and get

$$tl = \frac{m}{s}, \qquad t = \frac{m}{s}\frac{\sin\alpha}{np_n}. \tag{152 a, b}$$

Since all standard gear work is done on the diametral-pitch system and since the determining dimensions of the spur tooth

come over into the normal section of the helical tooth, we substitute π/p_d for p_n and have three relations as follows:

$$d = \frac{n}{p_d} \sec \alpha, \; l = \frac{n}{p_d} \frac{\pi}{\sin \alpha}, \; t = \frac{m}{s} \frac{p_d}{n} \frac{\sin \alpha}{\pi}. \qquad (153\ a, b, c)$$

Of course p_d has no longer its definition meaning of *diameter/number of teeth;* but there does appear a quotient $n/p_d = d_0$, smaller than actual d, which we might call the diameter of the underlying spur gear. This is not to be confused with the larger diameter for tooth profile, Sect. 43 (*e*).

(*d*) **Determining the Change-gear Train.**—For the mechanism of Fig. 341, with $m/s = 10$ as in Eq. (97), take a double numerical example as follows:

Data		(*a*)	(*b*)
Number of teeth	n	10	50
Angle of helix with axis	α	60°	45°
Normal diametral pitch	p_d	5	5
Results			
Normal linear pitch	p_n	0.6283	0.6283
Pitch diameter by Eq. (153*a*)	d	4″	14.142″
Helical lead by Eq. (153*b*)	l	7.255″	44.43″
Value of change train by Eq. (153*c*)	t	1.3785	0.2251

For shop use the obvious and customary thing is a full instruction table, giving all the leads that can be cut with the change gears supplied: then it is simply a matter of using the nearest available gear setting, and the amount of unavoidable error is at once apparent. Lacking such a ready-made solution, the first scheme that suggests itself is an extension of the factoring method used for the screw-cutting train, with room for many ingenious expedients. Thus for $t = 1.3873$ we may write

$$t = \frac{13{,}783}{10{,}000} = \frac{77}{50} \frac{179}{200} \qquad (a1)$$

exactly, but with the last pair of numbers too big. As a close approximation,

$$t = \frac{1{,}378}{1{,}000} = \frac{26}{20} \frac{53}{50} = \frac{52}{40} \frac{53}{50}, \qquad (a2)$$

with an error of about 1 in 4,600. Dropping to

$$t = \frac{1{,}376}{1{,}000} = \frac{32}{20} \frac{43}{50} = \frac{64}{40} \frac{86}{100}. \qquad (a3)$$

we get numbers that are found among the regular change wheels in Table 5, but the error is now 1 in 600.

Again, try our second requirement $t = 0.2251$; writing

$$t = \frac{2{,}250}{10{,}000} = \frac{25}{100}\frac{90}{100} = \frac{25}{50}\frac{45}{100}, \qquad (b1)$$

we have a close approximation. If when dropping 1 from 2,251 we deduct also the proportional amount 4 from 10,000, there results the almost exact train,

$$t = \frac{2{,}250}{9{,}996} = \frac{25}{98}\frac{90}{102} = \frac{25}{51}\frac{45}{98}. \qquad (b2)$$

Of course, if we have numbers so highly composite as 225 and 1,000 the ratio can be reduced to very simple lowest terms and available wheels selected as is commonly done for screw threads, thus,

$$t = \frac{225}{1{,}000} = \frac{9}{40} = \frac{3}{5}\frac{3}{8} = \frac{24}{40}\frac{24}{64}. \qquad (b3)$$

Prob. 285. By the methods just developed find change gears for milling one of the following pairs of helical gears with the index head, primarily getting diameter and lead:

	(*a*)		(*b*)		(*c*)		(*d*)		(*e*)	
Teeth n ...	12	66	15	68	18	60	20	45	24	57
Pitch p_d....	10	10	8	8	9	9	10	10	12	12
Angle α....	60°	30°	65°	25°	60°	30°	55°	35°	45°	45°

(*e*) **Use of the Hob.**—The simplest case of hob action occurs in the operations of cutting a worm wheel or a straight spur gear, which differ only in the manner of feeding tool to work, as described in Sect. 35 (*c*). The essential gear train is outlined in Fig. 706, with hob H on gear G and worm W on master wheel M. Relations are the same as for the indexing mechanism of the gear cutter, Fig. 339, with one turn of the hob corresponding to the one or more definite turns of drive shaft 1; but here the motion is continuous during cut, there it is intermittent between cuts. Including the 2 to 1 ratio of the bevel pair F-E, the equation of relation is,

$$t = \frac{d}{f} \text{ or } \frac{d_1}{f_1}\frac{d_2}{f_2} = \frac{m}{2n}. \qquad (154)$$

For necessary flexibility of the machine the gear train must include several transmissive elements that do not appear in Fig. 706. Thus in order that the hob may be inclined so as to cut gear teeth it must be driven by the train in Fig. 707 or an

equivalent. If the cutter slide has vertical movement shaft 1 is vertical with bearings in the frame, shaft 2 turns in the base piece or slide saddle, the rest of the train is in the front piece which swivels on the saddle at axis 2.

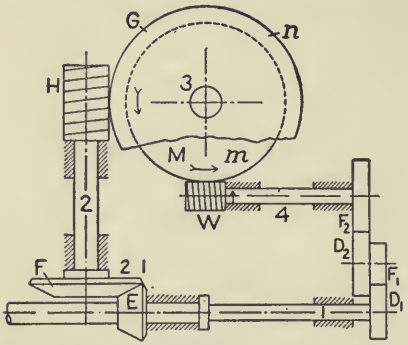

FIG. 706.—Hobbing train for spur gear or worm wheel.

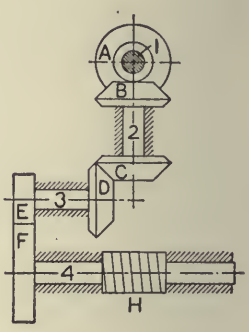

FIG. 707.—Flexible drive for hob.

Amount of hob inclination, or the angle which its axis must make with plane of wheel, is illustrated by Figs. 708 to 710. For the straight-tooth spur gear the angle is simply β_1 of the hob. Figures 709 and 710 show how a right-hand and a left-hand

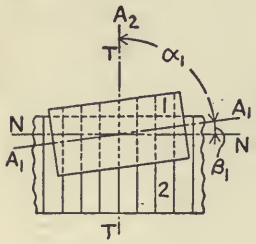

FIG. 708.

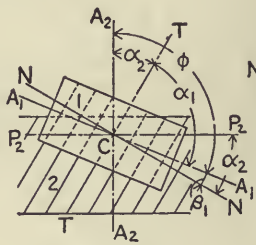

FIG. 709.

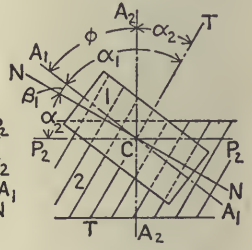

FIG. 710.

Angular setting of hob.

hob 1 will be set in order to cut a right-hand wheel 2: it is apparent that the inclination is $(\alpha_2 - \beta_1)$ when hob is of like hand with wheel, $(\alpha_2 + \beta_1)$ when they are unlike. Angle φ between axes is $(\alpha_1 + \alpha_2)$ in the first case, $(\alpha_1 - \alpha_2)$ in the second.

(*f*) **Hobbing Helical Teeth.**—If the hob is cutting a worm gear or a spur gear as in Fig. 708, or if in cutting a helical gear the feed motion is stopped and hob simply works with gear at a constant

level in Fig. 709, then the rapid relative motion due to the train of Fig. 706 is all that is required: we shall call this the principal indexing motion or component. But in order that the hob may be fed across the wheel face in an inclined direction there must be added a slow secondary motion of the kind produced in Fig. 341 and just now discussed in Art. (*d*). This requirement is illustrated by the enlarged detail section in Fig. 711, where a short portion HH of the hob tooth or thread is drawn as though it filled the tooth groove GG on the wheel.

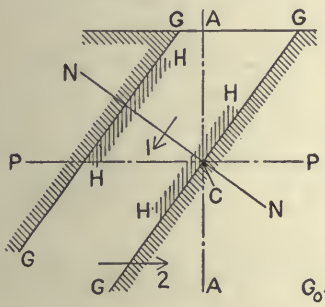

FIG. 711.

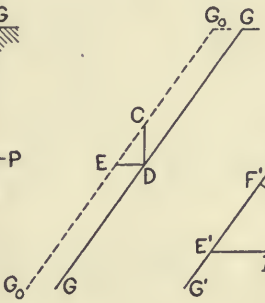

FIG. 712.

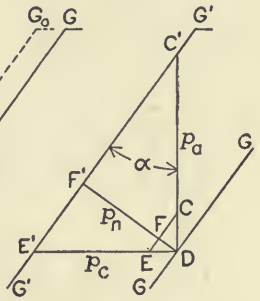

FIG. 713.

Feeding motion of hob and its components.

Motion arrow No. 1 shows the cutting advance of hob in gear blank; and since the feed motion of the machine is vertical, along line AA, the gear must move in the direction of arrow No. 2.

Just what is necessary at a place of contact like C of Fig. 711 is more clearly shown by Fig. 712, where G_0G_0 represents an initial position of tooth edge GG. During one turn of the gear blank the hob is fed downward a short distance CD, and at the end of this period one or the other of two things must be true, namely,

(*a*) With exactly n turns of the hob the gear tooth will have gone a little beyond G_0G_0, to GG; or

(*b*) With one exact turn of the wheel the hob will have made a little less than n full turns.

In Fig. 713 let GG and G′G′ be center lines or like edge lines of successive teeth, with the pitches designated as defined in Art. (*c*). Also let c be the quotient of vertical feed CD into axial pitch C′D or of circular advance ED into circular pitch E′D: this makes c the number of cuts per pitch, and the excess wheel motion per turn or cut is $1/c$ of a pitch or hob turn, which itself

is $1/n$ of a wheel turn in helical motion. Then the two conditions of working just stated will be,

$$(a)\ \left(1 + \frac{1}{cn}\right) \text{ turns of wheel to } n \text{ turns of hob;} \qquad (155)$$

$$(b)\ 1 \text{ turn of wheel to } \left(n - \frac{1}{c'}\right) \text{ turns of hob.} \qquad (156)$$

Which shall be taken as unit of motion, whether n turns of hob or 1 turn of wheel, will depend upon arrangement of machine, whether feed gearing is driven from hob arbor or from work spindle. By equating the turning ratios for cases (a) and (b) it can be shown that $c' - c = 1/n$.

Number of cuts c per tooth unit depends upon the allowable feed in direction CE, which ranges from about 0.01 in. for tough steel to as much as 0.15 in. for rough cutting in cast iron. Beside this true or tangential feed CE $= f_t$ we have the axial feed CD $= f_a$, which is what the gear train has to produce: the relation is

$$f_a = f_t \cos \alpha. \qquad (157)$$

The most direct measure of tooth unit is the normal pitch p_n; then for c we have,

$$c = \frac{DF'}{DF} = \frac{p_n}{f_a \sin \alpha} = \frac{p_n}{f_t \sin \alpha \cos \alpha} = \frac{2p_n}{f_t \sin 2\alpha}. \qquad (158)$$

(g) **Hobbing Train with Differential.**—Many machines for hobbing helical gears operate on the principle embodied in Fig. 714, where the indexing and helical motions are mechanically separate. Shaft 1 receives driving power and drives hob H at its own speed. The first change gearing C_1 is a plain indexing train (as in Fig. 706) and for a given tooth number n it would be set the same for either spur or helical teeth; but since shaft 4 drives the worm through a jack-in-the-box differential B there is a factor 2 that must enter into calculations. Rate of drive for the whole feed-and-helical train (or rate of feed) is fixed by the second change gearing C_2. Finally through change train C_3 and the worm-and-bevel member 9 the necessary additional motion is given to index worm W. The two motion components are separated to the extent that if clutch K is thrown out, stopping the whole feed train, the hob and gear will turn together without interference; while with the machine stopped the feed train can be turned by hand so as to move the hob freely in tooth grooves already cut.

In this mechanism the unit of motion is very decidedly n turns of the hob, Case (a) of the last article: then for the index worm, with $m = 120$,

$$n \times t_1 \times 2 = m, \quad nt_1 = 60, \quad t_1 = \frac{60}{n}, \tag{159}$$

just like Eq. (154). For the feed-and-helical train assume that the worm-and-wheel pairs 6-12 and 8-9 have the same ratio, so

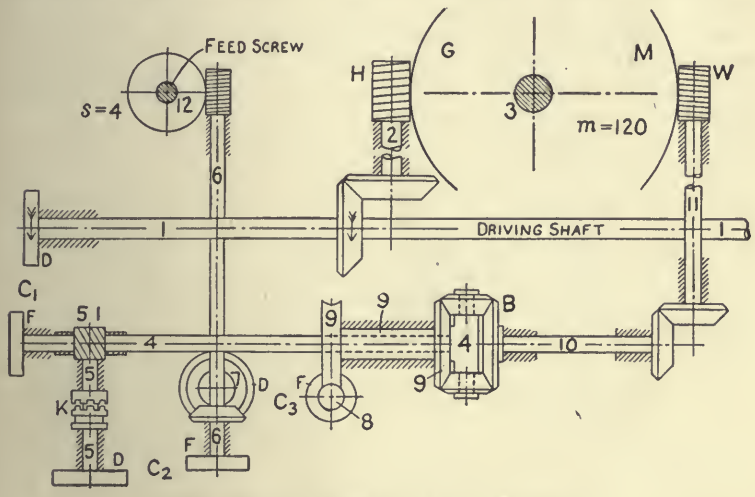

FIG. 714.—Hobbing train with differential.

that t_3 is the turning ratio from feed screw 12 to member 9 and to any extra motion that 9 gives 10; then if screw 12 turns n_f the turning of piece 9 will be $t_3 n_f$. Now from Fig. 713 with 705,

$$\frac{\text{CD}}{\text{ED}} = \frac{l}{\pi d}; \quad \text{CD} = f_a = \frac{n_f}{s}, \quad \text{ED} = \frac{\pi d}{cn} = \pi d \frac{t_3 n_f}{m};$$

the second value for ED expressing the fact that the extra turn-fraction $1/cn$ for master wheel M and gear blank G, required according to Eq. (155), is due to the extra motion from n_f of feed screw 12 though the differential train of value t_3. Then with $s = 4$ threads per inch on the feed screw and $m = 120$,

$$l = \pi d \frac{\text{CD}}{\text{ED}} = \frac{1}{t_3} \frac{m}{s} \text{ or } lt_3 = 30, \tag{160}$$

a formula similar to Eq. (97).

Example 15. For Fig. 714 get value of t_3 and select change gears to hob a wheel with 96 teeth, $p_d = 8$, $\alpha = 45°$.

From Eq. (153),

$$d = \frac{96}{8 \times 0.70711} = 16.970'', \; l = \frac{96 \times \pi}{8 \times 0.70711} = 53.31'',$$

then

$$t_3 = \frac{30}{53.31}; \; \frac{3{,}000}{5{,}328} = \frac{60\,50}{74\,72};$$

the error being 3 in 5,331 or about 1 in 1,800.

This determination may be checked as follows: Let screw 12 make just one turn, giving a feed movement of 0.25 in. for f_a or CD in Fig. 713. Then worm shaft 10 turns 3,000 ÷ 5,328 = 0.56306 and wheel M turns 0.56306 ÷ 120 = 0.0046922. The gear circumference is here the same as lead l, since α is 45 deg.; then 53.31 × 0.0046922 = 0.25014 in. for the exact linear advance of the pitch-cylinder surface.

Prob. 286. Starting in Fig. 714 with the indicated direction of turning for shaft 1, mark directions all over the train and answer the following questions: What shall be the "hand" (right or left) of feed screw 12 and of worm 9? What must be done to the train in order to cut a left-hand helix with a left-hand hob? In this mechanism and in Figs. 712 and 713, will any change in arrangement and in relations be needed for cutting a right-hand helix with a left-hand hob?

Prob. 287. Determine diameter, lead and all three change trains for one of the following helical gears, to be cut with the mechanism of Fig. 714:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
Teeth n	70	62	66	84	16	87	24	72
Pitch p_d	5	5	6	6	4	4	6	6
Angle α	40	50	36	54	60	30	55	35
Hand	right	right	left	left	right	right	left	left
Feed f_t	0.08	0.08	0.06	0.06	0.05	0.05	0.06	0.06

To make t_2 definite we must fix proportions for worm and wheel 6–12: make the ratio such that with t_2 equal to 1 feed f_a will be 0.5 in.

(*h*) **Direct Hobbing Train.**—We have seen that in hobbing helical teeth there must be an overindexing movement of gear relative to hob (with gear and hob of opposite hand, an underindexing). This can be put into the direct train instead of being added by means of a differential, and a suitable mechanism for so doing is laid out in Fig. 715. Change gearing C_1, which determines the indexing motion or the relative motion of hob and worm, is so selected as to give about the amount of extra advance needed for a desired rate of feed; then feed-drive train C_2 is carefully adjusted to the exact helical motion required. Since the feed train 7-8-9 is driven from work shaft 4, the obvious unit motion is one turn of this shaft, case (*b*) of Art. (*f*).

Using the ratio stated in Eq. (156), but with plain c in place of c', and working backward from worm shaft 6 to hob 3, we have,

$$\frac{m}{2t_1} = n - \frac{1}{c}, \; t_1 = \frac{m}{2}\frac{c}{cn-1} = \frac{c \times 60}{cn-1} = \frac{60}{n-1/c}, \quad (161)$$

as against $t_1 = 60/n$ for pure indexing. In the feed train, including the 1 to 2 bevel-gear pair to feed screw 9, one turn of shaft 7 moves the cutter slide

$$f_a = 1 \times \frac{t_2}{2} \times \frac{1}{s}, \text{ and } f_a = \frac{p_a}{c} = \frac{p_n}{c \sin \alpha};$$

therefore, with $s = 4$,

$$t_2 = \frac{8p_n}{c \sin \alpha} = \frac{8\pi}{cp_d \sin \alpha} = \frac{25.1327}{cp_d \sin \alpha}. \quad (162)$$

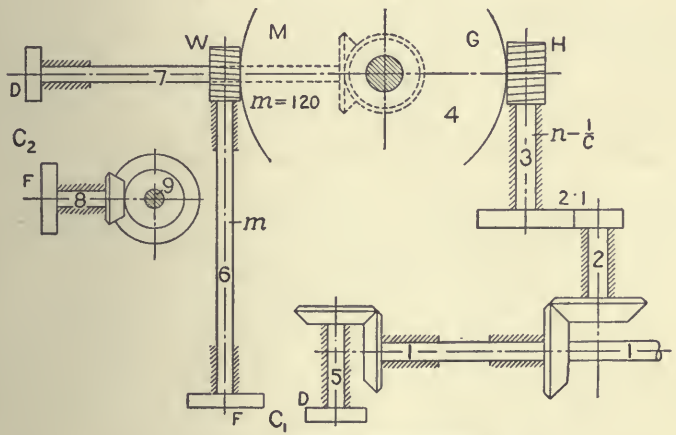

FIG. 715.—Direct hobbing train.

The methods to be used in applying Eqs. (161 and 162), with the difficulties that will be encountered, can best be seen from some definite examples, considering separately the two steps of the process.

(*i*) **Selection of First Change Train.**—Take three examples as follows, assuming data and first computing certain primary quantities:

Data		(*a*)	(*b*)	(*c*)
Number of teeth	n	71	25	96
Nominal diametral pitch	p_d	6	5	8
Angle of helix with axis	α	45°	60°	30°
Tangential feed per revolution	f_t	0.05″	0.055″	0.035″

Preliminary Computations

Pitch diameter of gear, Eq. (153)	d	16.735″	10″	13.856″
Normal linear pitch	p_n	0.5236″	0.6283″	0.3927″
Axial pitch, $p_n/\sin\alpha =$	p_a	0.7405″	0.7255″	0.7854″
Axial feed, $f_t \cos\alpha =$	f_a	0.035″	0.0275″	0.030″
Cuts per tooth unit	c	21	26	26

Having thus determined the value of c that corresponds with the desired feed—which need not be exactly realized, since the feed is not very definitely prescribed by conditions—the next step is to satisfy Eq. (161). For case (a), taking first $n = 71$ with $c = 21$ and using plain t because there is no present need of distinction,

$$t = \frac{21 \times 60}{1{,}491 - 1} = \frac{21 \times 60}{1{,}490} = \frac{21 \times 60}{10 \times 149}, \qquad (a1)$$

with too high a prime number in 149. Try the next lowest number 20 for c, getting the usable gears,

$$t = \frac{20 \times 60}{1{,}419} = \frac{20\,60}{33\,43} = \frac{40\,60}{43\,66}; \qquad (a2)$$

the second combination of factors gives a better balance of gear sizes.

For example (b), with $n = 25$ and indicated $c = 26$,

$$\frac{26 \times 60}{674} = \frac{26 \times 60}{11 \times 59} = \frac{52\,120}{44\,59} \qquad (b1);$$

$$\frac{25 \times 60}{624} = \frac{25\,60}{16\,39} \quad (b2); \qquad \frac{27 \times 60}{674} = \frac{27\,60}{2\,337} \qquad (b3);$$

The procedure is, try c, $c - 1$, $c + 1$, $c - 2$, etc., until a workable combination is secured. Evidently denominator $(cn - 1)$ is the critical quantity; and for example (c), $n = 96$, $c = 26$, we condense the trials to

$c = 26$, $cn - 1 = 2{,}495 = 5 \times 499$;
25, $2{,}399 =$ prime;
27, $2{,}591 =$ prime;
24, $2{,}303 = 47 \times 49$;
28, $2{,}687 =$ prime.

Out of five possible integral numbers $c = 24$ is the only one that can be used.

It is not essential that c be an integer. Thus for $n = 75$ and indicated $c = 8$, trials with 8, 7, 9, 6 and 10 all fail; but the number 8.4 which makes both $c \times 60$ and cn integral gives,

$$t = \frac{8.4 \times 60}{630 - 1} = \frac{18 \times 28}{17 \times 37} = \frac{54\,56}{51\,74}. \qquad (163)$$

Thus to make t_1 conform exactly to a simply expressed value of factor c is a matter of computational convenience rather than necessity. Having chosen gears for train C_1 that will give nearly the c wanted, we can get the resulting decimal value to as many places as needed from Eq. (161) put into the form,

$$c = \frac{t_1}{nt_1 - 60}, \tag{164}$$

and carry this over into Eq. (162).

(*j*) **Determining the Feed Train.**—The real difficulty in the use of the whole train of Fig. 715 is to select gears that will give a very close approximation to the value of t_2 computed after c and t_1 have been fixed. In our three problems we now have,

From Index Train	(*a*)	(*b*)	(*c*)
First change train $C_1 \ldots . t_1 = \frac{d_1}{f_1}\frac{d_2}{f_2}$	$\frac{40}{43}\frac{60}{66}$	$\frac{52}{44}\frac{120}{59}$	$\frac{32}{47}\frac{45}{49}$
Corresponding value of c	20	26	24
Desired value of t_2, by Eq. (162) . . .	0.29618	0.22324	0.26180

The machine which is represented by Fig. 715 (the Gould & Eberhardt gear hobber) has an extensive set of change gears, containing all the numbers from 24 to 97, with 120 (isolated) and enough duplicates to make 2-24, 2-30, 3-40, 2-45, 5-60, 2-80, 2-90 and 2-96, or 87 gears in all—these being used in both trains C_1 and C_2. The instruction book gives full directions for the index train C_1 but does not attempt to cover by tables the very wide field of choice for C_2 and t_2. Instead is provided a table giving decimal equivalents of all the common fractions that can be got with the numbers from 1 to 100: duplicates being omitted there are 3,120 values in this table. A factoring table is also provided and is a great help in computations of the kind already made for C_1 as well as in those that follow.

For $t_2 = 0.29618$ the nearest single fractional equivalent is $8/27 = 0.29630$, with a discrepancy of 12 in 29,618 or 1 in 2,470. A better guide to the compound of two ratios is got by splitting the decimal into the parts $0.29618 = 0.2 + 0.1 \times 0.9618$, and for 0.9618 finding $76/79 = 0.9620$. Then we may write,

$$t_2 = 0.2\,\frac{76}{79} = \frac{2}{10} + \frac{76}{790} = \frac{234}{790} = \frac{13 \times 18}{10 \times 79} = \frac{52}{80}\frac{36}{79}. \tag{a}$$

For case (*b*), $t_2 = 0.22323$, we get exactly,

$$t_2 = 0.22323 = 0.2\frac{23}{99} = \frac{221}{990} = \frac{13 \times 17}{30 \times 33} = \frac{26}{60}\frac{34}{66}. \quad (b1)$$

A very close approximation is found by the somewhat more obvious method of Art. (*d*), thus

$$t_2 = \frac{22{,}325}{100{,}000} = \frac{893}{4{,}000} = \frac{19}{50}\frac{47}{80} \text{ or } \frac{38}{100}\frac{47}{80}; \quad (b2)$$

but in 19 and in 100 these requirements run outside of the gear list.

When applying the compound-fraction method to the third example, $t_2 = 0.26180$, the closest approximation,

$$t_2 = 0.26180 = 0.2\frac{55}{89} = \frac{233}{890}, \quad (c1)$$

runs into high prime numbers and does not solve. The next best,

$$t_2 = 0.28182 = 0.2\frac{34}{55} = \frac{144}{550} = \frac{4 \times 36}{10 \times 55} = \frac{32}{80}\frac{35}{55}, \quad (c2)$$

solves readily. The direct-factoring method works out in this case also, as

$$t_2 = \frac{2{,}618}{10{,}000} = \frac{1{,}309}{5{,}000} = \frac{17 \times 77}{50 \times 100}. \quad (c3)$$

Prob. 288. For the hobbing machine of Fig. 715 select change wheels for the same gear that you used in Prob. 287. Get t_1 and c, and go as far as you can without the table of factors and decimal equivalents: then having those tables get the best approximation attainable.

Prob. 289. Compute the linear, circumferential error on the pitch cylinder of your gear due to an error of 1 in 1,000 on the lead, making width of face equal to 3 times the normal pitch.

Prob. 290. Eliminating c between Eqs. (161) and (162), derive the following overall relation for Fig. 715,

$$t_2 = \frac{25.1327nt_1 - 1{,}507\,96}{t_1 p_d \sin \alpha}. \quad (165)$$

(*k*) **Substitutes for the Helical Train.**—An obvious scheme is to move the cutter slide in the direction of the inclined tooth rather than squarely across the gear face, as outlined in Fig. 716: here we look toward the hob from the side of the gear blank, which is merely suggested by dotted lines. The hob is inclined on slide 2 as if for cutting a spur gear—see Fig. 708—then this slide is swivelled to angle α and fed in the direction of line CE on Figs. 712 and 713. A serious practical difficulty is incurred because it now becomes necessary to carry both hob drive and feed-screw drive through the same axis of swivelling (No. 2 of Fig. 707), a troublesome problem in transmission. Using a rack

cutter, in the planer generating method of Figs. 595 and 596, this mechanical difficulty disappears and it is the regular thing thus to give the cutter an inclined stroke.

With such inclined stroke and consequent component of motion tangential to wheel (left to right in Fig. 716) the hob tends to move out of its cutting zone—and has done so at lower position H′ in Fig. 716. For this type of operation, then. the hob must

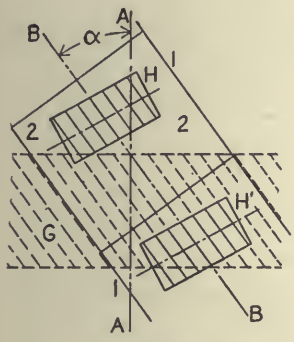

Fig. 716.

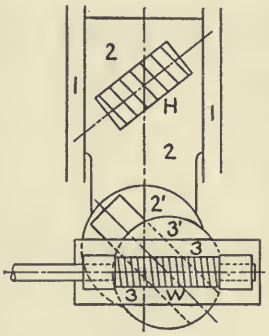

Fig. 717.

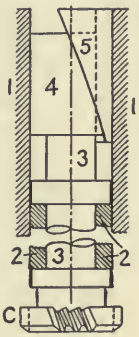

Fig. 718.

Use of inclined or helical guides.

have increased length, which means greater difficulty in producing an accurate tool. If, however, the tooth angle is not permitted to exceed 30 deg. the added length of hob is not likely to be a serious matter unless the gear is very wide of face. With the rack cutter, which can easily be ground true after hardening, the question becomes merely one of holding capacity of tool slide.

A possible equivalent to Fig. 716, avoiding the need of an extra long hob, is outlined in Fig. 717. Here the cutter slide (or another slide geared to move with it) carries an inclined guide that gives lengthwise displacement to the index worm within its end bearings. Obviously, it would be extremely difficult to construct and to operate such a device without an excessive amount of lost motion and inaccuracy.

In the system of generating by a wheel cutter, Fig. 594, slant-tooth spur gears of an established or adopted angle can easily be made by using a special cutter and giving it a helical motion as indicated by Fig. 718. Driving motion is imparted to sleeve 2, between collars on cutter ram or spindle 3; and the latter carries helical shoe 4 which works with fixed guide 5. Since spur-helical gears are of opposite hand, a pair of cutters and of guides must

be provided for each angle that is to be cut. The only element of geometrical inaccuracy is due to the very slight change of cutter diameter with resharpening and resultant small change in effective helix angle. If the two cutters of a pair are ground down together, this will not affect gear pairs produced simultaneously; but it will interfere somewhat with interchangeability between gears produced at different times in the lifetime of the respective cutters.

(*l*) **Worm-wheel Cutting with Tangential Feed.**—The matter now to be considered does not strictly conform to the title of this section, because the teeth of worm wheels are not strictly helical. In type of mechanism involved it is, however, very closely associated with what has preceded; and to the problem of tooth production it bears a similar relationship, being concerned wholly with the proper relative movement of tool and gear blank, not at all with the generated profiles of teeth.

The method outlined in Fig. 591 (*b*) has the disadvantage that the feed motion (exceedingly slow and against a heavy resistance) must be given in a direction (radially) in which rigidity is highly important: so that in this respect it is better to be able to clamp the axes of cutter and gear blank at correct distance, then advance the tool along its own axis or tangentially to the work. The two forms of tool that can be operated on this principle are shown in Figs. 719 and 720. View (*a*) of Fig. 719 outlines a worm that is made into a hob by cutting eight gashes: but the actual working element is a single bar tool or fly cutter F, fast in cutter bar B. What each individual tooth of a common hob does is simply to revolve in one fixed plane. Suppose that tool F holds the position of tooth 1 (on the hob) during n turns of bar B or one turn of the gear blank; evidently it will have the same cutting effect as that tooth. Now advance it, *along the hob thread*, to tooth position 2 and make another n turns, and so on until it reaches the far end of the equivalent hob. How the fly cutter starts, with its tip just clear of the gear blank, is seen in Fig. 721. Actually the advance per n turns will be much less than a hob-tooth space, to keep the full-depth cuts from becoming too heavy. The operation is slow, but it requires only a simple and inexpensive tool and is capable of great accuracy.

The parallel method for rapid production is to use the tapered hob shown in Fig. 720. As this rotates it is very slowly screwed forward along its own thread, doing the heaviest cutting with

the ends of its stub teeth: the last coil or more of full teeth has only the function of taking a light finishing cut, hence is subject to but little wear.

Two early mechanisms for using these tools are shown by Figs. 721 and 722. In the Gibson machine gear A is the last of the

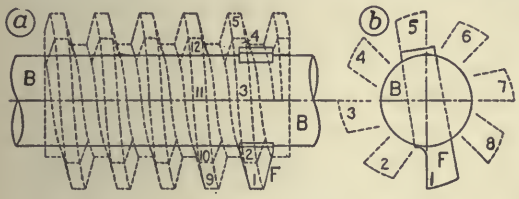

FIG. 719.—Outline of hob, with fly cutter.

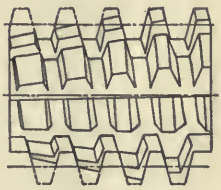

FIG. 720.—Tapered hob for tangential feed.

indexing change train, which is driven by the worm shaft (not drawn). This gear A, sleeve nut B and gear C form a rigid member. Through a rolling differential C-D-E, just like that in

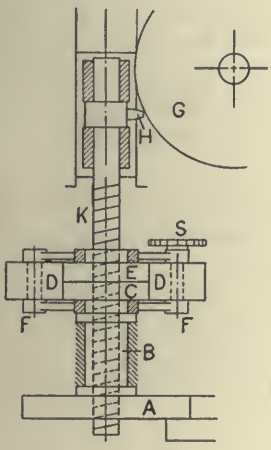

FIG. 721.—Simple fly-cutter train.

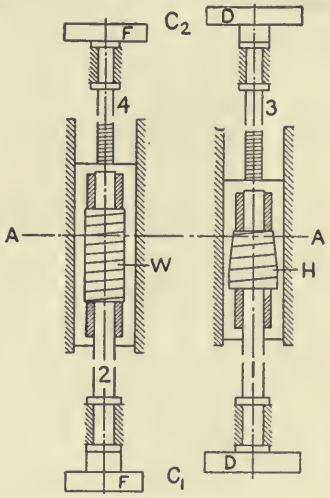

FIG. 722.—Train for tapered hob.

Fig. 327, screw bar K is driven by gear E which is splined to it. Here, however, train C-D-E in arm FF normally rotates without internal motion, K turning exactly with B: but at one place in each turn of the work shaft (or of gear blank G) a pin is raised so as to engage star wheel S, operating the differential and slightly advancing fly tool H. The helix in which H thus

advances will have the same pitch or lead as screw K, and this fact imposes a very narrow limitation upon the performance of the machine.

Figure 722 has both worm W and hob H in slides, which are moved axially by screws 3 and 4; then change train C_1 takes the simple indexing function, while train C_2 independently produces a proper relative movement of worm and hob, in amounts proportional to the diameters of master wheel and gear blank. It will be easier to get exact ratios if both hob and worm are in the same pitch system, with either circular or diametral pitch as determinant: but even then the question of normal as against axial pitch of the hob will intrude itself.

(*m*) **General Problem of the Fly-cutter Machine.**—If we separate the helical motion required by either Fig. 719 or Fig. 720 into its two components of angular and axial advance, the resulting kinematical problem is just like that of hobbing helical gears, and will be met as in Figs. 714 and 715. Whether by means of a differential or by modifying the indexing train, fly cutter or hob will be given a small extra rotation: and the feed screw, acting like screw 3 in Fig. 722, will give the tool slide a coordinate movement along the axis of the cutter shaft. Symbol c can be used as in Arts. (*f*) to (*h*), taking it to mean the number of parts into which one extra turn of the hob is divided—this number being 8 in Fig. 719 (*b*). The sketching of mechanisms to embody these principles will be left to the reader, in the problems that follow; he will also supply any needed diagram for the following discussion of the modified index train.

Let t be the value of the change-gear train from worm shaft (m teeth in master wheel) to hob shaft (n teeth in wheel to be cut). For exact indexing call the train ratio t_0, then $mt_0 = n$; and with an extra movement of $1/c$ turn for the hob, $mt = n + 1/c$. From these relations,

$$\frac{1}{c} = m(t - t_0). \qquad (166)$$

As a concrete example, let $m = 120$, $n = 75$, so that

$$t_0 = \frac{75}{120} = \frac{3}{4}\frac{5}{6} = \frac{48}{64}\frac{50}{60} = 0.62500. \qquad (a)$$

Now either of the fractions 48/64 or 50/60 will be slightly increased by adding 1 to both of its terms; thus we might get,

$$t = \frac{49}{65}\frac{50}{60} = 0.62806, \quad \frac{1}{c} = 120 \times 0.00306 = 0.367, \quad (b1)$$

$$t = \frac{48}{64}\quad\frac{51}{61} = 0.62704, \quad \frac{1}{c} = 120 \times 0.00204 = 0.245, \quad (b2)$$

but both of these values are too large for use. Noting that the effect of such a small addition (or subtraction) is greater as there is more difference in size between the terms of the original fraction, we may make opposing changes, thus

$$t = \frac{49}{65}\frac{49}{59} = 0.62608, \quad \frac{1}{c} = 120 \times 0.00108 = 0.130, \quad (c)$$

and get a suitable value for the taper hob. The method is supplementary to that of Art. (*h*), but does not give so flexible a choice as does an equation like (161).

Prob. 291. Following the general scheme of Fig. 714, sketch a suitable mechanism for a fly-cutter machine, arranging for a very slow working of the two elements (differential and feed-screw) that produce the helical movement of tool. Derive equations or relations that will direct choice of change wheels.

Prob. 292. Establish definite kinematic proportions for the gear train of the last problem, using $m = 120$, fixing ratios in transmission elements, and choosing change gears to cut a wheel of 75 teeth, with c equal to about 48.

Prob. 293. Adapt the mechanism of Fig. 715 to the driving of a taper hob, sketching the new arrangement in full. Then select change gears for the same job as in Prob. 292, but with c taken as 18. Derive any general formulas that will be serviceable

SECTION 42. BEVEL-GEAR TEETH

(*a*) **Tooth Profiles for Bevel Gears.**—The mere suggestion and outline in Figs. 723 and 724 is convincing of the fact that in the transition from spur gears to bevel gears there is a change from rolling cylinders to rolling cones for cycloidal teeth, from base cylinders to base cones for involute teeth. These cones all have common apex with the pitch cones at the intersection of axes; and since there must be a common length of element all points on the circumferences of their bases, and any curves traced by such points, will be on a sphere of radius OC. Describing line and plane DE now become the circumference and the plane of a "great circle" of the sphere, turning on center O; and it is by this turning that element length OC is transferred from pitch cones to base cones.

It is, of course, impossible to develop a spherical surface; but since any set of tooth profiles will be within a narrow zone of the

sphere, a very close approximation is secured by substituting for actual sphere the cone tangent to it along the pitch circle of a wheel. Thus in Fig. 725 a pair of bevel gears with pitch cones

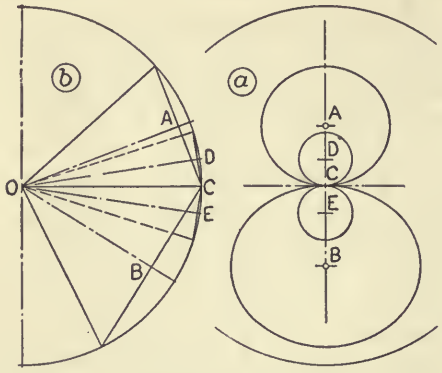

FIG. 723.

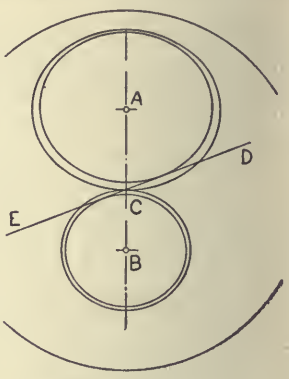

FIG. 724.

Generating elements for bevel-gear teeth.

OCA and OCB have the exact profiles of inner and outer ends of their teeth on spheres SCS and scs. Taking tangent cones PCA and QCB, pca and qcb, we can readily develop these from

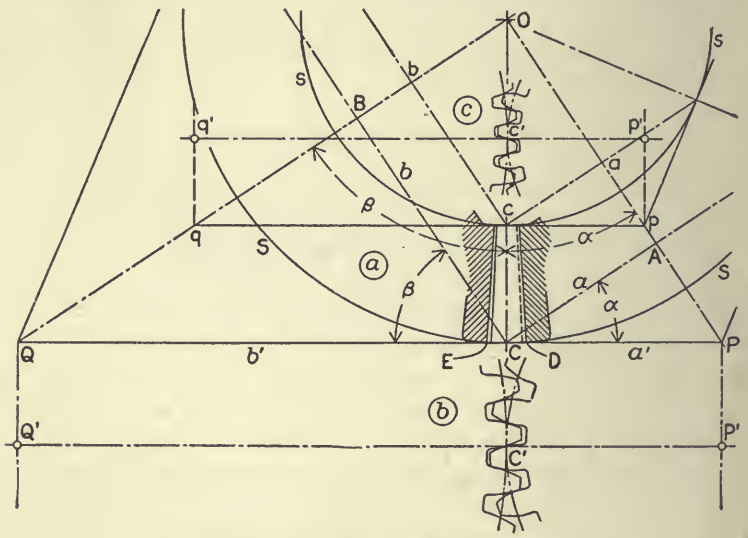

FIG. 725.—Tooth profiles for bevel gears.

centers P′ and Q′, p′ and q′, and lay out the teeth as if on spur gears of radii PC and QC, pc and qc.

For templates that will guide the shaping of wooden teeth on patterns for cast gears, or that will be used to test the correctness of cut teeth, profiles at both ends will be required, as drawn in Fig. 725. But in designing bevel gears for strength and in the various processes of cutting their teeth especial emphasis is laid upon form and dimensions at the outer ends, diagram (*b*) of Fig. 725. The spur gears to which these belong would have higher tooth numbers than the actual bevel gears, in the ratio of radii a' and b' to a and b. Letting α and β stand for the apex half-angles AOC and BOC of the pitch cones, which makes them the base angles ACP and ACQ of the tangent cones—this for the common case of perpendicular axes—and letting n and n' be the actual and the equivalent-spur-gear tooth numbers of any gear, we have

$$\begin{aligned} &\text{For wheel A,} \quad n' = n \sec \alpha, \\ &\text{For wheel B,} \quad n' = n \sec \beta. \end{aligned} \tag{167}$$

It is quite usual to designate bevel-gear teeth by diametral pitch, referring always to these outer-end profiles.

(*b*) **Milling Bevel-gear Teeth.**—The ordinary straight bevel-gear tooth (compare Figs. 692 and 693) has a conoidal surface, generated by a straight line which passes always through apex O and is guided by the profile at some location C. All cross sections of a tooth are geometrically similar, but they vary in size, hence in absolute shape of curves. Because of this variation it is not possible to produce such teeth exactly by means of any formed tool; but an approximation good enough for some easy conditions of service can be made, and we shall now consider briefly the use of milling cutters, paralleling the spur-gear process in Sect. 35 (*b*) and Sect. 36.

The problem is visualized in Fig. 726, which is an end view of a tooth space, looking right along the pitch-cone element at C. Since the cutter must move parallel to the bottom of this space, the cutting angle at which the gear blank must be held, relative to the direction of feed movement, is not AOC or BOC in Fig. 725, but AOD or BOE. Because the cutter must go through the narrow end of the tooth space it has to be thinner than a spur-gear cutter for the same pitch: standard practice is to allow for a tooth length Cc, Fig. 725, not to exceed one-third of the apex distance CO, while 0.25 to 0.30 is a common range for this ratio, in pairs containing low-number wheels.

Figure 727 shows the cutter adjusted to one side of the big-end profile and advancing along corner K. A similar cut must be taken on the other side of the space, but even then the teeth will be left too wide or too thick at the small end, as appears from

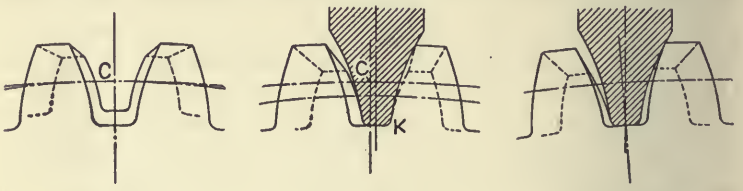

FIG. 726. FIG. 727. FIG. 728.

Use of milling cutters on bevel gears.

the correct dotted outline above K. On small gears this error may be corrected by filing the teeth: but by tipping the gear so as to accommodate the cutter curve as closely as possible to the small-end profile and at the same time to swing the big end clear of it, Fig. 728, this excess of metal can be milled off. Counting the first straight roughing-out operation, this calls for a total of

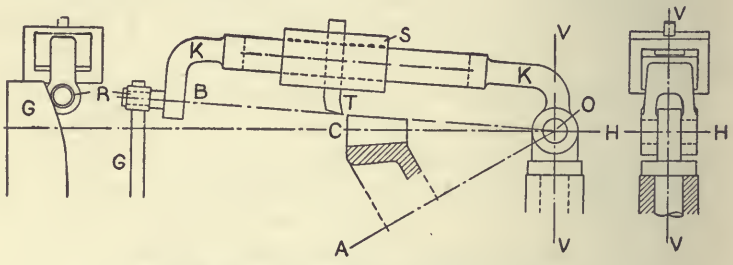

FIG. 729.—Mechanism for line-generating teeth.

five cuts in each space, or around the wheel—and even then it leaves some "swell" at the middle of the tooth. Further, it requires a skilful manipulation of the holding device on which the blank is mounted, to be worked out by trial or prescribed by careful instructions.

(*c*) **The Template or Copying Method.**—The conoidal form of tooth leads naturally to a line-generating or template-copying process, for which the mechanism is outlined in Fig. 729. Arm KK, able to swing on both of axes VV and HH, has a spheric motion without being free to turn on radius line OB. Slide S, suitably driven, carries the point of tool T back and forth along

line BO: the tooth must cut with a very limited portion of its edge, but can be rounded below the exact generating "point," to the radius of fillet at bottom of tooth space. Guide plate or template G is parallel to an enlarged profile of the tooth—enlarged and hence easy to lay out accurately because it is so far from center O. With right and left tools and a reversal of template G both sides of a tooth space can be shaped, after a preliminary rough slotting. The method is very effective for large gears of coarse pitch, especially when a number of them are to be

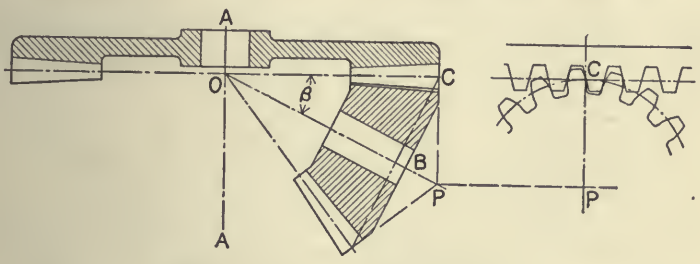

Fig. 730.—Crown gear and pinion.

made in one size. Provision must of course be made for slowly traversing roller R along guide G, also for indexing the gear between cuts.

(*d*) **The Crown Gear and Its Tooth.**—In preparing to adapt the rack-generating process of Sect. 35 (*e*) to bevel gears, we must think of the rack as having, not a straight pitch line, but a plane pitch surface. Then it is readily apparent that the plane form in spheric rolling is the special cone of 180-deg. apex angle, which when evolved into a wheel is called a crown gear. Such a wheel, in mesh with a pinion, is sectioned in Fig. 730. Necessarily the axis angle AOB is more than 90 deg., so that the crown gear cannot be used in the common type of right-angle drive: but while it very seldom appears as an actual wheel, the crown gear is of great practical importance as the foundation of the process of tooth generation for bevel gears. Its plane pitch surface puts it in class with the straight rack of the spur-gear system; and the natural thing is to choose the tooth form with straight profile and plane face, not only because this is the simplest form but even more because it is the only one that does not change along a conoidal tooth.

The form of contact locus associated with this plane crown-gear tooth is developed in Fig. 731. On front view (*a*) of the sphere

"equatorial" line BB represents the pitch cone of the crown gear having "polar" diameter AA as its axis. Line DD is an edge-on view of the great-circle plane of a straight-profile tooth, with PP for its polar axis. All spherical normals to profile DD are great-circle quadrants running from P. The one shown crosses pitch circle BB at C′, which projects to E on top view (*b*). When the wheel turns so as to bring C′ or E to C, point K will take position on the (fixed) locus of contact—compare Sect. 37 (*e*). By getting the equivalent arc FG on the small circle through H

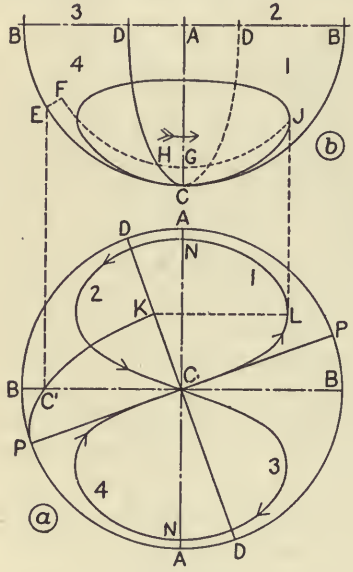

FIG. 731.—The octoid tooth.

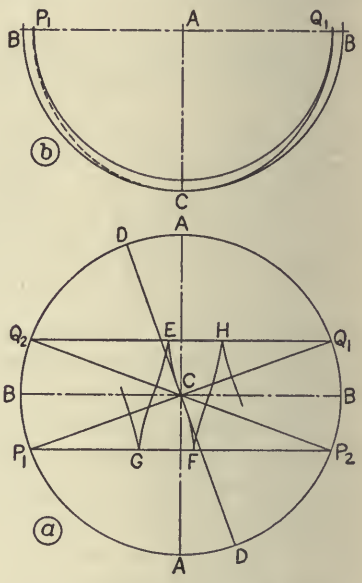

FIG. 732.—The spherical involute.

(K projected), then making arc HJ equal to FG, we have the point in view (*b*) and can bring it down to line (circle) KL in view (*a*). The full locus takes the figure-of-eight form shown, whence the name "octoid" as sometimes applied to this system of gear teeth. The numbers on the quadrants of this curve relate it to the quadrants in the rotation of circle DD about axis AA, as designated on the top view.

(*e*) **The Spherical Involute.**—The shape of the contact locus derived in Fig. 731 suggests serious doubt as to whether its tooth is an involute form. In clear analogy to the planar curve, the involute on a sphere will be generated by a point of a great circle whose plane rolls on a base cone. Such a curve cannot, however,

run out indefinitely, but must zigzag across a zone of the sphere, between conjugate base circles like P_1P_2 and Q_1Q_2 in Fig. 732. The crown-gear case is simplest, because its pitch circle BB lies midway between the base circles, hence the profiles are symmetrical with respect to their pitch points. Let describing plane or circle PQ roll so that its tangency point P_1 will move clockwise as viewed from above, this motion causing point C of P_1Q_2 to rise toward E. Since the periphery PQ is longer than PP, this rolling causes C to swing somewhat to the right of straight profile

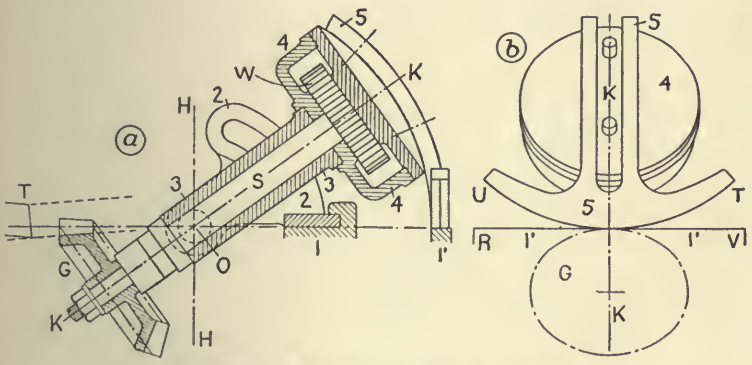

FIG. 733.—Mechanism for Bilgram process.

DD. If CE were the face of a tooth we should have solid tooth to the left, open space to the right; so that the face would be slightly concave, with compensation in the convexity of flank CF. Obviously the plane-face crown-gear tooth has departed from the involute system.

The contact locus for curve EF, Fig. 732, is circle P_1Q_1, fixed in the position drawn, while E and F are interference limits, realized when E turns to Q_1 or F to P_1. The difference between locus PQ and its branch of the octoid curve in Fig. 731—compare the rather similar difference in Fig. 604 between original and adjusted rack-face profiles—indicates the important fact that teeth generated by the plane crown-gear tooth will not be subject to interference of the kind that arises with involute teeth on spur gears.

(*f*) **Generating Bevel-gear Teeth.**—The geometrical features of the Bilgram bevel-gear planer are shown in very sketchy fashion by Fig. 733. On frame 1 is formed a turntable base upon which piece 2 rotates about vertical axis HH. Spindle bearing 3 can be clamped in 2 at a desired cone angle, with apex

at O. Head piece 4 turns on 3 and carries the indexing mechanism, which is only suggested by worm wheel W. During any cut spindle S is held rigid with 4, and except for indexing action it continually takes the motion of 4. Sector 5 is bolted to head 4 and rolls on a fixed flat track 1′, this rolling being constrained by two steel tapes fastened at R and T, U and V: a set of these sectors, each to be used for a moderate range of cone angle, forms a part of the machine's equipment. By means of a suitable drive gearing the whole arrangement on axis KK is given a conic rolling motion about axis HH, turning gear G beneath a fixed crown gear, of which one tooth is described by the cutting edge of tool T.

In the Gleason machine there is kinematic change or inversion, in that gear axis KK is held fixed and the crown wheel is turned back and forth. Instead of steel bands and arc, toothed sectors are used to turn the gear blank; and there are two cutting tools, related as shown by Fig. 735 and shaping both sides of a tooth at the same time. Completely automatic operation of either machine is secured by means of ingenious mechanism, best appreciated by seeing it at work.

To see just how the tool must move, consider Fig. 734. Drawing (*a*) is an end view of the crown-gear tooth, looking straight along the central pitch line at C; the tip excess for cutting clearance is marked off by a cross line. In (*b*) profile A is the cutting section of the tool, narrowed from B so that it will pass through the small end of the tooth space. Here we look straight along a pitch line tangent to cutting tooth and to gear tooth at C: the face of the gear tooth has just been shaped and further rolling in the direction of the arrow will generate its flank. The fact now to be stressed is that the plane in which active tooth edge EF travels must pass through the apex point back of C; but in this plane the tool does not move parallel to pitch line CC, view (*c*), but as determined by bottom line BB. This brings it to position D, view (*b*), at the inner, small end of the tooth.

(*g*) **Variation of Tool and Tooth.**—The single solid tool of Fig. 734 represents the tooth of a crown gear and works in a space of the bevel gear, one side at a time. The two tools of Fig. 735 embody a crown-gear space, of which both sides act at once upon the bevel-gear tooth. These tools are carried by separate slides, adjustable in the pitch plane of the crown gear as to the

angle appearing in top view (*b*). To diminish risk of interference at small end of tooth they make alternating strokes.

The crown-gear tooth is not restricted to a radial direction, but may be set at a slant across its annular pitch strip; this will generate the kind of wheels outlined in Fig. 693 (*b*), which will mesh just as perfectly as do the corresponding spur gears. Further, the tooth based on a plane pitch surface and having a straight-line profile in its normal section is not itself compelled

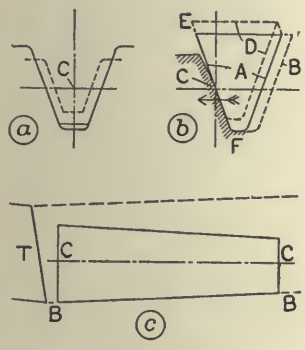

FIG. 734.—Action of single tool: Bilgram.

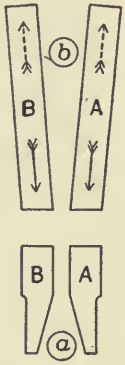

FIG. 735.—Use of two tools: Gleason.

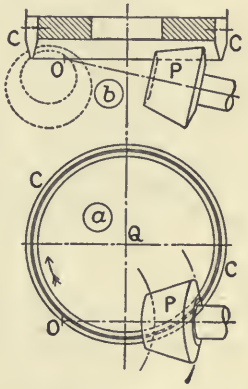

FIG. 736.—Generating a "spiral" or curved-tooth bevel gear.

to be straight: most simply it can be bent into a circular form, on the side of a rotary cutter C in Fig. 736. The apparently solid tooth ring in view (*a*) is of course broken into a number of cutting tools, inserted in body of disc: with multiple tool faces and continuous motion this cutter has much higher speed capacity than the planing tool—the practical difference being that it can cut from the solid while for the other type of tool there must be a preliminary slotting out of groove. It functions like the single tool of Fig. 734, in that two cuts must be made, for the two sides of the generated tooth space. The scheme can be used for either spur or bevel gears, but the application in Fig. 736 is the more common. Note that drawing (*b*) is really a combination top and side view.

The possibilities of the planer-generating process are illustrated by the combination outlined in Fig. 737, which was exhibited by Mr. Bilgram in 1893. Wheels A and B are bevel gears of normal form. Keeping the axes at right angles but having diameters

less and greater than B, gears D and E are so cut that they work perfectly with A. There is an apparent apex at Q in each figure, but the true apex of pitch cones must of course be at O′. The real effect is to make teeth A-D and A-E have variable addenda, complicated by the fact that the straight-line elements run toward Q, not O′: consequently the geometrical conditions

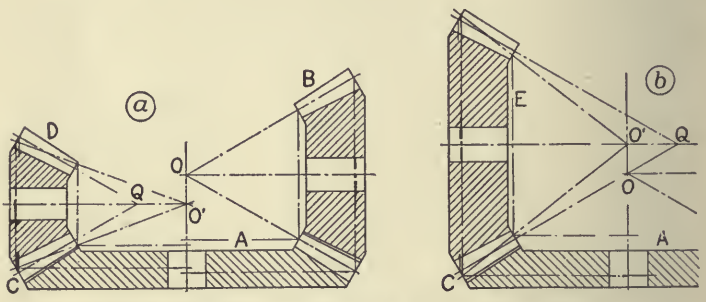

FIG. 737.—Bevel wheels with varying addendum.

for setting the tool slide are much less simple than in Fig. 734. The problem is too intricate and special to be worked out here, but is left to the student or reader who may have a highly specialized interest in the subject.

(*h*) **Teeth for Skew Bevel Wheels.**—Desiring to apply the process of rack-generation to this type of gear, our first step must be to see what form of surface corresponds with the plane in the straight rack of the spur system and the circular rack of the bevel system. To do this it will be better to proceed from the point reached in Sect. 40 (*c*) rather than from the approximation in Sect. 40 (*h*). We are concerned now with the general case of pitch surfaces that touch each other in straight lines, called their pitch lines. Referring back to the idea introduced by Figs. 684 and 685, we see that in spur gearing circles on the wheels and straight lines on the rack roll together, both continually normal to the parallel pitch lines. In bevel gearing the lines of rolling on the plane wheel change to circles, still normal to the now converging pitch lines. On skew-bevel hyperboloids the rolling circles are oblique to the pitch lines, whence the component of crosswise slip along those lines: but a surface can be rolled between them which retains the property of continual perpendicularity of rolling lines to pitch lines and which, at any place of contact, has no predominant curvature toward

either hyperboloid. This surface, logically to be called the *skew rack*, is the helicoid shown in Fig. 738. Note that for the particular pair of circles determined by the common perpendicular to axes—the case of Fig. 689—the helix of rolling is a straight line, DD in Fig. 738.

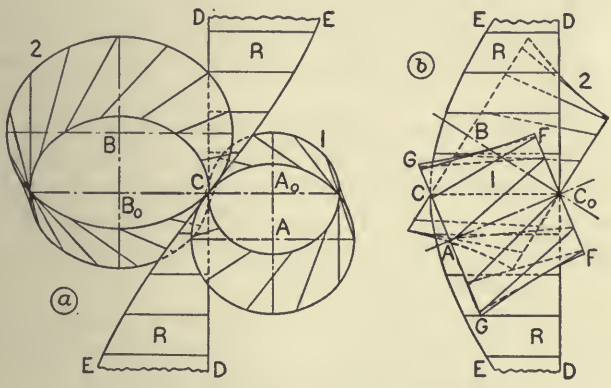

FIG. 738.—Pitch hyperboloids and skew rack.

Digressing for a moment from the main subject, it is of interest to discuss (in the mathematical sense) this concept of the helicoid as general form of rack surface. Considering dimensions of a helicoidal strip, zero lead with infinite radius gives the common straight rack and zero lead with finite radius the crown gear. Infinite lead with finite radius also produces the straight, plane rack, but rather as it is applied in the study of helical gears than as used with spur gears. Note that the approximation in Fig. 698 consists in replacing the helicoidal strip by a plane determined by the contact pitch line and by a tangent to the helix at middle of wheel strip, or through point of contact of cones.

A scheme for applying the rack-generating method to skew-bevel gearing would contain the following elements:

1. To hold the skew rack fixed and roll the wheel upon it will be better than the reverse arrangement, since it would be difficult to transmit tool drive to a moving skew rack.

2. Tool edge (carrying straight tooth profile) must advance with a twisting motion, turning on its pitch point as this follows a radius of the helicoid. Such motion can be attained by using

a round-bar slide, with a radial arm against a fixed helical guide; but a special guide will be necessary for each rate of twist. The tool will cut a constant root depth, since it will be too complicated to provide for moving as in Fig. 734.

3. The rolling of pitch hyperboloid on skew rack can be secured by putting two narrow sectors in place of (parts of) circles FF and GG in Fig. 738 and rolling them on toothed portions of the corresponding helices. Much ingenuity will be required to devise arrangements that will definitely and smoothly constrain this motion.

4. A different rigging—consisting of helix elements, rolling arcs and constraining parts—will be needed for each wheel to be cut. This lack of flexibility in the mechanism, or large requirement of specialized parts, is the really prohibitive difficulty.

The reason why no one has developed a practical machine for cutting skew-bevel teeth is obvious.

(*i*) **Helical Bevel Gears.**—The process of planing generation, as described in Arts. (*f*) and (*g*), may freely be varied by inclining the teeth. Simple suggestion of such a scheme has been made in Fig. 693 (*b*), while the process outlined in Fig. 736 goes a step farther and uses variable inclination. Teeth thus produced have just as good contact as straight teeth. Yet another possibility is based on the two-cone approximation of Figs. 698 and 700, for gears whose axes do not intersect: as indicated by Fig. 698, the two wheels would be cut by the same planing tool but at different inclinations. Like the pitch-cylinders of so-called spiral gears, for crossing axes, these pitch cones have only point contact. Discussion of this more general form would logically follow that of the cylindrical pair, but its probable practical value is so small that a mere note of the possibility of the scheme is sufficient.

Prob. 294. Within practical working range the difference between contact loci as shown in Figs. 731 and 732 is very small. Assuming that it is negligible, or that teeth generated by the plane-face crown-gear tooth have all the properties of involutes, consider for bevel gears the question of variation in depth of mesh, which has been presented for spur gears in Sect. 33(*g*). Take two cases,

(*a*) Move gears apart or together by slightly varying the axis angle.

(*b*) Move gears in or out on axes, keeping their teeth together lengthwise but varying the distance from intersection point.

(*c*) Further, what will be the effect of slightly twisting the pair, so that axes cross without quite intersecting?

SECTION 43. HELICAL TEETH AND GEARS

(*a*) **Use of Helical Teeth.**—Inclined teeth are used on spur or parallel-axis gears in order more readily to secure smooth and quiet running under severe conditions as to pressure and speed. If teeth could be made and maintained exact in spacing and geometrically correct in form, this requirement would be fully met by straight or axial teeth. Properly made helical teeth have line contact, just as do straight ones, but an instantaneous line of contact is no longer parallel to the axes or at a constant height across the tooth faces. Instead it runs at a slant, so that in successive sections all conjugate points of a pair of profiles are in contact at the same time. Because of this simultaneous use of the whole tooth curve there is less disturbance of action due to small initial defects of form and less tendency toward unequal wear such as might cause irregular transmission.

For cylindrical gears with helical teeth, connecting axes that cross without intersecting, a truly descriptive title would be "crossed-helical," in contrast with "spur-helical." The incorrect name "spiral gears" is, however, so firmly fixed in shop usage that it may be accepted as a convenient short designation. In such gear pairs the teeth have only point or spot contact, hence they may be used only in comparatively light service and are of secondary importance. Line contact can be secured either by going to the correct but difficult skew bevel form or by the method of the worm and worm wheel. The latter will be explained and discussed in Part II.

Because of the added obliquity due to slant of teeth the effective tangential component T is a relatively smaller part of total pressure P than with straight teeth—referring now particularly to the spur-helical pair and anticipating the showing of Art. (*d*). This causes a somewhat larger frictional loss in tooth action and slightly increased pressures at the bearings. Axial component or end thrust can readily be neutralized by the common expedient of using double or herringbone gears, as in Fig. 61 (*b*).

(*b*) **Contact of Helical Teeth.**—The helical tooth has simplest form and action when generated by a plane-tooth rack that rolls with its pitch cylinder. Consider the contact of a parallel-axis pair as shown in Figs. 739 and 740. The series of profiles in Fig. 739 is to be regarded, not as representing successive

positions of a pair of teeth (like Figs. 555 and 556), but as a series of sections, by the parallel planes indicated in Fig. 740, of a pair standing in one position. The rack-tooth profile, central on pitch line P_3, is drawn in as a geometrical intermediate—compare Fig. 625. To save drawing detail the tooth profiles are shown only from base circle D to addendum circle F.

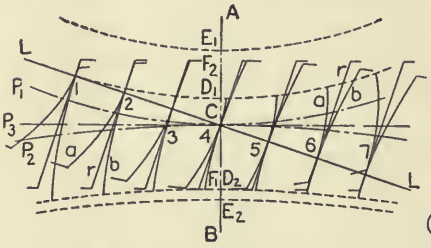

FIG. 739.—Successive sections of teeth.

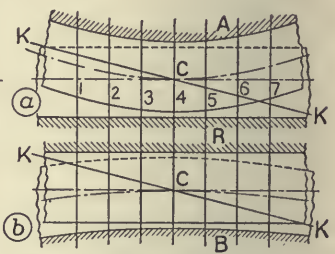

FIG. 740.—Cross sections and line of contact.

Relatively half-size cross sections of the gear rims, viewed in a direction perpendicular to the axis but with the cutting planes following the slant of a tooth space, are given in Fig. 740, both looking from left toward right in Fig. 739. Section (*a*) shows wheel A on the rack, section (*b*) the rack on wheel B. The drawing is made for a 12- and 30-tooth pair and the curvature due to obliquity of section is much greater for small wheel A. Remembering now that contact-locus line LL in Fig. 739 is really the edge-on view of a plane tangent to the pitch cylinders, it is at once evident that the intersection of this plane locus of tooth contact with the plane surface of the rack tooth must be a straight line, projecting as the same KK in both parts of Fig. 740. Since each tooth touches the rack-tooth surface in this line, they there touch each other, or line KK persists even if the rack is left out.

The existence of this straight contact line, less simple than the corresponding element of cylinder in straight-tooth spur gearing and of cone in common bevel gearing but yet easy enough to locate, is a highly important and useful fact. Its quantitative geometry, most effective when brought to the shape of simple formulas for its angle of inclination in the several useful projections, will be left to the closer study of helical teeth in Part II. Here we go no farther than to show a rather extreme example, in Fig. 741. This drawing is made from a 21- and 208-tooth pair for a steam-turbine "reduction," with 45-deg. slant of teeth.

As seen in top view (*b*) and cross section (*c*)—line AA being parallel to the axis—the contact lines marked K have a big overlap, so that their combined length is much in excess of the width of the gear rim.

In the case of crossed helical gears the intermediate rack-tooth face will touch each wheel tooth in the line of its intersection

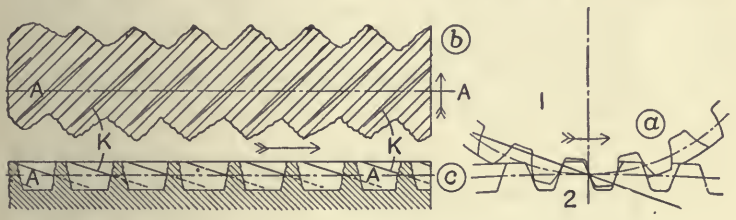

FIG. 741.—A multiple-contact helical pair.

with the contact-locus plane (tangent to base cylinder) of that wheel. Now, however, the two locus planes are separate, each being parallel to its own wheel axis. For this reason the two contact lines on the rack-tooth face will have in common only the one point where they intersect, and only there will the gear

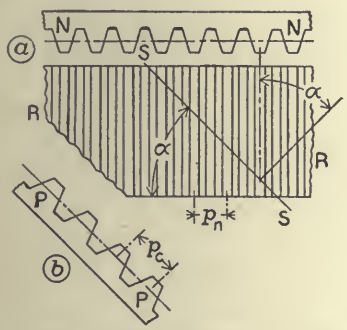

FIG. 742.—Inclined section of rack.

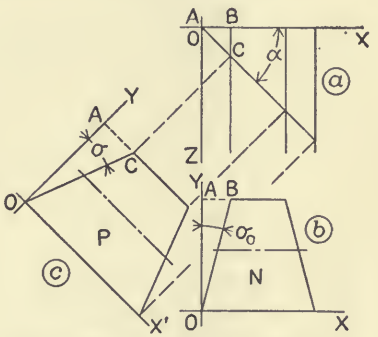

FIG. 743.—Relation of obliquities.

teeth touch. In space the locus of this tooth-contact point is the line of intersection of the two locus planes for contact of rack with gear.

(*c*) **The Slant-tooth Rack.**—Since in its determining dimensions wheel tooth agrees with rack tooth these dimensions can most readily be made determinate by fixing the proportions of the slant-tooth rack. For this purpose, in Fig. 742, compare oblique section PP (by plane SS at inclination α) with normal section

NN. The outstanding geometrical results from inclination of section plane are increase of pitch and of profile obliquity σ. For the pitch we have already, from Sect. 41 (*c*),

$$p_c = \frac{p_n}{\cos \alpha} = p_n \sec \alpha. \tag{168}$$

As to obliquity, consider the enlarged detail of one tooth in Fig. 743, where σ_0 is the primary obliquity in normal section N and σ is the effective obliquity in slanted section P. Taking altitude OA as base or unit, AB is tan σ_0 and AC is tan σ; then from top view (*a*) comes the relation

$$\tan \sigma = \tan \sigma_0 \sec \alpha. \tag{169}$$

Table 11 shows how σ varies with α for the primary obliquity $\sigma_0 = 14° 28.7'$, or the angle whose sine is exactly 0.25. For high values of the slant angle obliquity σ becomes very large; but when α goes above 60 deg. we enter the region of worm proportions, in which axial dimensions (in a plane perpendicular to SS of Fig. 742) become more important than circular. To get angle σ' in this plane, simply substitute complementary angle β for α as argument in Table 11.

TABLE 11

α	σ	α	σ	α	σ
5°	14° 32′	35°	17° 30′	65°	31° 25′
10°	14° 42′	40°	18° 38′	70°	37° 3′
15°	14° 58′	45°	20° 4′	75°	44° 56′
20°	15° 22′	50°	21° 53′	80°	56° 5′
25°	15° 54′	55°	24° 14′	85°	72° 21′
30°	16° 36′	60°	27° 19′	90°	90°

$\sigma_0 = 14° 28.7'$ $\quad$ $\sin \sigma_0 = 0.25$ $\quad$ $\log \tan \sigma_0 = 9.41196$

(*d*) **Tooth-pressure Effects.**—Slant angle α has a big influence upon tooth pressure, increased obliquity σ a much smaller influence. This matter is illustrated by Figs. 744 and 745, the first for the spur-helical gear but covering also the spiral pair with shafts at right angles, the second for the general case of any angle between shafts. View (*a*) of Fig. 744 has its force diagram in the pitch plane of the rack, or in a plane tangent to the pitch cylinder of a gear, and shows that with a required driving force T is asso-

ciated a considerably larger total tangential pressure T_t, in the relation

$$T_t = T \sec \alpha. \tag{170}$$

The other component A is the force along the axis, but is also the effective force on the mating gear in a square spiral pair. Views (b) and (c) show that the radial component N is given by either

$$N = T_t \tan \sigma_0 \quad \text{or} \quad N = T \tan \sigma, \tag{171}$$

hence is greater than the value $T \tan \sigma_0$ that it would have with straight teeth. For a given driving force T the total tooth pressure is now,

$$P = T \sec \alpha \sec \sigma_0. \tag{172}$$

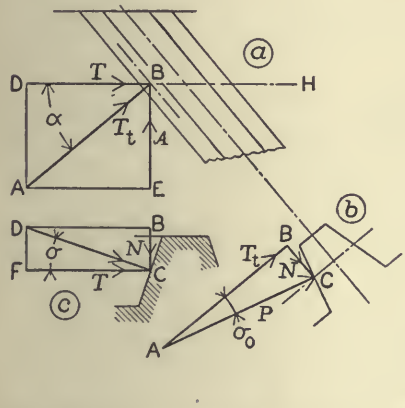

FIG. 744.—Tooth-pressure relations.

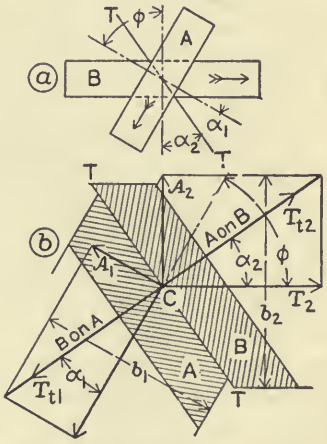

FIG. 745.—Pair with oblique axes.

In Fig. 745 view (a) shows the pair of wheels, with TT as the line of tooth inclination in the plane tangent to pitch cylinders, force T_t being normal to this line: here the acute axis angle φ equals $\alpha_1 + \alpha_2$. View (b) represents pitch-cylinder sections of two teeth, conceived as developed into the tangent plane, with a force parallelogram for each tooth, like that in Fig. 744 (a). These diagrams have the total tangential pressures T_{t1} and T_{t2} equal (and opposite), but the T and A components are not the same for the two wheels.

The rack alone, whether it works with a wheel or is taken as a geometrical intermediate after the manner of Figs. 739 and 740, completely determines direction of tooth pressure. For the pres-

ent we disregard friction and consider pressure to be normal to surfaces at contact.

Prob. 295. For a helical gear with normal obliquity of 14.5 deg. and tooth angle $\alpha = 45$ deg., find total pressure P and radial force N for each 100 lb. of effective driving force T. If the addendum is the module of normal pitch p_n what is its value in terms of p_c on these gears?

Prob. 296. In a pair of rolling-mill pinions like Fig. 61 (*b*) the effective pitch-circle obliquity is 22.5 deg. and the tooth angle is 30 deg. Compute the normal or primary obliquity and get P and N for each 1,000 lb. of T.

Prob. 297. For a square spiral pair (axes at 90 deg.) choose a line of tooth contact TT, then draw a velocity diagram like Fig. 694 and a force diagram like Fig. 744; prove that the products (work rates) T_1v_1 and T_2v_2 are equal.

Prob. 298. Write expressions for T_1 and T_2, Fig. 745, in terms of P and the α's, then resolve a common normal velocity v_n after the manner of Fig. 694 and prove that the product Tv is the same for both wheels. If it were desirable to equalize the tooth lengths TT in Fig. 745 (*b*) for the two wheels, what relation would exist between the rim breadths b_1 and b_2?

(*e*) **Section Normal to Active Teeth.**—Advancing now from rack to wheel teeth, important and useful results can be got by sectioning the helical pair in a plane determined by the common perpendicular O_1O_2 to the axes and by the tooth normal NN at point C of pitch-cylinder contact. In Figs. 746 and 747 both types of mating are considered: top view (*a*) carries the data, section (*b*) shows results. The pitch cylinders, extended as necessary, are sectioned in ellipses whose lengthened major axes are easily determined by the projections indicated. With each pair is drawn the normal profile of the rack that will generate or work with the teeth of either gear.

To make a complete and true oblique section of a lengthened wheel and all its teeth would be a very troublesome bit of drawing. But fortunately all that we need for an analysis of tooth action is a range of one or two pitches to either side of contact point C; and over this short range we may substitute for the ellipse its circle of curvature (at end of short axis) and conceive the tooth profiles as if on a spur gear of that size. If a and b stand for the major and minor half-axes OG and OH, the radius of curvature at end of b is $R = a^2/b$. Here OH or b is the actual pitch radius r of the wheel, while OG or a is $r/\cos\alpha$: consequently the value of R, used to strike arcs from Q_1 and Q_2, is

$$R = \frac{r}{\cos^2\alpha}. \tag{173}$$

Graphically this is given by length CF in top view (*a*), cut from midplane line CD by projecting line EG, the double projection from CD to CE, then from CE to CF, bringing in cos α twice as a divisor.

The same pitch-arc curvature in the normal section, with a better idea of its continuity, can be obtained by considering the

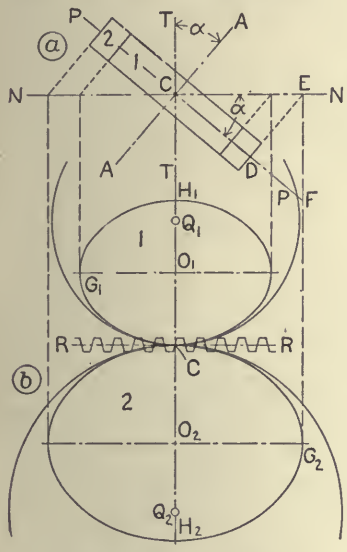

FIG. 746.—Spur pair.

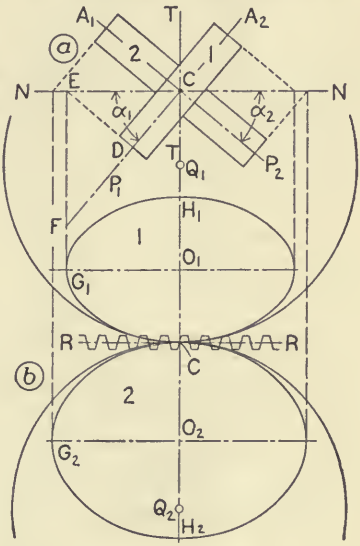

FIG. 747.—"Spiral" pair.

Oblique sections of wheels, normal to teeth at place of contact.

sectioning surface that is normal to all the elements of a helical tooth. Such a surface is now a helicoidal strip, in which the tooth profiles will lie just as bevel-gear profiles really lie in a spherical strip in Fig. 725. Determination of the properties of the normal helix on the pitch cylinder is not very difficult, and leads to the result expressed by Eq. (173); but it has no added practical utility and such a duplication of results would be wholly a matter of geometrical interest.

(*f*) **Relations for Spiral Gears.**—Applying Eq. (168) to a pair of gears like Fig. 747 we first get the circular pitches,

$$p_{c_1} = \frac{p_n}{\cos \alpha_1}, \qquad p_{c2} = \frac{p_n}{\cos \alpha_2}.$$

whence, in what is either an application or an independent development of Eq. (151), the tooth numbers are,

$$n_1 = \frac{2\pi r_1}{p_{c1}} = \frac{2\pi r_1}{p_n} \cos \alpha_1, \; n_2 = \frac{2\pi r_2}{p_n} \cos \alpha_2.$$

Letting N_1 and N_2 stand for the respective turning speeds, the turning ratio is,

$$\frac{N_2}{N_1} = \frac{n_1}{n_2} = \frac{r_1 \cos \alpha_1}{r_2 \cos \alpha_2} \tag{174a}$$

$$= \frac{r_1}{r_2} \cot \alpha_1 \quad \text{or} \quad \frac{r_1}{r_2} \tan \alpha_2, \tag{174b}$$

expression (*a*) being general, as for Fig. 745, while the (*b*) forms are limited to the case of axes at right angles.

Writing α_1 and α_2 on Figs. 694 to 696 and working through linear velocities, we should have,

$$v_1 = \frac{v_n}{\cos \alpha_1}, \; N_1 = \frac{v_1}{2\pi r_1}, \; v_2 = \frac{v_n}{\cos \alpha_2}, \; N_2 = \frac{v_2}{2\pi r_2};$$

whence, as before,

$$\frac{N_2}{N_1} = \frac{r_1 \cos \alpha_1}{r_2 \cos \alpha_2} \tag{174'}$$

For the approximate process of milling helical gears by means of the same formed cutters that are made for spur gears, the tooth number corresponding to artificial radius R is to be used. Letting n_s be the number on this related spur gear, with n on the actual wheel of radius r, we have

$$n_s = \frac{2\pi R}{p_n}, \; n = \frac{2\pi r}{p_c};$$

then from Eqs. (168) and (173),

$$\frac{n_s}{n} = \frac{R}{r} \frac{p_c}{p_n} = \frac{1}{\cos^3 \alpha} \tag{175}$$

The method thus indicated is of quite sufficient accuracy for most spiral pairs. It is for spur-helical and worm pairs, used in heavier transmission, that more precise methods of production and analysis are needed.

(*g*) **Variation of Axis Distance.**—A question now to be considered is: Do gears with helical teeth of involute form, generated by the plane-face rack tooth, retain the flexibility as to axis distance or depth of mesh that is shown in Sect. 33 (*g*) and so fully applied in Sect. 38? A cursory view of conditions as depicted in Fig. 739 indicates that for spur-helical gears the answer must be, Yes. In each pair of profiles there shown full flexibility exists, and the only question is whether, with change of pitch

cylinders, correct relative spacing is maintained. To this a simple and positive answer is found by viewing the developments of pitch-cylinder helices in Fig. 748: the gears are rolled upon a common tangent plane, 1 above, 2 below, from a common axial tangent A_1A_2. Leads l_1 and l_2, related by the ratio of diameters or of circumferences CD and CF, are fixed and unchangeable for the respective gears: compare Eq. (150*b*). If the axes are

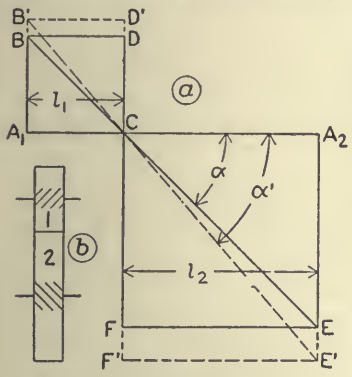

FIG. 748.—Spur-helical pair.

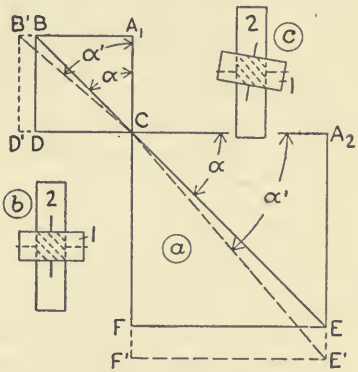

FIG. 749.—Crossed-helical pair.

Effect of variation of axis distance.

separated, increasing circumferences by similar amounts as in Fig. 579, the new helices CB′ and CE′ will agree in direction, and on their cylinders will roll together.

For the spiral or crossed-helical pair there is evidently a like requirement that pitch-cylinder helices shall have and maintain common direction at tangency: but Fig. 749 shows that now coincident changes of diameter will produce contrary effects. True action can be preserved only by changing the axis angle, as suggested in view (*c*). Of course proportions are here much exaggerated, but it is clear that even small changes of distance will seriously affect conditions of contact.

Example 16. Given tooth numbers $n_1 = 12$ and $n_2 = 30$, with slant angle $\alpha = 40$ deg., compute circular pitch, pitch diameters and normal curvature diameters for this pair of spur-helical gears, getting them in modules.

$\cos \alpha = 0.76604$ $\qquad p_n = 3.142$ mod.

$\sec \alpha = 1.3054$ $\qquad p_c = p_n \sec \alpha = 4.101$ mod.

For spur gears of the same tooth numbers the diameters would be $d_{10} = 12$ and $d_{20} = 30$ mod.; then with $\sec \alpha = 1.3054$ as factor, Eq. (153*a*),

$$d_1 = 15.665,\ d_2 = 39.163 \text{ mod.}$$

For both D and n_3 the factor is $\sec^3 \alpha = 2.2246$, to be applied to d_0 and n, whence

$$D_1 = n_{s1} = 26.695,\ D_2 = n_{s2} = 66.738.$$

Example 17. With $n_1 = 12$, $\alpha_1 = 50$ deg., $n_2 = 30$, $\alpha_2 = 40$ deg., make similar computations for a spiral pair with axes at 90 deg.

$$\alpha_1 = 50°,\ \cos\alpha_1 = 0.64279,\ \sec\alpha_1 = 1.5557,\ \sec^3\alpha_1 = 3.7653;$$
$$\alpha_2 = 40°,\ \cos\alpha_2 = 0.76604,\ \sec\alpha_2 = 1.3054,\ \sec^3\alpha_2 = 2.2246;$$
$$p_{c1} = 4.8875,\ d_1 = 18.668,\ D_1 = 45.184 = n_{s1};$$
$$p_{c2} = 4.1101,\ d_2 = 39.163,\ D_2 = 66.738 = n_{s2}.$$

Check against numerical error by getting rotation ratio m from the computed diameters, using Eq. (174), thus

$$m = \frac{d_1}{d_2}\tan 40° = \frac{18.668}{39.163} \times 0.83910 = 0.39998;$$

from the tooth numbers it must be 0.4.

Prob. 299. A spur-helical pair has 15 and 32 teeth of 45-deg. angle: compute same quantities as in Example 16.

Prob. 300. A spiral pair has $n_1 = 15$, $n_2 = 32$, $\alpha_1 = 60°$, $\alpha_2 = 30°$: compute pitches, d's and D's, then check as in Example 17.

Prob. 301. A spur-helical pair must fit an axis distance of 37 mod. and is to have the tooth numbers 20 and 36: find the necessary tooth angle α, then get the exact pitch diameters and the lead of each pitch-cylinder helix.

Prob. 302. A spur-helical pair is to span an axis distance of 42 mod. and to have $n_2 = 2n_1$: determine suitable tooth numbers, then find angle α and compute pitch and normal-curvature diameters.

Prob. 303. For a pair spiral the (perpendicular) distance between axes is 40 mod. and we wish to get a turning ratio of about 2.4 to 1 with diameters about as 1 to 2: determine consistent tooth slant and numbers, then take the latter integral, adjust angle, and compute obliquity and normal-curvature radius for each wheel; its exact pitch diameter will be found in the main determination.

Prob. 304. A spiral pair is to connect shafts 10 in. apart and at a (projected) angle of 75 deg., which are to have a turning ratio of 2 to 1. Using teeth of 6 normal d.p., find suitable numbers, angles and diameters.

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